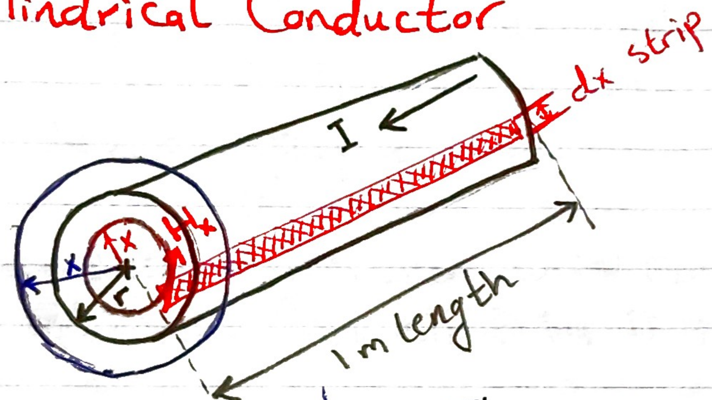
Inductance

» For Calculating Inductance we need to go to four steps:-

1. Magnetic Field Intensity H, from Ampere's Law
2. Magnetic Flux Density B, ($B = \mu H$)
3. Flux Linkages, (λ)
4. Inductance From Flux Linkages per ampere. ($L = \lambda / I$)

Solid Cylindrical Conductor



A Internal Flux Linkage   **B** External Flux Linkage

» The magnetic field intensity H_x, around a circle of radius x, is constant and tangent to the circle. The Ampere's Law relating H_x to the current I_x is given by:

$$\oint H_{tan}\, dl = I_{enclosed}$$

(محيط الدائرة)

$$\int_0^{2\pi x} H_x \cdot dl = I_x$$

I_x is the Current enclosed at radius x.

$$\boxed{H_x = \frac{I_x}{2\pi x}} \quad \cdots \quad (1)$$

A Internal Inductance

» A simple expression can be obtained for the internal flux linkage by neglecting the skin effect and assuming uniform current density throughout the conductor cross section, i.e.

مقطع

$$\frac{I}{\pi r^2} = \frac{I_x}{\pi x^2} \Rightarrow I_x = \left(\frac{x}{r}\right)^2 I$$

• uniform current density

from (1) $$H_x = \frac{I_x}{2\pi x}$$

$$H_x = \frac{I}{2\pi r^2}\, x$$

» For a nonmagnetic conductor with constant permeability μ_o, the magnetic flux density is given by:

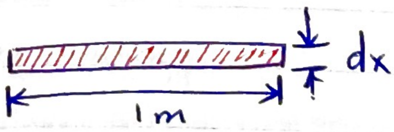
$$B_x = \mu_o H_x$$

$$B_x = \mu_o \left[\frac{I}{2\pi r^2}\, x\right]$$

$\mu_o \equiv$ permeability free space $\equiv 4\pi * 10^{-7}$ H/m

» The differential flux $d\phi$ for a small region of thickness dx and one meter length of the conductor is

$$d\phi_x = B_x \underbrace{dx \cdot 1}_{\text{area of strip}}$$



⊙ The flux $d\phi_x$ links only the fraction of the conductor from the center to radius x.
Thus, on the assumption of uniform current density, only the fraction $\frac{\pi x^2}{\pi r^2}$ of the total current is linked by the flux, i.e.,

$$d\lambda_x = \left(\frac{x^2}{r^2}\right) d\phi_x$$

$$d\lambda_x = \left(\frac{x^2}{r^2}\right) d\Phi_x$$

$$= \left(\frac{x^2}{r^2}\right)\left[B_x\, dx\right]$$

$$= \frac{x^2}{r^2}\left[\frac{\mu_0 I x}{2\pi r^2}\right] dx$$

$$d\lambda_x = \frac{\mu_0 I x^3}{2\pi r^4}\, dx$$

- $B_x = \mu_0 \left[\frac{I x}{2\pi r^2}\right]$
- $d\phi_x = B_x\, dx$

» The total flux linkage

$$\lambda_{int} = \int_0^r d\lambda = \frac{\mu_0 I}{2\pi r^4}\int_0^r x^3\, dx$$

$$= \frac{\mu_0 I}{8\pi} \text{ Wb/m}$$

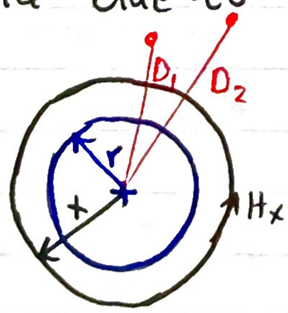
By def^n, for nonmagnetic material, the inductance L is the ratio of its total magnetic flux linkage to the current I, given by $L = \lambda / I$.

The Inductance due to the internal flux linkage is

$$L_{int} = \frac{\mu_0}{8\pi} = \frac{1}{2} * 10^{-7} \text{ H/m}$$

Note that L_{int} is independent of the conductor radius r.

$\boxed{B}$ Inductance due to external flux linkage



$$\oint H_{tan}\, dl = I_{enclosed}$$

$$\int_0^{2\pi x} H_x\, dl = I$$

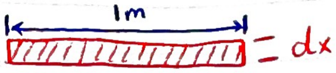
» $H_x (2\pi x) = I$

$$H_x = \frac{I}{2\pi x} \text{ A/m} \qquad x > r$$

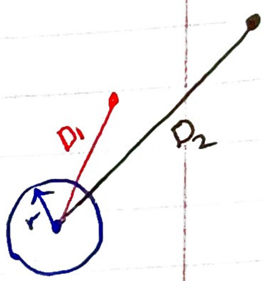
$$\gg B_x = \mu_0 H_x = 4\pi * 10^{-7} \left[\frac{I}{2\pi x}\right]$$

$$= 2 * 10^{-7} \frac{I}{x}$$

$$d\phi = B_x \cdot dx \cdot 1 = 2 * 10^{-7} \frac{I}{x} dx$$



» Total Flux Linkages between any two points



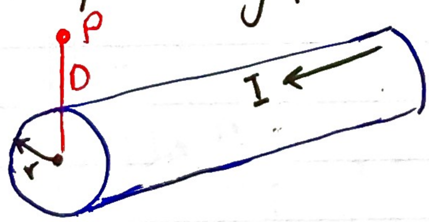
$$\lambda_{12} = \int_{D_1}^{D_2} d\lambda = 2 * 10^{-7} I \int_{D_1}^{D_2} \frac{1}{x} dx .$$

$$\lambda_{12} = \lambda_{ext} = 2 * 10^{-7} I \ln \frac{D_2}{D_1}$$

» The inductance between two points external to a Conductor is

$$\boxed{L_{ext} = 2 * 10^{-7} \ln \frac{D_2}{D_1} \quad H/m}$$

Total Flux Linkage up to any point P for this conductor Carrying current I.



