

## CH600 Three-phase Induction Motor

\*

① The distinguishing feature of induction motor is that no field circuit current is required to run the machine ① advantage

② The construction of Induction motor.

- it has the same stator construction of synchronous motor, with different  $\downarrow$  construction  
Rotor

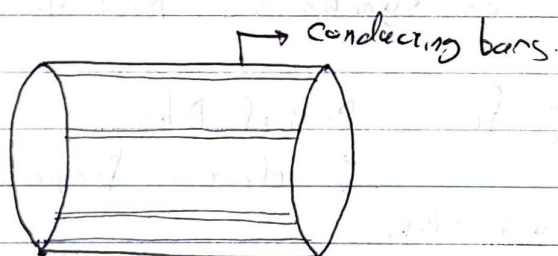
- The induction motor has two rotor types:

① Squirrel cage rotor

② Wound rotor.

① Cage Rotor

it consists of series conducting bars laid into slots carved by large short rings at either end.

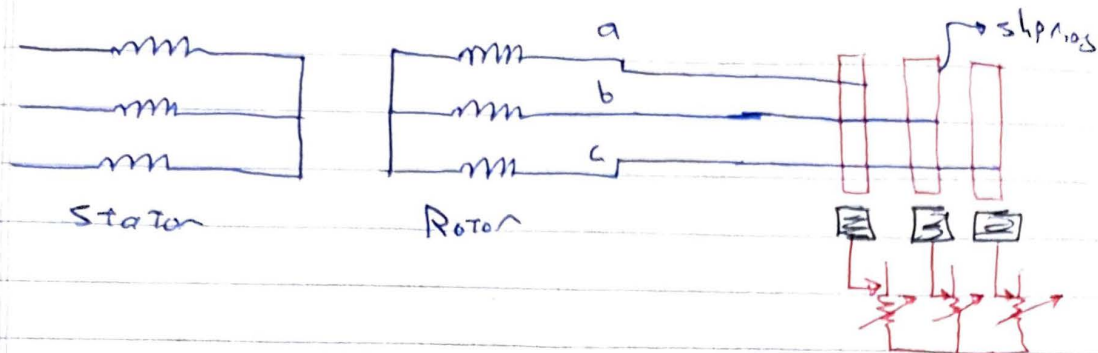


in the face of rotor and shorted at either

② Wound Rotor

its consists of a set of 3 $\phi$  windings that are mirror to the stator winding. The winding are usually Y-connected. The terminal a, b, c are tied to the sliprings on the rotor's shaft.

The terminal are shorted via brushes.

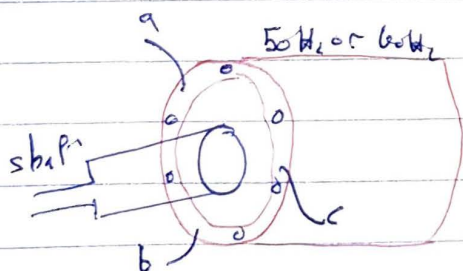


- Notes:**
- In wound rotor Type, an external resistor can be added to the rotor circuit to modify the Torque-speed characteristic.
  - The wound Rotor is more expensive & it requires much more maintenance.

### \* Operating Principle of Induction motors.

- ⊕ A set of three- $\phi$  voltage is applied to the ~~power~~ stator windings to cause a set of 3- $\phi$  current to flow in stator windings.
- ⊕ The current will produce a rotating stator magnetic field  $\vec{B}_s$  which rotates at synchronous speed  $\omega_s$ . where  $\omega_s$  is given by
 
$$\omega_s = \frac{120}{P} f_e$$

$P$ : # of Poles  
 $f_e$ : electric frequency



- ⊕ The Rotating magnetic field will pass over the conducting ~~part~~ bars and induces voltage on them.
 
$$e_{ind} = (\vec{V} \times \vec{B}_s) \cdot \vec{L}$$

where  $\vec{V}$  is relative velocity of bars with respect to the speed of  $\vec{B}_s$

- ⊗ The induced voltage will produce a rotor current  $\vec{I}_r$
- ⊗  $\vec{I}_r$  produces  $\vec{B}_r$
- ⊗  $\vec{B}_r$  interact with  $\vec{B}_s$  to produce the induced Torque in the machine  

$$T_{ind} = K \vec{B}_r \times \vec{B}_s$$

Note: The Rotor can speed up to synchronous speed, but it can never reach it [to keep emf to]

