Problem 12-13

†12-14 Figure P12-4 shows a system with two weights on a rotating shaft. $W_1 = 15$ lb @ 30° at a 4-in radius and $W_2 = 20$ lb @ 270° at a 6-in radius. Determine the radii and angles of the balance weights needed to dynamically balance the system. The balance weight in plane 3 weighs 15 lb and in plane 4 weighs 30 lb.

†12-15 Figure P12-5 shows a system with two weights on a rotating shaft. $W_1 = 10$ lb @ 90° at a 3-in radius and $W_2 = 15$ lb @ 240° at a 3-in radius. Determine the magnitudes and angles of the balance weights needed to dynamically balance the system. The balance weights in planes 3 and 4 are placed at a 3-in radius.

†12-16 Figure P12-6 shows a system with three weights on a rotating shaft. $W_1 = 9$ lb @ 90° at a 4-in radius, $W_2 = 9$ lb @ 225° at a 6-in radius, and $W_3 = 6$ lb @ 315° at a 10-in radius. Determine the magnitudes and angles of the balance weights needed to dynamically balance the system. The balance weights in planes 4 and 5 are placed at a 3-in radius.

†12-17 Figure P12-7 shows a system with three weights on a rotating shaft. $W_2 = 10$ lb @ 90° at a 3-in radius, $W_3 = 10$ lb @ 180° at a 4-in radius, and $W_3 = 8$ lb @ 315° at a 4-in radius. Determine the magnitudes and angles of the balance weights needed to dynamically balance the system. The balance weight in plane 1 is placed at a radius of 4 in and in plane 5 of 3 in.

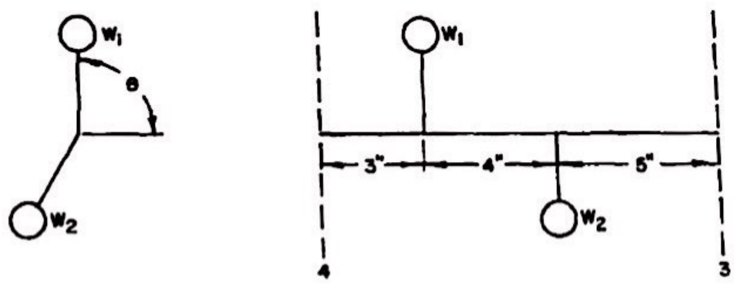

FIGURE P12-5

Problem 12-15

† These problems are suited to solution using *Mathcad*, *Matlab*, or *TKsolver* equation solver programs.

‡ These problems are suited to solution using program FOURBAR which is on the attached CD-ROM.

