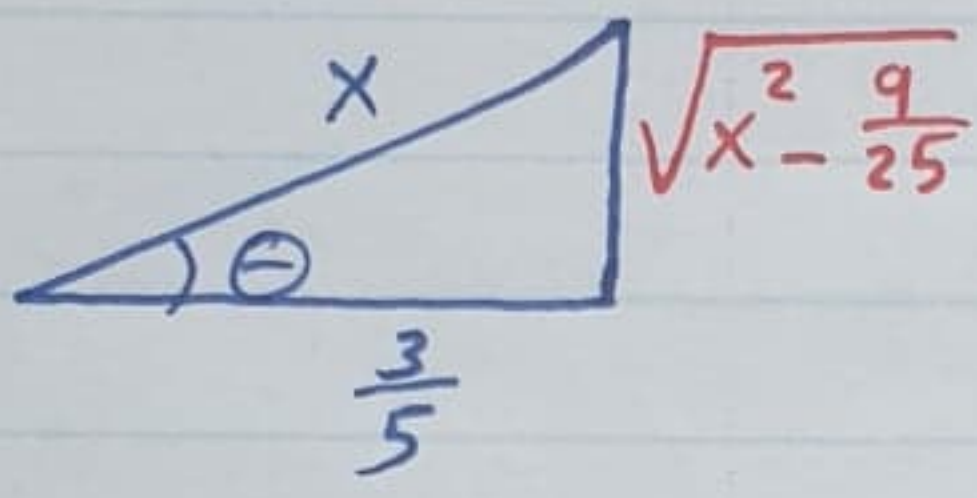


Discussion 8.3

⑩ $\int \frac{5 dx}{\sqrt{25x^2 - 9}} = \int \frac{5 dx}{\sqrt{25(x^2 - \frac{9}{25})}} = \int \frac{dx}{\sqrt{x^2 - \frac{9}{25}}}$

$x = \frac{3}{5} \sec \theta$
 $dx = \frac{3}{5} \sec \theta \tan \theta d\theta$

$\sqrt{x^2 - \frac{9}{25}} = \frac{3}{5} \tan \theta$



$= \int \frac{\frac{3}{5} \sec \theta \tan \theta d\theta}{\frac{3}{5} \tan \theta} = \int \sec \theta d\theta$

$= \ln |\sec \theta + \tan \theta| + C$

$= \ln \left| \frac{x}{\frac{3}{5}} + \frac{\sqrt{x^2 - \frac{9}{25}}}{\frac{3}{5}} \right| + C$

$= \ln \left| \frac{5x}{3} + \frac{5}{3} \sqrt{x^2 - \frac{9}{25}} \right| + C$

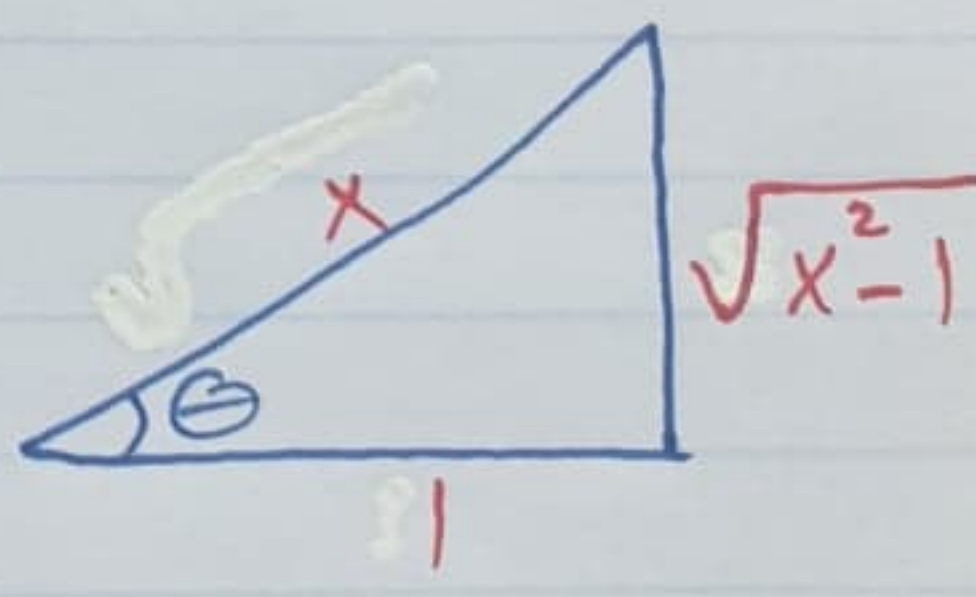
⑫ $\int \frac{x^2 dx}{(x^2 - 1)^{\frac{5}{2}}}, x > 1$

$x = \sec \theta$
 $dx = \sec \theta \tan \theta d\theta$
 $\sqrt{x^2 - 1} = \tan \theta$
 $(x^2 - 1)^{\frac{5}{2}} = \tan^5 \theta$

$= \int \frac{\sec^2 \theta \sec \theta \tan \theta d\theta}{\tan^5 \theta}$

$= \int \frac{\sec^3 \theta}{\tan^4 \theta} d\theta = \int \frac{1}{\cos^3 \theta} \frac{\cos^4 \theta}{\sin^4 \theta} d\theta$

$= \int \frac{\cos \theta}{\sin^4 \theta} d\theta$ $u = \sin \theta$
 $du = \cos \theta d\theta$



$= \int \frac{du}{u^4} = \int u^{-4} du = \frac{u^{-3}}{-3} + C = -\frac{1}{3} \frac{1}{\sin^3 \theta} + C$

$= -\frac{1}{3} \csc^3 \theta + C = -\frac{1}{3} \frac{x^3}{(x^2 - 1)^{\frac{3}{2}}} + C$