Speed of  $\vec{B}_s$ :  $\omega_s$

speed of  $\vec{B}_r$ :  $\omega_r$

speed of Rotor:  $\omega_m$

, where  $\omega_m < \omega_s$

### ⊗ Concept of slip

slip speed  $\omega_a$  is the difference between  $\omega_s$  & rotor speed

$$\omega_a = \omega_s - \omega_m \quad \text{OR} \quad \omega_a = \omega_s - \omega_m$$

synchronous speed
mechanical speed

slip ( $\alpha$ ) is the Ratio of slip speed and synchronous speed

$$\alpha = \frac{\omega_a}{\omega_s} \Rightarrow \boxed{\omega_a = \alpha \omega_s}$$

$$\alpha = \frac{\omega_a}{\omega_s} \Rightarrow \boxed{\omega_a = \alpha \omega_s}$$

$$\alpha = \frac{\omega_a}{\omega_s} = \frac{\omega_s - \omega_m}{\omega_s} = \frac{\omega_s - \omega_m}{\omega_s} \Rightarrow \alpha \omega_s = \omega_s - \omega_m$$

$$\boxed{\omega_m = \omega_s (1 - \alpha)}$$

$$\omega_s \alpha = \omega_s - \omega_m$$

$$\omega_m = \omega_s (1 - \alpha)$$

## Rotor frequency ( $f_r$ )

when the Rotor is locked ( $\omega_m = 0, a = 1$ )  $\Rightarrow f_r = f_e$   
when the Rotor turns at  $\omega_s$  ( $a = 0$ )  $\Rightarrow f_r = 0$   
at any other speed than  $\omega_s$ , the rotor frequency is proportional to the slip

$$f_r = a f_e$$

$$\omega_s = \frac{120}{P} f_e \Rightarrow \left[ f_e = \frac{P \omega_s}{120} \right], f_r = (a) \left( \frac{P}{120} \right) \omega_s$$

$$f_r = \frac{P}{120} \omega_m = \frac{P}{120} (\omega_s - \omega_m)$$

Ex

A 208 V, 10 hp, four pole, 60 Hz,  $\Delta$ -connected induction motor has a full load slip of 5%.

- what is the synchronous speed of this motor?
- what is the Rotor speed of this motor?
- what is the Rotor frequency of this motor at rated speed?
- what is the shaft torque of this motor at rated load?

Solution a)  $\omega_s = \frac{120}{P} f_e = 1200 \text{ rpm}$

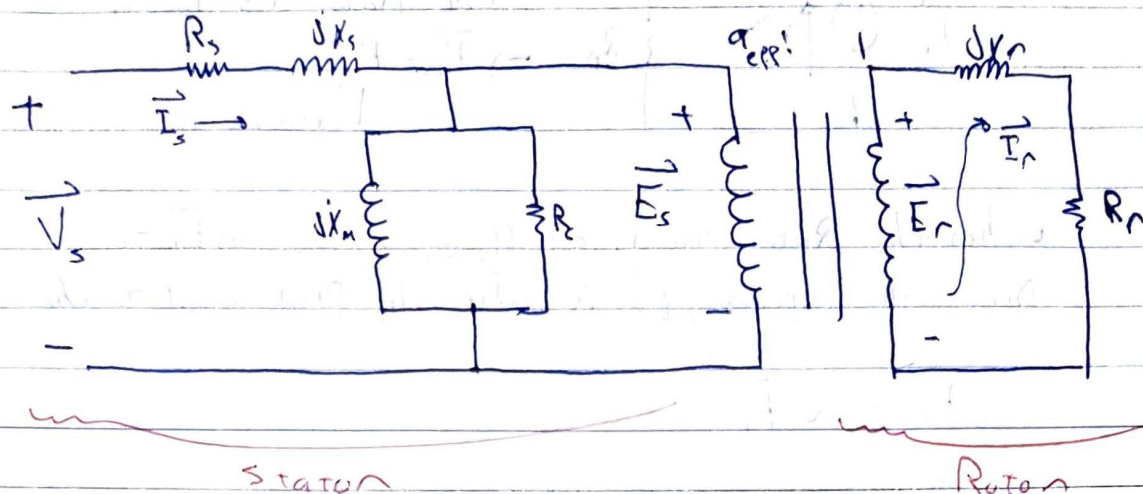
b)  $\omega_m = (1 - a) \omega_s = (1 - 0.05)(1200) = 1140 \text{ rpm}$

c)  $f_r = a f_e = (0.05)(60) = 3 \text{ Hz}$

d)  $T_r = \frac{P_r}{\omega_m} = \frac{(10)(746)}{\frac{(1140)(2\pi)}{60}} = 41.7 \text{ N.m}$