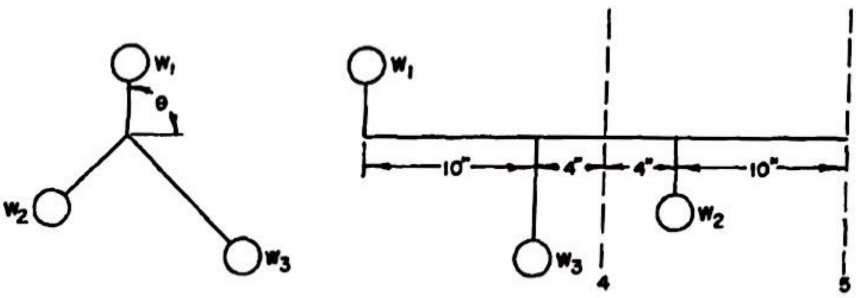

FIGURE P12-6

Problem 12-16

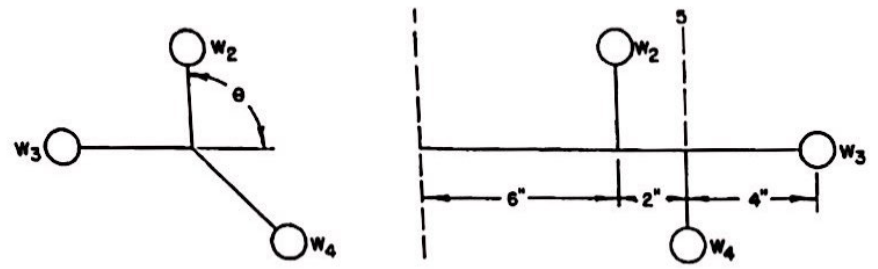

FIGURE P12-7

Problem 12-17

PROBLEM 12-15

Statement: Figure P12-5 shows a system with two weights on a rotating shaft. For the given data below, determine the magnitudes and angles of the balance weights needed to dynamically balance the system.

Given: Weights and radii:

$W_1 := 10 \cdot lbf$ $\quad r_1 := 3 \cdot in$ $\quad \theta_1 := 90 \cdot deg$ $\quad l_1 := 3 \cdot in$

$W_2 := 15 \cdot lbf$ $\quad r_2 := 3 \cdot in$ $\quad \theta_2 := 240 \cdot deg$ $\quad l_2 := 7 \cdot in$

Distance between correction planes: $l_B := 12 \cdot in$

Correction weight radii: Plane 4 $R_A := 3 \cdot in$ Plane 3 $R_B := 3 \cdot in$

Solution: See Figure P12-5 and Mathcad file P1215.

1. Resolve the position vectors into *xy* components in the arbitrary coordinate system associated with the freeze-frame position of the linkage chosen for analysis.

$R_{1x} := r_1 \cdot cos(\theta_1)$ $\quad R_{1x} = 0.000\, in$ $\quad R_{1y} := r_1 \cdot sin(\theta_1)$ $\quad R_{1y} = 3.000\, in$

$R_{2x} := r_2 \cdot cos(\theta_2)$ $\quad R_{2x} = -1.500\, in$ $\quad R_{2y} := r_2 \cdot sin(\theta_2)$ $\quad R_{2y} = -2.598\, in$

2. Solve equations 12.4e for summation of moments about *O*, which is at plane 4.

$$mR_{Bx} := \frac{-(W_1 \cdot R_{1x}) \cdot l_1 - (W_2 \cdot R_{2x}) \cdot l_2}{l_B \cdot g} \qquad mR_{Bx} = 13.125\, in \cdot lb$$

$$mR_{By} := \frac{-(W_1 \cdot R_{1y}) \cdot l_1 - (W_2 \cdot R_{2y}) \cdot l_2}{l_B \cdot g} \qquad mR_{By} = 15.233\, in \cdot lb$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane *B* (3). Also, solve for the weight required at the given radius.

$\theta_B := atan2(mR_{Bx}, mR_{By})$ $\quad \theta_B = 49.252\, deg$

$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2}$ $\quad mR_B = 20.108\, in \cdot lb$

$W_3 := \frac{mR_B \cdot g}{}$ $\quad W_3 = 6.70\, lbf$

$\theta_B := atan2(mR_{Bx}, mR_{By})$ $\theta_B = 49.252\ deg$

$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2}$ $mR_B = 20.108\ in \cdot lb$

$W_3 := \frac{mR_B \cdot g}{R_B}$ $W_3 = 6.70\ lbf$

4. Solve equations 12.4c for forces in x and y directions in plane A (4).

$mR_{Ax} := \frac{-W_1 \cdot R_{1x} - W_2 \cdot R_{2x}}{g} - mR_{Bx}$ $mR_{Ax} = 9.375\ in \cdot lb$

$mR_{Ay} := \frac{-W_1 \cdot R_{1y} - W_2 \cdot R_{2y}}{g} - mR_{By}$ $mR_{Ay} = -6.262\ in \cdot lb$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A (4).

$\theta_A := atan2(mR_{Ax}, mR_{Ay})$ $\theta_A = -33.741\ deg$

$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2}$ $mR_A = 11.274\ in \cdot lb$

$W_4 := \frac{mR_A \cdot g}{R_A}$ $W_4 = 3.76\ lbf$

PROBLEM 12-16

Statement: Figure P12-6 shows a system with three weights on a rotating shaft. For the given data below, determine the magnitudes and angles of the balance weights needed to dynamically balance the system.