$$\lambda_p = \underbrace{\frac{1}{2} * 10^{-7} I}_{\text{internal F.L.}} + \underbrace{2 * 10^{-7} I \ln \frac{D}{r}}_{\text{external F.L.}}$$

note:-
$\ln(a * b) = \ln(a) + \ln(b)$

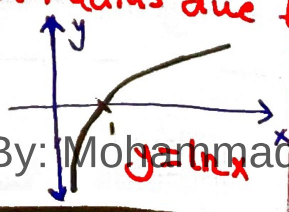
using $\frac{1}{2} = 2 \ln e^{\frac{1}{4}}$

$$\lambda_p = 2 * 10^{-7} I \left(\ln e^{\frac{1}{4}} + \ln \frac{D}{r}\right) = 2 * 10^{-7} I \ln \frac{D}{e^{-\frac{1}{4}} r}$$

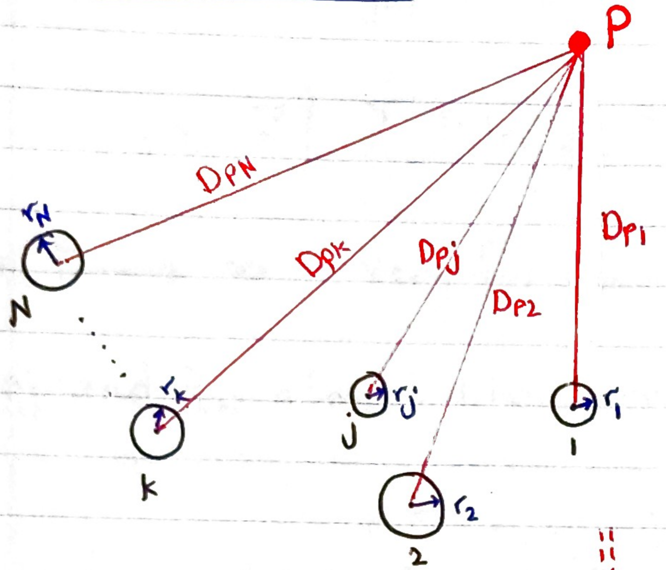
$$= 2 * 10^{-7} I \ln \frac{D}{r'}$$

where $r' = e^{-\frac{1}{4}} r = 0.7788 r \triangleq$ effective radius due to internal flux

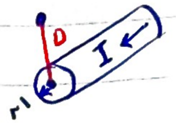
$$\boxed{L_P = \frac{\lambda_P}{I} = 2 * 10^{-7} \ln\left(\frac{D}{r'}\right) \quad H/m}$$



Composite Conductor :-



note :- $\lambda_p = 2*10^{-7} I \ln \frac{D}{r'}$



$$I_1 + I_2 + I_3 + \cdots + I_N = 0$$

$$\sum_{j=1}^{N} I_j = 0$$

$$\lambda_{kpk} = 2*10^{-7} I_k \ln \frac{D_{pk}}{r'_k}$$

$\lambda_{kp1} = 2*10^{-7} I_1 \ln \frac{D_{p1}}{D_{k1}}$

$\lambda_{kpk} = 2*10^{-7} I_k \ln \frac{D_{pk}}{D_{kk}}$; where $D_{kk} = r'_k$

↳ Flux Linkages for the conductor k up to a point p due to the current flowing in conductor k.

$\lambda_{kp} \rightarrow$ Flux Linkages for the conductor k up to a point p due to the currents flowing in conductors 1, 2, ... N.

$$\lambda_{kp} = \lambda_{kp1} + \lambda_{kp2} + \cdots + \lambda_{kpN}$$

$$= 2*10^{-7} \sum_{j=1}^{N} I_j \ln \frac{D_{pj}}{D_{kj}} \quad , \text{ where } D_{kk} = r'_k$$

$$= 2*10^{-7} \sum_{j=1}^{N} I_j \ln \frac{1}{D_{kj}} + 2*10^{-7} \underbrace{\sum_{j=1}^{N} I_j \ln D_{pj}}$$

$j=1 \rightarrow j=N-1$ $\quad j=N$

$$= 2*10^{-7} \left[\sum_{j=1}^{N} I_j \ln \frac{1}{D_{kj}} + \underbrace{\sum_{j=1}^{N-1} I_j \ln D_{pj}}_{j=1 \rightarrow j=N-1} + \underbrace{I_N \ln D_{pN}}_{j=N} \right]$$

where

$$I_N = -(I_1 + I_2 + \cdots + I_{N-1}) = -\sum_{j=1}^{N-1} I_j$$

$$\lambda_{kp} = 2*10^{-7}\left[\sum_{j=1}^{N} I_j \ln \frac{1}{D_{kj}} + \sum_{j=1}^{N-1} I_j \ln D_{pj} - \overbrace{\left(\sum_{j=1}^{N-1} I_j\right)}^{I_N} \ln D_{pN}\right]$$

$$= 2*10^{-7}\left[\sum_{j=1}^{N} I_j \ln \frac{1}{D_{kj}} + \sum_{j=1}^{N-1} I_j \ln \frac{D_{pj}}{D_{pN}}\right]$$

As $P \longrightarrow \infty$ very far away

D_{pj} and D_{pN} almost the same $(D_{pj} = D_{pN}) \Longrightarrow \left[\ln \frac{D_{pj}}{D_{pN}} = \ln 1 = 0\right]$

$$\boxed{\lambda_k = 2*10^{-7} \sum_{j=1}^{N} I_j \ln \frac{1}{D_{kj}}} \quad **$$