## Equivalent circuit of Induction Motors:

its very similar to the per-phase equivalent circuit of Transformer.



$R_s$  % Resistance of stator winding

$R_r$  % Resistance of rotor bar or windings

$X_s$  % stator Leakage Reactance

$X_r$  % rotor Leakage Reactance

$X_m$  % Magnetizing Reactance

$R_c$  % Resistance of core accounting for eddy current & hysteresis

$a_{eff}$  % its the effective turns ratio of the motor which couples between  $\vec{E}_s$  &  $\vec{E}_r$ .

Note % we can easily find it in the core of wound Rotor (approx)

$\vec{E}_s$  % Internal stator Voltage

$\vec{E}_r$  % rotor induced voltage

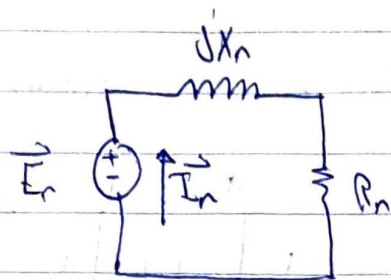
$\vec{I}_r$  % rotor current

$\vec{I}_s$  % Stator or source phase current

$\vec{V}_s$  % Stator or source Phase Voltage

Frequency of rotor is same as stator frequency - eq circuit is same

## Rotor Circuit Model



For  $\vec{E}_r \neq 0$

The Rotor is locked

$$\Rightarrow \vec{E}_r = \vec{E}_{r0}$$

$\hookrightarrow$  maximum Motor Voltage

when the Rotor turns at  $\omega_s \Rightarrow \alpha = 0 \Rightarrow \vec{E}_r = 0$

Any other speed,  $\vec{E}_r$  is directly proportional to the slip

$$\boxed{\vec{E}_r = \alpha \vec{E}_{r0}}$$

For  $\vec{X}_r$

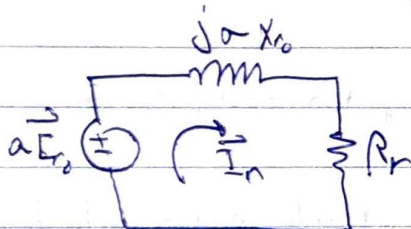
$$X_r = \omega L_r = 2\pi f_r L_r = 2\pi (\alpha f_e) L_r = \alpha X_{r0}$$

