12.2 The Power Rule

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تكامل بالتعويض باستخدام

$\underline{\underline{12.1}} \Rightarrow \int x^n \, dx = \dfrac{x^{n+1}}{n+1} + c$ ✓

$$\int [f(x)]^n f'(x)\, dx$$

المشتقة موجودة

التعويض

$y = f(x)$

$y' = f'(x)$

$\dfrac{dy}{dx} = f'(x)$

$dy = f'(x)\, dx$

$\dfrac{dy}{f'(x)} = dx$

$$\int y^n \cancel{f'(x)} \frac{dy}{\cancel{f'(x)}}$$

$$\int y^n \, dy = \frac{y^{n+1}}{n+1} + c$$

$$= \frac{[f(x)]^{n+1}}{n+1} + c$$

$\underline{\underline{Exp}}$ ①

$$\int (\underbrace{2x^2+4}_{y=f})^4 \underbrace{4x}_{f'} \, dx$$

$y = f(x)$

$= 2x^2 + 4$

$y' = 4x$

$$\int y^4 \, \cancel{4x} \, \frac{dy}{\cancel{4x}}$$

$$\frac{dy}{dx} = 4x$$

$$dy = 4x \, dx$$

$$\frac{dy}{4x} = dx$$

$$\int y^4 \, dy = \frac{y^5}{5} + C$$

$$= \frac{(2x^2+4)^5}{5} + C$$

$$F \Rightarrow F' = \frac{\cancel{5}}{\cancel{5}}(2x^2+4)^4 (4x)$$

$$= (2x^2+4)^4 \, 4x$$

(2) $\int 3\sqrt{\underset{y=f}{3x+1}} \; dx$

$f' = y'$

$y = 3x+1$

$y' = 3$

$$\int \cancel{3}\sqrt{y} \, \frac{dy}{\cancel{3}}$$

$$\frac{dy}{dx} = 3$$

$$dy = 3\,dx$$

$$\frac{dy}{3} = dx$$

$$\int y^{\frac{1}{2}} \, dy = \frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$$

$$= \frac{y^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2}{3}\sqrt[2]{y^3} + c$$

$$= \frac{2}{3}\sqrt{3x+1} + c$$

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3 $\int (\underbrace{3x^2 + 6x}_{y})(\underbrace{x+1}_{\approx y'})\, dx$

$y = 3x^2 + 6x$

$y' = 6x + 6$

$\frac{dy}{dx} = 6(x+1)$

$dy = 6(x+1)dx$

$\frac{dy}{6(x+1)} = dx$

Method 1

$$\int y \cancel{(x+1)} \frac{dy}{6\cancel{(x+1)}}$$

$$\frac{1}{6}\int y\, dy = \frac{1}{6}\frac{y^2}{2} + c$$

$$= \frac{1}{12}(3x^2+6x)^2 + c$$

Method 2

$$\int (3x^2 + 6x)(x+1)\, dx$$

$$\int (3x^3 + 3x^2 + 6x^2 + 6x)\, dx$$

$$\int (3x^3 + 9x^2 + 6x)\, dx = 3\frac{x^4}{4} + \frac{9x^3}{3} + \frac{6x^2}{2} + c$$

$\int (3x^2 + 9x + 6x)$ $\qquad = \frac{3x^4}{4} + \frac{9x^3}{3} + \frac{6x^2}{2}$

$$= \frac{3}{4}x^4 + 3x^3 + 3x^2 + C$$

$$\frac{1}{12}(3x^2 + 6x)^2 + C \overset{?}{=} \frac{3}{4}x^4 + 3x^3 + 3x^2 + C$$

$$\frac{1}{12}\left(9x^4 + 2(3x^2)(6x) + 36x^2\right) + C =$$

$$\frac{\cancel{9}^{3}}{\cancel{12}_{4}}x^4 + \frac{\cancel{36}^{3}x^3}{\cancel{12}} + \frac{\cancel{36}^{3}}{\cancel{12}}x^2 + C = \frac{3}{4}x^4 + 3x^3 + 3x^2 + C$$