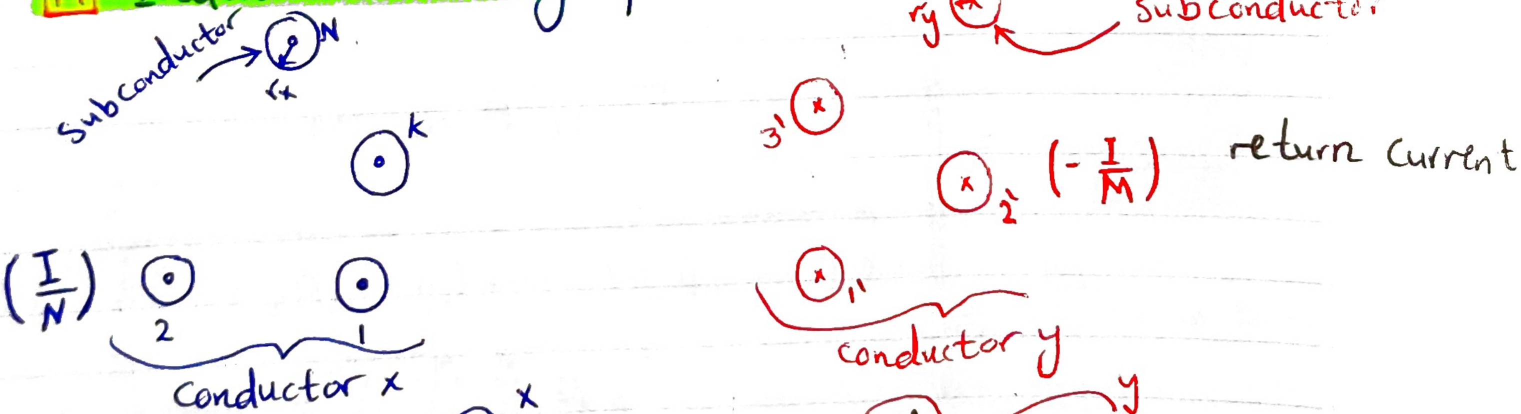
↳ Total Flux Linkages for the Conductor k.

Inductance

Inductance of Single-Phase Lines [A]

Inductance of 3 ϕ T.L [B]

[A] Inductance of Single-phase Lines



$$\phi_k = 2*10^{-7}\left[\overbrace{\frac{I}{N}\sum_{m=1}^{N} \ln \frac{1}{D_{km}}}^{x} - \overbrace{\frac{I}{M}\sum_{m=1'}^{M} \ln \frac{1}{D_{km}}}^{y}\right] \quad \text{using } **$$

↳ The total flux for any subconductor k in conductor x.

$$\phi_k = 2*10^{-7}\left[\frac{I}{N}\sum_{m=1}^{N}\ln\frac{1}{D_{km}} - \frac{I}{M}\sum_{m=1'}^{M}\ln\frac{1}{D_{km}}\right]$$

Since only the fraction $\frac{1}{N}$ of the total conductor current I is linked by this flux, the flux linkage (λ_k) of sub conductor k is

$$\lambda_k = \frac{\phi_k}{N} = 2*10^{-7} I\left[\frac{1}{N^2}\sum_{m=1}^{N}\ln\frac{1}{D_{km}} - \frac{1}{NM}\sum_{m=1'}^{M}\ln\frac{1}{D_{km}}\right]$$

The total flux linkage of conductor x is:-

$$\lambda_x = \sum_{k=1}^{N}\lambda_k$$

$$= 2*10^{-7} I \sum_{k=1}^{N}\left[\frac{1}{N^2}\sum_{m=1}^{N}\ln\frac{1}{D_{km}} - \frac{1}{NM}\sum_{m=1'}^{M}\ln\frac{1}{D_{km}}\right]$$

$$= 2*10^{-7} I \ln \prod_{k=1}^{N}\frac{\left(\prod_{m=1'}^{M} D_{km}\right)^{\frac{1}{NM}}}{\left(\prod_{m=1}^{N} D_{km}\right)^{\frac{1}{N^2}}}$$

>>> $L_x = 2*10^{-7}\ln\frac{D_{xy}}{D_{xx}}$ H/m/conductor

>>> $L_y = 2*10^{-7}\ln\frac{D_{xy}}{D_{yy}}$ H/m/conductor

where:

Geometric Mean Distance between x and y

$$D_{xy} = GMD_{x,y} = \sqrt[MN]{\prod_{k=1}^{N}\prod_{m=1'}^{M} D_{km}}$$

$$= \sqrt[MN]{(D_{11'}D_{12'}D_{13'}\cdots D_{1M})\cdots\cdots(D_{N1'}D_{N2'}\cdots D_{NM})}$$

$$D_{xx} = GMR_x = \sqrt[N^2]{\prod_{k=1}^{N}\prod_{m=1}^{N} D_{km}}$$

$$= \sqrt[N^2]{(D_{11}D_{12}D_{13}\cdots D_{1n})\cdots\cdots(D_{N1}D_{N2}\cdots D_{NN})}$$

Geometric Mean Radius of Conductor x

$$D_{yy} = GMR_y = \sqrt[M^2]{\prod_{k=1'}^{M}\prod_{m=1'}^{M} D_{km}}$$

$$= \sqrt[M^2]{(D_{1'1'}D_{1'2'}\cdots D_{1'M})\cdots(D_{M1'}D_{M2'}\cdots D_{MM})}$$

Geometric Mean Radius of Conductor y

Note:

- $\frac{1}{N^2}\left(\ln\frac{1}{a}+\ln\frac{1}{b}+\ln\frac{1}{c}\right) - \frac{1}{NM}\left(\ln\frac{1}{x}+\ln\frac{1}{y}+\ln\frac{1}{z}\right)$

$= \frac{1}{N^2}\left[\ln\frac{1}{abc}\right] - \frac{1}{NM}\left(\ln\frac{1}{xyz}\right)$

$= \ln\frac{1}{(abc)^{\frac{1}{N^2}}} - \ln\frac{1}{(xyz)^{\frac{1}{NM}}}$

$= \ln\frac{\frac{1}{(abc)^{\frac{1}{N^2}}}}{\frac{1}{(xyz)^{\frac{1}{NM}}}}$

$= \ln\frac{(xyz)^{\frac{1}{NM}}}{(abc)^{\frac{1}{N^2}}}$

- $\ln A^{\alpha} = \alpha\ln A$
- $\sum\ln A_k = \ln\prod A_k$

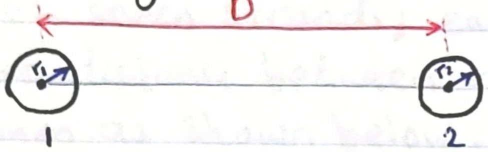
note that:-

$D_{11} = D_{22} = D_{33} = \cdots = D_{NN} = r'_x$

$D_{1'1'} = D_{2'2'} = D_{3'3'} = \cdots = D_{MM} = r'_y$

>>> $L = L_x + L_y$ H/m/circuit

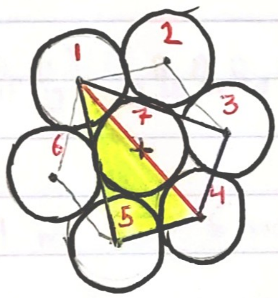
» if we have Single-phase two-wire line



$L_1 = 2 * 10^{-7} \ln \frac{D}{r_1'}$ H/m $\quad L_2 = 2 * 10^{-7} \ln \frac{D}{r_2'}$ H/m

$r_1' = 0.7788 r_1 \quad r_2' = 0.7788 r_2'$

Example: A stranded conductor consists of seven identical strands each strand having a radius r as shown in Figure below, determine the GMR of the conductor interms of r.



