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Exercises Chapter 2 :

Q.5 let $A = \{1,2\}$. List all numbers of $P(A)$

$P(A) = \{ \{1\}, \{2\}, \{1,2\}, \emptyset \}$

Q.10 Determine which of the following statments are true

a) $\emptyset \in \{\emptyset\}$ $\Rightarrow$ true.
b) $\emptyset \subset \{\emptyset\}$ $\Rightarrow$ true.
c) $\pi/4 \in \{\pi/4\}$ $\Rightarrow$ true.
d) $\emptyset \in \{\pi/4\}$ $\Rightarrow$ false.
e) $\pi/4 \in \{\{\pi/4\}\}$ $\Rightarrow$ false
f) $\{\pi/4\} \subseteq \{\{\pi/4\}\}$ $\Rightarrow$ false
g) $\{\pi/4\} \in \{\{\pi/4\}\}$ $\Rightarrow$ true
h) $\emptyset \subseteq \{\{\pi/4\}\}$ $\Rightarrow$ true
i) $\{\pi/4\} \subseteq \{\pi/4, \{\pi/2\}\}$ $\Rightarrow$ true

Q.11 Determine which of the following statments are true :

a) The empty set is subset of every set $\Rightarrow$ true
b) If A is a proper subset of $\emptyset$, then $A = \{17\}$ $\Rightarrow$ False.
c) If $A \subseteq B$, then $A = B$ $\Rightarrow$ False
d) If $A = B$, then $A \subseteq B$ $\Rightarrow$ true

e) Since $\emptyset$ is a number of $\{\emptyset\}$, $\emptyset = \{\emptyset\}$ $\Rightarrow$ false

f) There is a set that is a member of every set $\Rightarrow$ false

g) $\{\{3,5,7\}\} \in N \cup P(N)$ $\Rightarrow$ ~~true~~ false

h) $\{\{3,5,7\}\} \subseteq N \cup P(N)$ $\Rightarrow$ true

$\boxed{Q.15}$ for each of the following subsets of R, either list all the members of the set or describe the set in plain English :-

a) $\{x \in R : x > 3,\ x^2 < 5, \text{ and } x \neq 4\}$

The members of the set is $\emptyset$

b) $\{x \in R : x > 3 \text{ or } -x > 3\}$

$(-\infty, -3) \cup (3, \infty)$

c) $\{x \in R : x > 3 \text{ and } -x > 3\}$

$\emptyset$

d) $\{x \in R : x > 3 \text{ and } -x < 3\}$

$(3, \infty)$

e) $\{x \in R : x^2 \neq x\} \Rightarrow R - \{0, 1\}$

(Q.18) If A and B are sets and $A \subseteq B$, then $P(A) \subseteq P(B)$ which, if any are correct? justify your answer.

- proof 1 : let $x \in P(A)$, Then $x \in A$ Since $A \subset B$, $x \in B$ There fore $x \in P(B)$ so $P(A) \subseteq P(B)$

- Proof 2 : let $A = \{1, 2\}$ and $B = \{1, 2, 3\}$. Then $P(A) = \{\emptyset, \{1\}, \{2\}, A\}$ and $P(B) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, B\}$. There fore $P(A) \subseteq P(B)$.

- Proof 3 : let $x \in A$. Since $A \subseteq B$, $x \in B$. Since $x \in A$ and $x \in B$ $\{x\} \in P(A)$ and $\{x\} \in P(B)$. There fore $P(A) \subseteq P(B)$

$\Rightarrow$ Proof 1 is wrong, suppose $x \in \emptyset$, $\emptyset \in P(A)$ but $\emptyset \notin A$ so all this proof is wrong.

$\Rightarrow$ Proof 2 this example is true but we must prove in general

$\Rightarrow$ proof 3 is true, using Direct method. A, B are sets
Suppose that $A \subseteq B$ and $x \in P(A)$
$\Rightarrow x \in A \xrightarrow[\text{Since } A \subseteq B]{} x \in B \Rightarrow \{x\} \subseteq P(A) \Rightarrow P(A) \subseteq P(B)$
$\{x\} \subseteq P(B)$

Q.20 we give two proofs of the following conjecture: If A and B are sets such that $P(A) \subseteq P(B)$ then $A \subseteq B$: which, if either, is correct? Justify.

. Proof 1: let $x \in P(A)$, Then $x \subseteq A$. since $A \subseteq B$, $x \subseteq B$
There fore $x \in P(B)$, so $P(A) \subseteq P(B)$.

. Proof 2: let $x \in A$, Then $\{x\} \in P(A)$. Since $P(A) \subseteq P(B)$
$\{x\} \in P(B)$. Therefore $x \in B \Rightarrow A \subseteq B$.