Given: Weights and radii:

$W_1 := 9 \cdot lbf$	$r_1 := 4 \cdot in$	$\theta_1 := 90 \cdot deg$	$l_1 := -14 \cdot in$
$W_2 := 9 \cdot lbf$	$r_2 := 6 \cdot in$	$\theta_2 := 225 \cdot deg$	$l_2 := 4 \cdot in$
$W_3 := 6 \cdot lbf$	$r_3 := 10 \cdot in$	$\theta_3 := 315 \cdot deg$	$l_3 := -4 \cdot in$

Distance between correction planes: $l_B := 14 \cdot in$

Correction weight radii: Plane 4 $R_A := 3 \cdot in$ Plane 5 $R_B := 3 \cdot in$

Solution: See Figure P12-6 and Mathcad file P1216.

1. Resolve the position vectors into *xy* components in the arbitrary coordinate system associated with the freeze-frame position of the linkage chosen for analysis.

$R_{1x} := r_1 \cdot cos(\theta_1)$ $R_{1x} = 0.000\ in$ $R_{1y} := r_1 \cdot sin(\theta_1)$ $R_{1y} = 4.000\ in$

$R_{2x} := r_2 \cdot cos(\theta_2)$ $R_{2x} = -4.243\ in$ $R_{2y} := r_2 \cdot sin(\theta_2)$ $R_{2y} = -4.243\ in$

$R_{3x} := r_3 \cdot cos(\theta_3)$ $R_{3x} = 7.071\ in$ $R_{3y} := r_3 \cdot sin(\theta_3)$ $R_{3y} = -7.071\ in$

2. Solve equations 12.4e for summation of moments about *O*, which is at plane 4.

$$mR_{Bx} := \frac{-(W_1 \cdot R_{1x}) \cdot l_1 - (W_2 \cdot R_{2x}) \cdot l_2 - (W_3 \cdot R_{3x}) \cdot l_3}{l_B \cdot g} \qquad mR_{Bx} = 23.031\ in \cdot lb$$

$$mR_{By} := \frac{-(W_1 \cdot R_{1y}) \cdot l_1 - (W_2 \cdot R_{2y}) \cdot l_2 - (W_3 \cdot R_{3y}) \cdot l_3}{l_B \cdot g} \qquad mR_{By} = 34.788\ in \cdot lb$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane *B* (5). Also, solve for the weight required at the given radius.

$\theta_B := atan2(mR_{Bx}, mR_{By})$ $\theta_B = 56.493\ deg$

$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2}$ $mR_B = 41.721\ in \cdot lb$

$$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} \qquad mR_B = 41.721\ in \cdot lb$$

$$W_5 := \frac{mR_B \cdot g}{R_B} \qquad W_5 = 13.91\ lbf$$

4. Solve equations 12.4c for forces in *x* and *y* directions in plane *A* (4).

$$mR_{Ax} := \frac{-W_1 \cdot R_{1x} - W_2 \cdot R_{2x} - W_3 \cdot R_{3x}}{g} - mR_{Bx} \qquad mR_{Ax} = -27.274\ in \cdot lb$$

$$mR_{Ay} := \frac{-W_1 \cdot R_{1y} - W_2 \cdot R_{2y} - W_3 \cdot R_{3y}}{g} - mR_{By} \qquad mR_{Ay} = 9.822\ in \cdot lb$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane *A* (4).

$$\theta_A := atan2(mR_{Ax}, mR_{Ay}) \qquad \theta_A = 160.194\ deg$$

$$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2} \qquad mR_A = 28.989\ in \cdot lb$$

$$W_4 := \frac{mR_A \cdot g}{R_A} \qquad W_4 = 9.66\ lbf$$

PROBLEM 12-17

Statement: Figure P12-6 shows a system with three weights on a rotating shaft. For the given data below, determine the magnitudes and angles of the balance weights needed to dynamically balance the system.