(4) $\displaystyle\int \frac{x^2 - 1}{(x^3 - 3x)^3}\,dx$ $\leftarrow \approx y'$

$\underbrace{x^3 - 3x}_{y}$

$y = x^3 - 3x$

$y' = 3x^2 - 3$

$\frac{dy}{dx} = 3(x^2 - 1)$

$dy = 3(x^2 - 1)dx$

$\frac{dy}{3(x^2 - 1)} = dx$

$$\int \frac{\cancel{x^2 - 1}}{y^3} \; \frac{dy}{3\cancel{(x^2 - 1)}}$$

$$\frac{1}{3}\int \frac{dy}{y^3} = \frac{1}{3}\int y^{-3}\,dy$$

$$3 \int \quad y^3 \quad 3 \int$$

$$= \frac{1}{3} \frac{y^{-2}}{-2} + C$$

$$= \frac{-1}{6} \frac{1}{y^2} + C$$

$$= \frac{-1}{6} \frac{1}{(x^3 - 3x)^2} + C$$

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(5) $\int (\underbrace{x^2 + 3}_{y})^2 \, dx$

بدو

$2x$

$y = x^2 + 3$ does not work

$y' = 2x$

$(a+b)^2 = a^2 + 2ab + b^2$

$$\int (x^4 + 2(x^2)(3) + 9) \, dx$$

$$\int (x^4 + 6x^2 + 9) \, dx$$

$$\frac{x^5}{5} + \frac{\cancel{6}^{\,2} x^3}{\cancel{3}} + 9x + C$$

$$\frac{x^5}{5} + 2x^3 + 9x + C$$

Exp Given $\overline{MR} = \frac{-30}{(2x+1)^2} + 30$

x : # of units sold

1) Find total revenue

2) Find total revenue when one unit is being sold

1) $R(x) = \int \overline{MR}\, dx = \int \left(\frac{-30}{(2x+1)^2} + 30 \right) dx$

y

$y = 2x+1$

$y' = 2$

$\frac{dy}{dx} = 2$

$dy = 2\, dx$

$\frac{dy}{2} = dx$

$$= \int \frac{-30}{(2x+1)^2}\, dx + \int 30\, dx$$

$$= \int \frac{\overset{-15}{\cancel{-30}}}{y^2} \frac{dy}{\cancel{2}} + 30x + c$$

$$= -15 \int y^{-2}\, dy + 30x + c$$

$$= -15 \frac{y^{-1}}{-1} + 30x + c$$

$\overline{-1}$

$$R(x) = 15\,\frac{1}{y} + 30x + C$$

$R(x) = px$

$\boxed{P(0) = 0}$

$$R(x) = \frac{15}{2x+1} + 30x + C$$

To find $C \Rightarrow$ we use $R(0) = 0$

$$R(0) = \frac{15}{2\boxed{0}+1} + 30\,\boxed{0} + C$$

$$0 = 15 + 0 + C$$

$$\boxed{-15 = C}$$

$$\boxed{R(x) = \frac{15}{2x+1} + 30x - 15}$$

2

$$R(1) = \frac{15}{2\boxed{1}+1} + 30\,\boxed{1} - 15$$

$$= \frac{15}{3} + 30 - 15$$

$$= 5 + 15$$

$= \$20$

Power Rule for Integration

$$\int y^n \, dy = \frac{y^{n+1}}{n+1} + C$$

$$\int [f(x)]^n \, f'(x) \, dx$$

$y = f(x)$

$y' = f'(x)$

$\frac{dy}{dx} = f'(x)$

$dy = f'(x) \, dx$

$$\int y^n \, dy$$

$$\frac{y^{n+1}}{n+1} + c$$

$n \neq -1$

$$\frac{(f(x))^n}{n+1} + C$$

If $n = -1 \Rightarrow \int x^{-1}\, dx = \int \frac{1}{x}\, dx$

$= \ln|x| + C$