$D_{12} = D_{16} = D_{17} = 2r$

$D_{14} = 4r$

$D_{13} = D_{15} = \sqrt{D_{14}^2 - D_{45}^2}$

$= \sqrt{16r^2 - 4r^2}$

$= \sqrt{12 r^2}$

$= 2\sqrt{3}\, r$

$$GMR = \sqrt[N^2]{(D_{11} D_{12} D_{13} D_{14} D_{15} D_{16} D_{17})(D_{21} D_{22} D_{23} \cdots D_{27}) \cdots\cdots (D_{71} \cdots)}$$

$$= \sqrt[49]{\underbrace{(r' . 2r . 2\sqrt{3} r . 4r . 2\sqrt{3} r . 2r . 2r)^6}_{1,2,3,4,5,6} \underbrace{(r')(2r)^6}_{7}}$$

$= 2.1767\, r$

» With large number of strands the calculation of GMR can become very tedious. (مضجر، ممل)

» Usually these are available in the manufacturer's data. (Tables)

» The design of a power line requires the value of resistance and reactance to find out the active and reactive power, and the voltage drop in the process of power transfer over the transmission line.

» Power losses should be limitted to around (5-10)% of the power transfered.

TABLE A.4 Characteristics of aluminum cable, steel, reinforced (Aluminum Company of America)—ACSR