$\Rightarrow$ Proof 1 is wrong because it proofs that $A \subseteq B$ then $P(A) \subseteq P(B)$, we can't start with what we want to prove. and we can't say that $(x \subseteq A)^{x} \longrightarrow (x \in A)$ ✓ $x \in B$

$\Rightarrow$ proof 2 is true, using Direct method:
Suppose A, B are sets and $P(A) \subseteq P(B)$, $x \in A$
$\Rightarrow \{x\} \in P(A) \xrightarrow{P(A) \subseteq P(B)} \{x\} \in P(B) \Rightarrow x \in B$
$\Rightarrow A \subseteq B$

(Q.21) $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 3, 6\}$, $B = \{3, 4, 5, 6\}$ and $C = \{3, 6, 9\}$, :=

a) $A \cup B = \{1, 3, 4, 5, 6\}$
b) $(A \cup B)' = \{2, 7, 8, 9, 10\}$
c) $A \cap B = \{3, 6\}$
d) $(A \cap B)' = \{1, 2, 4, 5, 7, 8, 9, 10\}$
e) $(A \cup B) \cap C = \{3, 6\}$
f) $(A \cap B) \cup C = \{3, 6, 9\}$
g) $(A')' = A = \{1, 3, 6\}$
h) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C) = \{3, 6\}$
i) $A' \cup B' = (A \cap B)' = \{1, 2, 4, 5, 7, 8, 9, 10\}$
j) $A' \cap B' = (A \cup B)' = \{2, 7, 8, 9, 10\}$

(Q.40) Prove that for any sets A and B, the following are true :-

a) $A - (A \cap B') = A \cap B$ (proof by Direct) :=

$x \in A - (A \cap B') \Rightarrow x \in A$ and $(x \notin A$ or $x \notin B')$ $\Rightarrow x \in B$

$\Rightarrow x \in A$ and $x \notin A$ or $x \in A$ and $x \notin B'$

$\times$ $\Rightarrow x \in A$ and $x \in B \Rightarrow \underline{\underline{A \cap B}}$

b) $(A' \cup B)' \cap A = A - B$

- $((A')' \cap B') \cap A = (A \cap B') \cap A = (B' \cap A) \cap A$
$= B' \cap (A \cap A) \Rightarrow \underline{B' \cap A} = A \cap B' = A - B$ ✓
- by direct : Suppose $x \in (\bar{A} \cup B) \cap A \Rightarrow x \in (\bar{A} \cup B)$ and $x \in A$
$\Rightarrow x \notin (\bar{A} \cup B)$ and $x \in A$
$\Rightarrow x \notin \bar{A}$ and $x \notin B$ and $x \in A$
$\Rightarrow x \in A$ and $x \notin B \Rightarrow x \in (A - B)$ ✓

c) $(\overline{A-B})' - (\overline{B-A})' = B-A$ by direct

$x \in (\overline{A-B}) - (\overline{B-A}) \Rightarrow x \in (\overline{A-B})$ and $x \notin (\overline{B-A})$

$\Rightarrow x \notin (A-B)$ and $x \in (B-A)$

$\Rightarrow x \notin A$ and ~~and~~ $x \in B$ and $x \in B$ and $x \notin A$

$\Rightarrow x \notin A$ and $x \in B$ $\Rightarrow x \in (B-A)$ ✓

$\boxed{Q.46}$ Prove or find a counterexample to each of the following:

a) $(A-B) \cup C = A - (B \cup C)$

• let $A = \{2,6,10\}$, $B = \{2,4\}$, $C = \{8,10\}$

$A - B = \{6,10\}$; $B \cup C = \{2,4,8,10\}$

$(A-B) \cup C = \{6,8,10\} \neq A - (B \cup C) = \{6\}$

b) $(A' \cup B) \cap (B' \cup C) \subseteq A' \cup C$

by direct $\Rightarrow$ Suppose $x \in (\bar{A} \cup B) \cap (\bar{B} \cup C)$

$\Rightarrow (x \in \bar{A}$ or $x \in B)$ and $(x \notin B$ or $x \in C)$

$\Rightarrow (x \in \bar{A}$ or $x \in B$ and $x \notin B)$ or $(x \in \bar{A}$ or $x \in B$ and $x \in C)$

$\Rightarrow (x \in \bar{A}$ and $x \notin B)(x \in B$ and $x \notin B)$ or $(x \in \bar{A}$ and $x \in C)$ or

$\times$ $(x \in B$ and $x \in C)$

$\Rightarrow x \in \bar{A}$ or $x \in C$

$\Rightarrow x \in (\bar{A} \cup C)$

Q.49 Let A and B be sets. Prove that if $P(A \cup B) = P(A) \cup P(B)$ then $A \subseteq B$ or $B \subseteq A$. Proof by contradiction :⟹