Given: Weights and radii:

$W_2 := 10 \cdot lbf$	$r_2 := 3 \cdot in$	$\theta_2 := 90 \cdot deg$	$l_2 := 6 \cdot in$
$W_3 := 10 \cdot lbf$	$r_3 := 4 \cdot in$	$\theta_3 := 180 \cdot deg$	$l_3 := 12 \cdot in$
$W_4 := 8 \cdot lbf$	$r_4 := 4 \cdot in$	$\theta_4 := 315 \cdot deg$	$l_4 := 8 \cdot in$

Distance between correction planes: $l_B := 8 \cdot in$

Correction weight radii: Plane 1 $R_A := 4 \cdot in$ Plane 5 $R_B := 3 \cdot in$

Solution: See Figure P12-7 and Mathcad file P1217.

1. Resolve the position vectors into *xy* components in the arbitrary coordinate system associated with the freeze-frame position of the linkage chosen for analysis.

$R_{2x} := r_2 \cdot cos(\theta_2)$	$R_{2x} = 0.000\, in$	$R_{2y} := r_2 \cdot sin(\theta_2)$	$R_{2y} = 3.000\, in$
$R_{3x} := r_3 \cdot cos(\theta_3)$	$R_{3x} = -4.000\, in$	$R_{3y} := r_3 \cdot sin(\theta_3)$	$R_{3y} = 0.000\, in$
$R_{4x} := r_4 \cdot cos(\theta_4)$	$R_{4x} = 2.828\, in$	$R_{4y} := r_4 \cdot sin(\theta_4)$	$R_{4y} = -2.828\, in$

2. Solve equations 12.4e for summation of moments about *O*, which is at plane 1.

$$mR_{Bx} := \frac{-(W_2 \cdot R_{2x}) \cdot l_2 - (W_3 \cdot R_{3x}) \cdot l_3 - (W_4 \cdot R_{4x}) \cdot l_4}{l_B \cdot g} \qquad mR_{Bx} = 37.373\, in \cdot lb$$

$$mR_{By} := \frac{-(W_2 \cdot R_{2y}) \cdot l_2 - (W_3 \cdot R_{3y}) \cdot l_3 - (W_4 \cdot R_{4y}) \cdot l_4}{l_B \cdot g} \qquad mR_{By} = 0.127\, in \cdot lb$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane *B* (5). Also, solve for the weight required at the given radius.

$$\theta_B := atan2(mR_{Bx}, mR_{By}) \qquad \theta_B = 0.195\, deg$$

$_{B \cdot g}$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B (5). Also, solve for the weight required at the given radius.

$$\theta_B := atan2(mR_{Bx}, mR_{By}) \qquad \theta_B = 0.195\ deg$$

$$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} \qquad mR_B = 37.373\ in \cdot lb$$

$$W_5 := \frac{mR_B \cdot g}{R_B} \qquad W_5 = 12.46\ lbf$$

4. Solve equations 12.4c for forces in x and y directions in plane A (1).

$$mR_{Ax} := \frac{-W_2 \cdot R_{2x} - W_3 \cdot R_{3x} - W_4 \cdot R_{4x}}{g} - mR_{Bx} \qquad mR_{Ax} = -20.000\ in \cdot lb$$

$$mR_{Ay} := \frac{-W_2 \cdot R_{2y} - W_3 \cdot R_{3y} - W_4 \cdot R_{4y}}{g} - mR_{By} \qquad mR_{Ay} = -7.500\ in \cdot lb$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A (1).

$$\theta_A := atan2(mR_{Ax}, mR_{Ay}) \qquad \theta_A = -159.444\ deg$$

$$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2} \qquad mR_A = 21.360\ in \cdot lb$$

$$W_1 := \frac{mR_A \cdot g}{R_A} \qquad W_1 = 5.34\ lbf$$