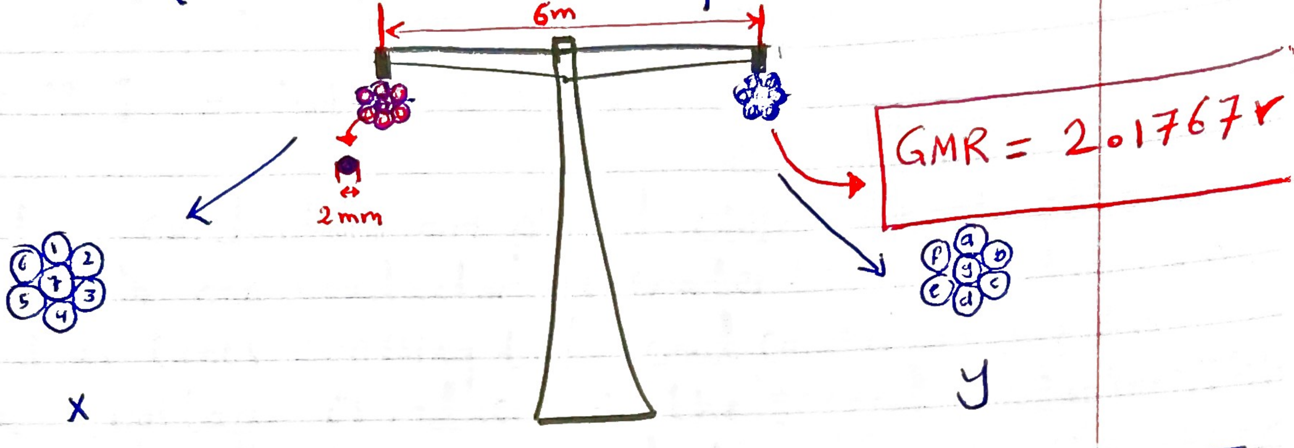
Code Word	Circular Mils Aluminum	Aluminum			Steel		Outside Diameter (inches)	Copper Equivalent[*] Circular Mils or A.W.G.	Ultimate Strength (pounds)	Weight (pounds per mile)	Geometric Mean Radius at 60 Hz (feet)	Approx. Current Carrying Capacity[†] (amps)	r_a Resistance (Ohms per Conductor per Mile) 25°C (77°F) Small Currents				r_a Resistance (Ohms per Conductor per Mile) 50°C (122°F) Current Approx. 75% Capacity[‡]				x_a Inductive Reactance (ohms per conductor per mile at 1 ft spacing all currents)	x'_a Shunt Capacitive Reactance (megohms per conductor per mile at 1 ft spacing)
				Strand Diameter (inches)		Strand Diameter (inches)							dc	25 Hz	50 Hz	60 Hz	dc	25 Hz	50 Hz	60 Hz	60 Hz	60 Hz
Joree	2 515 000	76		0.1819	19	0.0849	1.880		61 700		0.0621									0.0450	0.337	0.0755
Thrasher	2 312 000	76		0.1744	19	0.0814	1.802		57 300		0.0595									0.0482	0.342	0.0767
Kiwi	2 167 000	72	4	0.1735	7	0.1157	1.735		49 800		0.0570									0.0511	0.348	0.0778
Bluebird	2 156 000	84	4	0.1602	19	0.0961	1.762		60 300		0.0588									0.0505	0.344	0.0774
Chukar	1 781 000	84	4	0.1456	19	0.0874	1.602		51 000		0.0534									0.0598	0.355	0.0802
Falcon	1 590 000	54	3	0.1716	19	0.1030	1.545	1 000 000	56 000	10 777	0.0520	1 380	0.0587	0.0588	0.0590	0.0591	0.0646	0.0656	0.0675	0.0684	0.359	0.0814
Parrot	1 510 500	54	3	0.1673	19	0.1004	1.506	950 000	53 200	10 237	0.0507	1 340	0.0618	0.0619	0.0621	0.0622	0.0680	0.0690	0.0710	0.0720	0.362	0.0821
Plover	1 431 000	54	3	0.1628	19	0.0977	1.465	900 000	50 400	9 699	0.0493	1 300	0.0652	0.0653	0.0655	0.0656	0.0718	0.0729	0.0749	0.0760	0.365	0.0830
Martin	1 351 000	54	3	0.1582	19	0.0949	1.424	850 000	47 600	9 160	0.0479	1 250	0.0691	0.0692	0.0694	0.0695	0.0761	0.0771	0.0792	0.0803	0.369	0.0838
Pheasant	1 272 000	54	3	0.1535	19	0.0921	1.382	800 000	44 800	8 621	0.0465	1 200	0.0734	0.0735	0.0737	0.0738	0.0808	0.0819	0.0840	0.0851	0.372	0.0847
Grackle	1 192 500	54	3	0.1486	19	0.0892	1.338	750 000	43 100	8 082	0.0450	1 160	0.0783	0.0784	0.0786	0.0788	0.0862	0.0872	0.0894	0.0906	0.376	0.0857
Finch	1 113 000	54	3	0.1436	19	0.0862	1.293	700 000	40 200	7 544	0.0435	1 110	0.0839	0.0840	0.0842	0.0844	0.0924	0.0935	0.0957	0.0969	0.380	0.0867
Curlew	1 033 500	54	3	0.1384	7	0.1384	1.246	650 000	37 100	7 019	0.0420	1 060	0.0903	0.0905	0.0907	0.0909	0.0994	0.1005	0.1025	0.1035	0.385	0.0878
Cardinal	954 000	54	3	0.1329	7	0.1329	1.196	600 000	34 200	6 479	0.0403	1 010	0.0979	0.0980	0.0981	0.0982	0.1078	0.1088	0.1118	0.1128	0.390	0.0890
Canary	900 000	54	3	0.1291	7	0.1291	1.162	566 000	32 300	6 112	0.0391	970	0.104	0.104	0.104	0.104	0.1145	0.1155	0.1175	0.1185	0.393	0.0898
Crane	874 500	54	3	0.1273	7	0.1273	1.146	550 000	31 400	5 940	0.0386	950	0.107	0.107	0.107	0.108	0.1178	0.1188	0.1218	0.1228	0.395	0.0903
Condor	795 000	54	3	0.1214	7	0.1214	1.093	500 000	28 500	5 399	0.0368	900	0.117	0.118	0.118	0.119	0.1288	0.1308	0.1358	0.1378	0.401	0.0917
Drake	795 000	26	2	0.1749	7	0.1360	1.108	500 000	31 200	5 770	0.0375	900	0.117	0.117	0.117	0.117	0.1288	0.1288	0.1288	0.1288	0.399	0.0912
Mallard	795 000	30	2	0.1628	19	0.0977	1.140	500 000	38 400	6 517	0.0393	910	0.117	0.117	0.117	0.117	0.1288	0.1288	0.1288	0.1288	0.393	0.0904
Crow	715 500	54	3	0.1151	7	0.1151	1.036	450 000	26 300	4 859	0.0349	830	0.131	0.131	0.131	0.132	0.1442	0.1452	0.1472	0.1482	0.407	0.0932
Starling	715 500	26	2	0.1659	7	0.1290	1.051	450 000	28 100	5 193	0.0355	840	0.131	0.131	0.131	0.131	0.1442	0.1442	0.1442	0.1442	0.405	0.0928
Redwing	715 500	30	2	0.1544	19	0.0926	1.081	450 000	34 600	5 865	0.0372	840	0.131	0.131	0.131	0.131	0.1442	0.1442	0.1442	0.1442	0.399	0.0920
Flamingo	666 600	54	3	0.1111	7	0.1111	1.000	419 000	24 500	4 527	0.0337	800	0.140	0.140	0.141	0.141	0.1541	0.1571	0.1591	0.1601	0.412	0.0943
Rook	636 000	54	3	0.1085	7	0.1085	0.977	400 000	23 600	4 319	0.0329	770	0.147	0.147	0.148	0.148	0.1618	0.1638	0.1678	0.1688	0.414	0.0950
Grosbeak	636 000	26	2	0.1564	7	0.1216	0.990	400 000	25 000	4 616	0.0335	780	0.147	0.147	0.147	0.147	0.1618	0.1618	0.1618	0.1618	0.412	0.0946
Egret	636 000	30	2	0.1456	19	0.0874	1.019	400 000	31 500	5 213	0.0351	780	0.147	0.147	0.147	0.147	0.1618	0.1618	0.1618	0.1618	0.406	0.0937
Peacock	605 000	54	3	0.1059	7	0.1059	0.953	380 500	22 500	4 109	0.0321	750	0.154	0.155	0.155	0.155	0.1695	0.1715	0.1755	0.1775	0.417	0.0957
Squab	605 000	26	2	0.1525	7	0.1186	0.966	380 500	24 100	4 391	0.0327	760	0.154	0.154	0.154	0.154	0.1700	0.1720	0.1720	0.1720	0.415	0.0953
Dove	556 500	26	2	0.1463	7	0.1138	0.927	350 000	22 400	4 039	0.0313	730	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	0.420	0.0965
Eagle	556 500	30	2	0.1362	7	0.1362	0.953	350 000	27 200	4 588	0.0328	730	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	0.415	0.0957
Hawk	477 000	26	2	0.1355	7	0.1054	0.858	300 000	19 430	3 462	0.0290	670	0.196	0.196	0.196	0.196	0.216				0.430	0.0988
Hen	477 000	30	2	0.1261	7	0.1261	0.883	300 000	23 300	3 933	0.0304	670	0.196	0.196	0.196	0.196	0.216				0.424	0.0980
Ibis	397 500	26	2	0.1236	7	0.0961	0.783	250 000	16 190	2 885	0.0265	590	0.235				0.259				0.441	0.1015
Lark	397 500	30	2	0.1151	7	0.1151	0.806	250 000	19 980	3 277	0.0278	600	0.235		Same as dc		0.259		Same as dc		0.435	0.1006
Linnet	336 400	26	2	0.1138	7	0.0855	0.721	4/0	14 050	2 442	0.0244	530	0.278				0.306				0.451	0.1039
Oriole	336 400	30	2	0.1059	7	0.1059	0.741	4/0	17 040	2 774	0.0255	530	0.278				0.306				0.445	0.1032
Ostrich	300 000	26	2	0.1074	7	0.0835	0.680	188 700	12 650	2 178	0.0230	490	0.311				0.342				0.458	0.1057
Piper	300 000	30	2	0.1000	7	0.1000	0.700	188 700	15 430	2 473	0.0241	500	0.311				0.342				0.462	0.1049
Partridge	266 800	26	2	0.1013	7	0.0788	0.642	3/0	11 250	1 936	0.0217	460	0.350				0.385				0.465	0.1074

*Based on copper 97%, aluminum 61% conductivity

†For conductor at 75°C, air at 25°C, wind 1.4 miles per hour (2 ft/sec), frequency = 60 Hz

‡ "Current Approx. 75% Capacity" is 75% of the "Approx. Current Carrying Capacity in Amps" and is approximately the current which will produce 50°C conductor temp. (25°C rise) with 25°C air temp., wind 1.4 miles per hour.