Suppose $P(A \cup B) \Rightarrow P(A) \cup P(B)$ and $A \nsubseteq B$ and $B \nsubseteq A$

$\Rightarrow \exists\, a \in A,\ a \notin B$ and $\exists\, b \in B,\ b \notin A$

$\Rightarrow \{a,b\} \subseteq A \cup B \Rightarrow \{a,b\} \in P(A \cup B)$

but $\{a,b\} \notin P(A)$, $\{a,b\} \notin P(B) \Rightarrow \{a,b\} \notin P(A) \cup P(B)$

$\Rightarrow P(A \cup B) \neq P(A) \cup P(B)$ X

Q.50 We give a proof and a counterexample to the following Conjecture : "if $A \subseteq B$ and $C \subseteq D$ then $A \cap C \subseteq B \cap D$".

which, if either, of these is correct ?

- Counter example : let $A = \{1,2\}$, $B = \{1,2,3,4\}$, $C = \{\{1\},\{2\}\}$ and $D = \{1,2,3,4\}$

- Proof : let $x \in A \cap C$. Then $x \in A$ and $x \in C$. Since $A \subseteq B$ and $C \subseteq D$ $x \in B$, and $x \in D$. Therefore $x \in B \cap D$ so $A \cap C \subseteq B \cap D$.

- Proof 2 by direct : Suppose $A \subseteq B$ and $C \subseteq D$ (true)

  let $x \in A \cap C \Rightarrow x \in A$ and $x \in C$

  $\Rightarrow x \in B$ and $x \in D$, Since $A \subseteq B$ and $C \subseteq D$

  $\Rightarrow x \in (B \cap D)$ ✓

- Proof 1 : $A \cap C = \{1,2\}$

  $B \cap D = \{1,2,3,4\}$ $\subseteq$

  we must prove in general.

(Q.53) For each natural number n. let $A_n = \{7, n\}$. Find $\bigcup_{n\in N} A_n$ and $\bigcap_{n\in N} A_n$.

- $\bigcup_{n\in N} A_n = \{1,2,3,4,5,6,7,\ldots\} = N$
- $\bigcap_{n\in N} A_n = \{7\}$

(Q.54) for each natural number n, let $A_n = \{k\in N : k\geq n\}$ Find $\bigcup_{n\in N} A_n$ and $\bigcap_{n\in N} A_n$??

- $\bigcup_{n\in N} A_n =$ ~~[illegible]~~ $= N$
- $\bigcap_{n\in N} A_n = \emptyset$

(Q.55) for each natural number n, let $A_n = [-\frac{1}{n}, \frac{1}{n}]$ and $B_n = (-\frac{1}{n}, \frac{1}{n})$. Find :-

① $\bigcap_{n\in N} A_n = \{0\}$

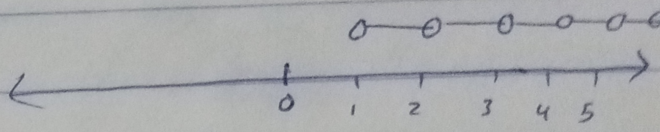
② $\bigcap_{n\in N} B_n = \cancel{\emptyset}\ \{0\}$

(Q.56) for each natural number n, let $A_n = [1-1/n,\ 1+1/n]$ find:

① $\bigcup_{n\in N} A_n = [0, 2]$

② $\bigcap_{n\in N} A_n = \{1\}$

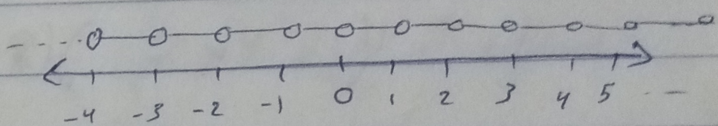
Q.57 a) for each natural number n, let $A_n = (n, n+1)$. find



① $\bigcup_{n \in \mathbb{N}} A_n = (1, \infty)$

② $\bigcap_{n \in \mathbb{N}} A_n = \emptyset$

b) for each real number x, let $A_x = (x, x+1)$. Find :-



① $\bigcup_{x \in R} A_x = (-\infty, \infty)$

② $\bigcap_{x \in R} A_x = \emptyset$

Q.61 for each natural number n, let
$A_n = (-1/n, (2n-1)/n)$ Find :

①

① $\bigcup_{n \in \mathbb{N}} A_n = (-1, 2)$

② $\bigcap_{n \in \mathbb{N}} A_n = \cancel{\emptyset}\ [0, 1)$

Q.62 for each real number x, let $A_x = \{3, -2\} \cup \{y \in R : y > x\}$
find :-

① $\bigcup_{x \in R} A_x = \mathbb{R}$

② $\bigcap_{x \in R} A_x = \{3, -2\}$