Example Power is transmitted over the live stranded conductor with seven strands; each strand 2 mm in diameter. The distance between the live and neutral wires is 6mm as shown below. Calculate the inductance and reactance of the line in mH per km.



$$GMD_{xy} = \sqrt[49]{(D_{1a}\, D_{1b}\, D_{1c}\, D_{1d}\, D_{1e}\, D_{1f}\, D_{1g})(D_{2a}\, D_{2b}\, D_{2c}\, D_{2d}\, D_{2e}\, D_{2f}\, D_{2g}) \cdots\cdots (D_{7a}\, D_{7b}\, D_{7c}\, D_{7d}\, D_{7e}\, D_{7f}\, D_{7g})}$$

$$= 5.99999971 \text{ m} \cong 6 \text{ m}$$

$$GMR_x = GMR_y = 2.1767r = (2.1767)(0.001)$$
$$= 0.0021767$$

$$L_x = 2 * 10^{-7} \ln \frac{D_{xy}}{D_{xx}} = 2 * 10^{-7} \ln \frac{6}{0.002177} \text{ H/m}$$

$$= 1.584 * 10^{-6} \text{ H/m}$$ per conductor

$$L = L_x + L_y = 3.168 * 10^{-6} \text{ H/m}$$

$X_L = wL = 2\pi f L \triangleq$ Reactance per meter length of conductor

$$= 2\pi(50)(L)$$

$$= 9.954 * 10^{-4}\ \Omega/m$$

$$= 0.9954\ \Omega/Km$$

Notes

» The flux Linkage $\lambda = L \cdot I$

» The voltage drop due to this flux Linkage is

$$V = ZI = j\omega LI = j\omega\lambda$$

» When two conductors are placed close to each other, current in one conductor generates the magnetic flux. These flux lines crossing the second conductor due to which a voltage is induced in the second conductor. This process of current in one conductor affecting the other conductor is the mutual inductance.

» If we define the two conductors as 1 and 2, then

$$M_{12} = \frac{\lambda_{12}}{I_2}$$

where
- M_{12} is the mutual inductance between conductors 1 and 2.
- λ_{12} is the flux linkage between conductors 1 and 2.
- I_2 is the current in conductor 2.

This in turn introduces the voltage drop in the first conductor which is defined by:

$$\boxed{V_1 = j\omega M_{12} I}$$

Inductance

Inductance of Single ϕ $\boxed{A}$ — Inductance of 3ϕ T.L. $\boxed{B}$

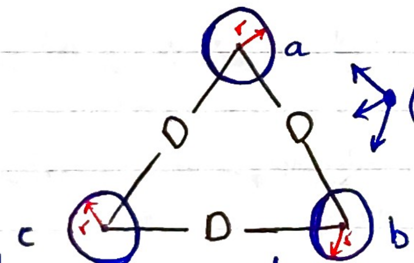
$\boxed{B}$ Inductance of 3ϕ T.L.

a) Symmetrical Spacing (Equilateral Spacing).
b) Asymmetrical Spacing.
c) Transposition.
d) Bundled Conductor.

Composite Conductor :-

$$\lambda_k = 2*10^{-7} \sum_{j=1}^{N} I_j \ln \frac{1}{D_{kj}}$$

a)) Three phase line with equilateral spacing.



((one meter length))

Assuming Balanced 3ϕ currents :- $I_a + I_b + I_c = 0$

$\Rightarrow$ The total flux linkage of phase a conductor is :-

$$\lambda_a = 2*10^{-7}\left(I_a \ln \frac{1}{r'} + I_b \ln \frac{1}{D} + I_c \ln \frac{1}{D}\right)$$

$$= 2*10^{-7}\left(I_a \ln \frac{1}{r'} + (I_b + I_c) \ln \frac{1}{D}\right)$$

$$= 2*10^{-7}\left(I_a \ln \frac{1}{r'} - I_a \ln \frac{1}{D}\right) = 2*10^{-7} I_a \ln \frac{D}{r'}$$

$$L_a = \frac{\lambda_a}{I_a} = 2*10^{-7} \ln \frac{D}{r'} \ H/m \quad = 0.2 \ln \frac{D}{D_s} \ mH/km$$

D_s: solid → r' ; stranded → GMR

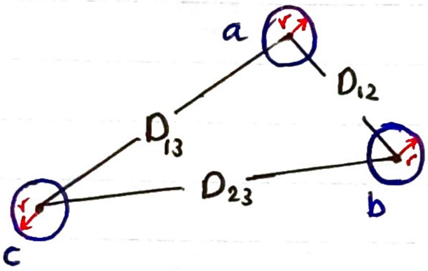
$$\lambda_a = \lambda_b = \lambda_c \Rightarrow L_a = L_b = L_c$$

** This means that the inductance per phase for 3ϕ circuit equilateral spacing is the same as for one conductor of single phase circuit.

b)) Asymmetrical Spacing:-

» Practical transmission lines cannot maintain symmetrical spacing of conductors because of construction considerations.

» With asymmetrical spacing, even with balanced currents, the voltage drop due to line inductance will be unbalanced.



$$\lambda_a = 2*10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D_{12}} + I_c \ln\frac{1}{D_{13}}\right)$$

$$\lambda_b = 2*10^{-7}\left(I_a \ln\frac{1}{D_{12}} + I_b \ln\frac{1}{r'} + I_c \ln\frac{1}{D_{23}}\right)$$

$$\lambda_c = 2*10^{-7}\left(I_a \ln\frac{1}{D_{13}} + I_b \ln\frac{1}{D_{23}} + I_c \ln\frac{1}{r'}\right)$$

Or in matrix form $\boldsymbol{\lambda} = \mathbf{L}\,\mathbf{I}$

where the symmetrical inductance matrix $\mathbf{L}$ is given by:

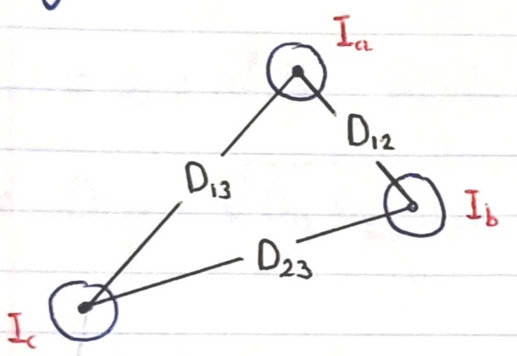
$$\mathbf{L} = 2*10^{-7}\begin{bmatrix} \ln\frac{1}{r'} & \ln\frac{1}{D_{12}} & \ln\frac{1}{D_{13}} \\ \ln\frac{1}{D_{12}} & \ln\frac{1}{r'} & \ln\frac{1}{D_{23}} \\ \ln\frac{1}{D_{13}} & \ln\frac{1}{D_{23}} & \ln\frac{1}{r'} \end{bmatrix}$$

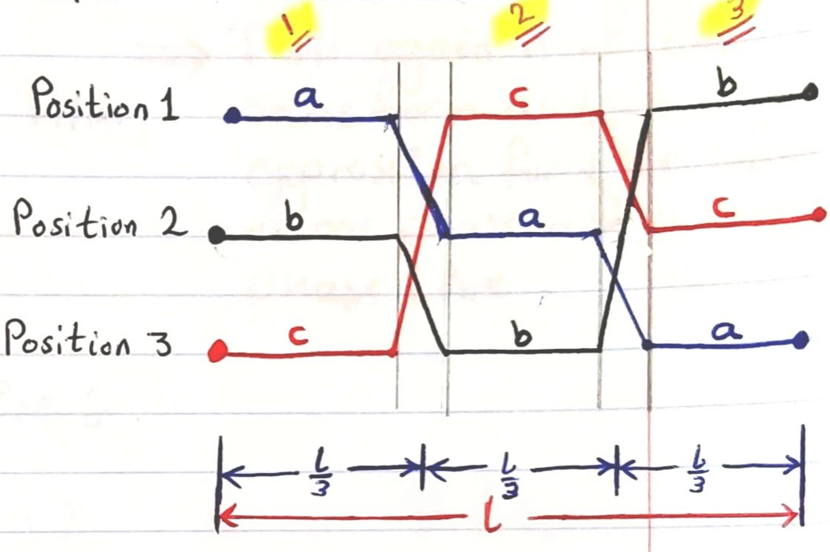
⇒ The phase inductances are not equal

c)) Three phase transposed Line:

» One way to regain symmetry and obtain per-phase model is consider transposition.

» The transposition consists of interchanging the phase configuration every one-third the length.





$$\lambda_{a1} = 2*10^{-7}\left[I_a \ln\frac{1}{D_s} + I_b \ln\frac{1}{D_{12}} + I_c \ln\frac{1}{D_{31}}\right]$$

r' GMR

$$\lambda_{a2} = 2*10^{-7}\left[I_a \ln\frac{1}{D_s} + I_b \ln\frac{1}{D_{23}} + I_c \ln\frac{1}{D_{12}}\right]$$

$$\lambda_{a3} = 2*10^{-7}\left[I_a \ln\frac{1}{D_s} + I_b \ln\frac{1}{D_{31}} + I_c \ln\frac{1}{D_{23}}\right]$$

$$\lambda_a = \frac{\lambda_{a1}\left(\frac{l}{3}\right) + \lambda_{a2}\left(\frac{l}{3}\right) + \lambda_{a3}\left(\frac{l}{3}\right)}{l} = \frac{\lambda_{a1} + \lambda_{a2} + \lambda_{a3}}{3}$$

$$= \frac{2*10^{-7}}{3}\left[3 I_a \ln\frac{1}{D_s} + I_b \ln\frac{1}{D_{12}D_{23}D_{31}} + I_c \ln\frac{1}{D_{12}D_{23}D_{31}}\right]$$

$$= \frac{2*10^{-7}}{3}\left[3 I_a \ln\frac{1}{D_s} - I_a \ln\frac{1}{D_{12}D_{23}D_{31}}\right]$$

$$\lambda_a = 2 * 10^{-7} I_a \ln \frac{\sqrt[3]{D_{12} D_{23} D_{31}}}{D_s}$$

$$L_a = \frac{\lambda_a}{I_a} = 2 * 10^{-7} \ln \frac{\sqrt[3]{D_{12} D_{23} D_{31}}}{D_s} \quad \text{H/m per phase}$$

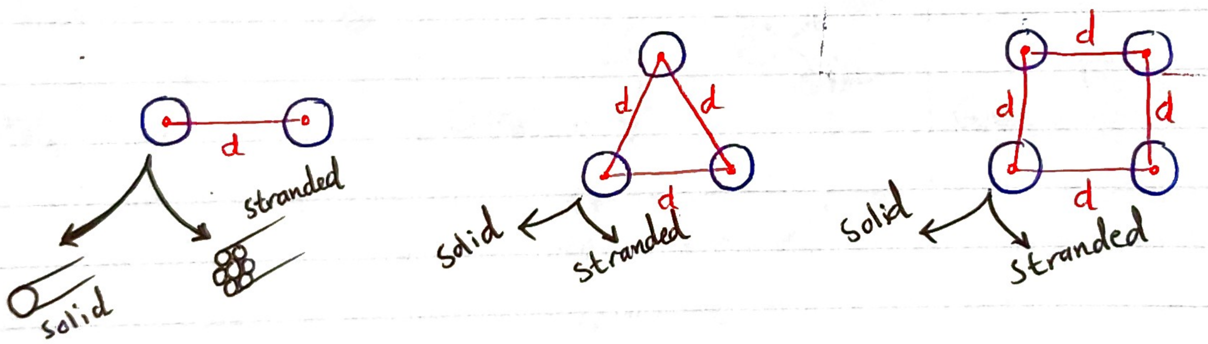
$$L_a = 2 * 10^{-7} \ln \frac{D_{eq}}{D_s} \quad \text{H/m}$$

$$= 2 * 10^{-7} \ln \frac{GMD}{D_s} \quad \text{H/m}$$

where $D_{eq} = \sqrt[3]{D_{12} D_{23} D_{31}}$

⇒ This again is of the same form as the expression for the induc of one phase of a single phase line.

d)) Bundled Conductor line :-



>> Extra-high voltage transmission lines are usually constructed with bundled conductors. Bundling reduces the line reactance, which improves the line performance and increase the power capability of the line. Bundling also reduces the voltage surface gradient, which in turn reduces corona loss, radio interference, and surge impedance. $(\sqrt{\frac{L}{C}})$

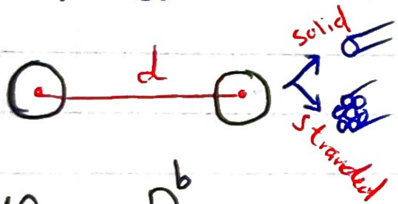
>> Typically, bundled conductors consists of two, three, or four subconducters symmetrically arranged in configuration as shown in Figure above.

» The subconductors within a bundle are separated at frequent intervals by spacer-dampers. spacer-dampers prevent clashing, provide damping, and connect the subconductors in parallel.

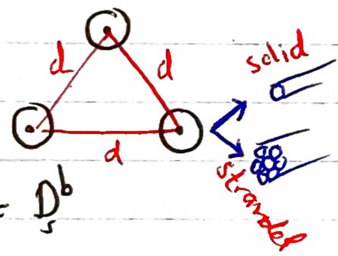
Bundling

Reduces Electric Field Strength on Conductor Surface → **Reduces Corona**

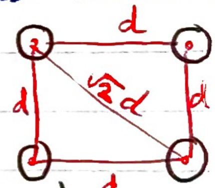
Increases Effective Radius (GMR) → **Reduces Inductance**



$$GMR_b = D_s^b = \sqrt[4]{(r' \cdot d)^2} = \sqrt[2]{r' \cdot d}$$



$$GMR_b = D_s^b = \sqrt[9]{(r' \cdot d \cdot d)^3} = \sqrt[3]{r' \, d^2}$$



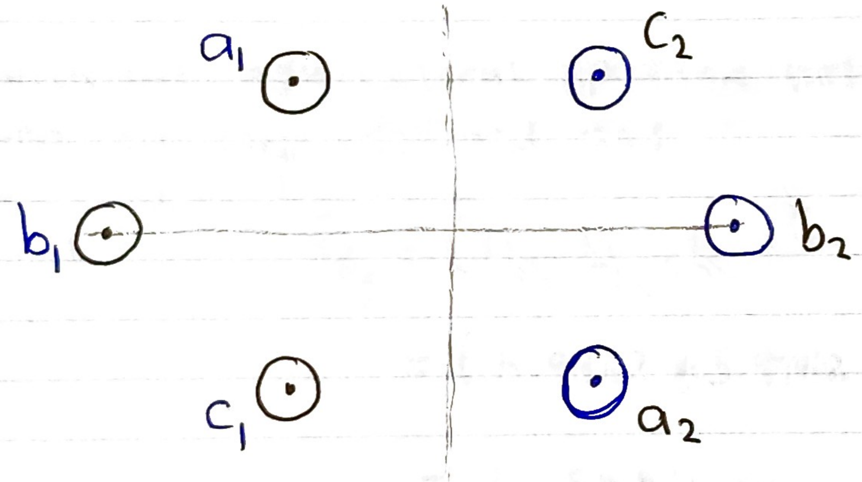
$$GMR_b = D_s^b = \sqrt[16]{(r' \cdot d \cdot d \cdot d\sqrt{2})^4} = 1.091 \sqrt[4]{r' \, d^3}$$

$$\boxed{L_a = 2 * 10^{-7} \ln \frac{D_{eq}}{D_s^b} \quad H/m}$$

>> Three-phase Lines - Parallel Circuits.

>> Three-phase Double-Circuit Lines.

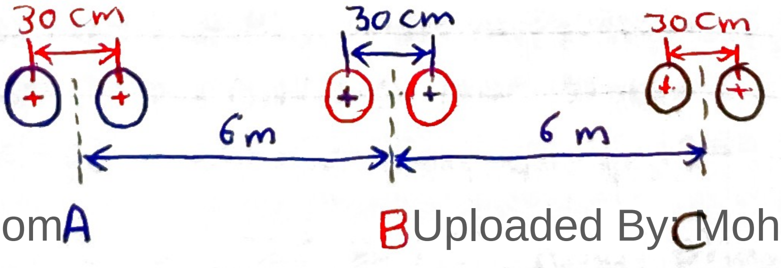
A three-phase double-Circuit line consists of two identical 3ϕ circuits. The circuits are operated with abc, cba in parallel. Because of geometrical differences between Conductors, voltage drop due to line inductance will be unbalanced. To achieve balance, each phase conductor must be transposed within its group and with respect to parallel 3ϕ line.



example The conductor configuration of a completely transposed 3-ϕ overhead transmission line with bundled conductor is shown below. All the Conductors have a radius of 0.74 cm with a 30 cm bundle spacing.

a)) Determine the inductance per-phase in mH/km and in mH/m.

b)) Find the inductive line reactance per phase in Ω/m at f = 50 Hz.



$$D_{ab} = \sqrt[4]{d_{13}\ d_{14}\ d_{23}\ d_{24}}$$

$$= (6 * 6.3 * 5.7 * 6)^{1/4} = 5.9962\ m$$

Similarly,

$$D_{bc} = 5.9962\ m$$

$$D_{ca} = \sqrt[4]{d_{15}\ d_{16}\ d_{25}\ d_{26}}$$

$$= (12 * 12.3 * 11.7 * 12)^{\frac{1}{4}} = 11.9981\ m$$

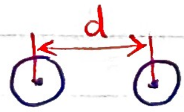
The equivalent equilateral spacing between the phases is given by D_{eq} defined as:-

$$D_{eq} = (D_{ab} \cdot D_{bc} \cdot D_{ca})^{\frac{1}{3}}$$

$$= (5.9962 * 5.9962 * 11.9981)^{\frac{1}{3}}$$

$$= 7.5559\ m$$

$$D_s^b = \sqrt[2]{r' d}$$



$$= (0.7788 * r * 30)^{\frac{1}{2}} = 4.1580\ cm$$

a)) Inductance per phase for the given system is:-

$$L = 2 * 10^{-7} \ln \frac{D_{eq}}{D_s^b} \quad H/m/phase$$

$$= 1.04049 * 10^{-6} \quad H/m/phase$$

$$= 1.04049 * 10^{-3} \quad mH/m/phase = 1.04049 \quad mH/km/phase$$

b)) The inductive line reactance per phase

$$X_L = 2\pi f L = 2\pi (50)(1.04049) * 10^{-6} \quad \Omega/m/phase$$

$$= 3.270 * 10^{-4} \quad \Omega/m/phase$$