$$\text{where } X_{r0} = 2\pi f_e L_r$$



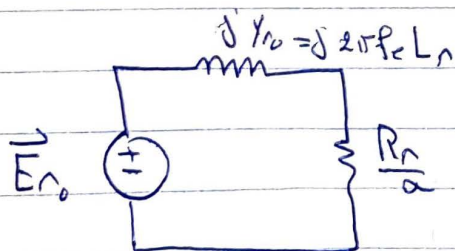
Maximum leakage rotor reactance

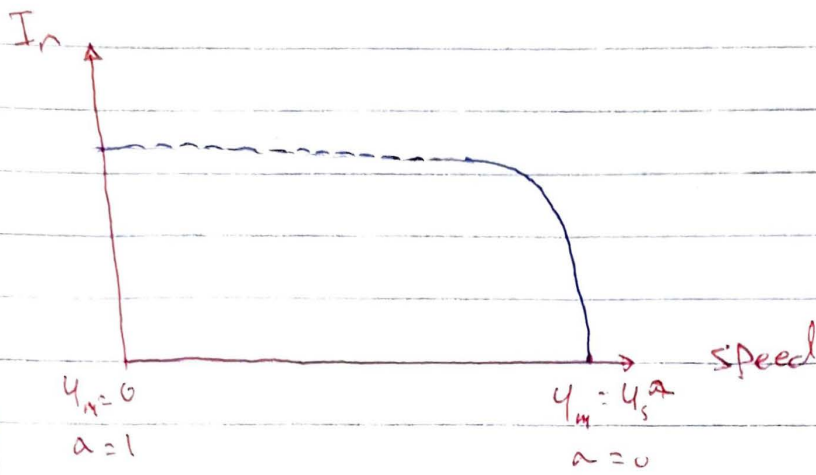
So



By applying KVL in the Rotor circuit

$$\vec{I}_r = \frac{\alpha \vec{E}_{r0}}{R_r + (jX_{r0})\alpha} \Rightarrow \vec{I}_r = \frac{\vec{E}_{r0}}{\frac{R_r}{\alpha} + jX_{r0}}$$





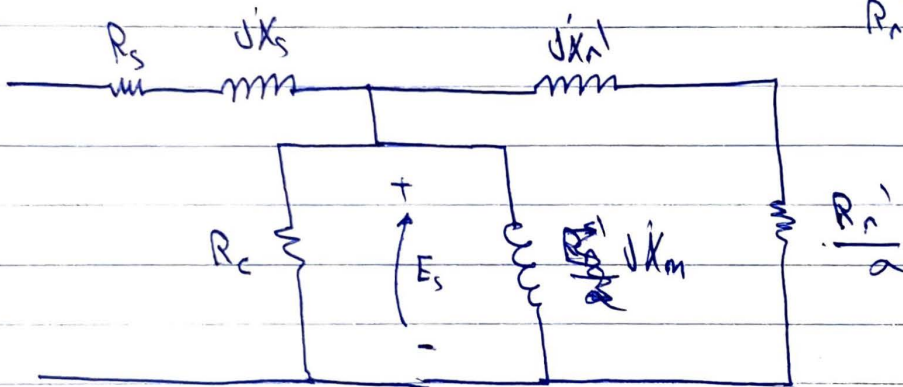
when  $a$  is small  $I_r = \frac{\vec{E}_r}{\frac{R_r}{a} + jX_r} \approx \frac{\vec{E}_r}{\frac{R_r}{a}} \approx \frac{a \vec{E}_r}{R_r}$

**Note** At starting the Rotor current is usually high compared with the motor nominal current.

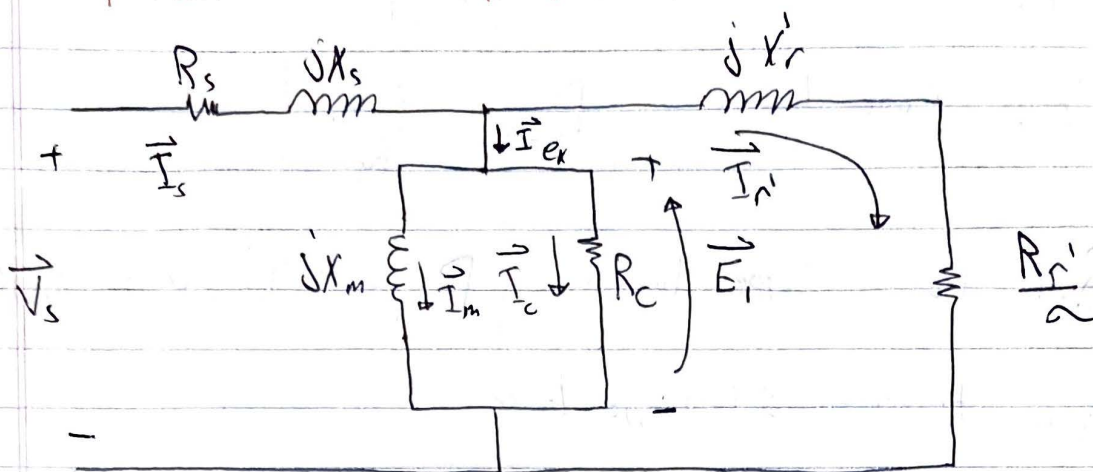
**Final equivalent circuit**

$$X_r' = \frac{X_r}{a} = a_{eff}^2 X_r$$

$$R_r' = \frac{R_r}{a} = a_{eff}^2 R_r$$



## Equivalent Circuit of Induction Motor.

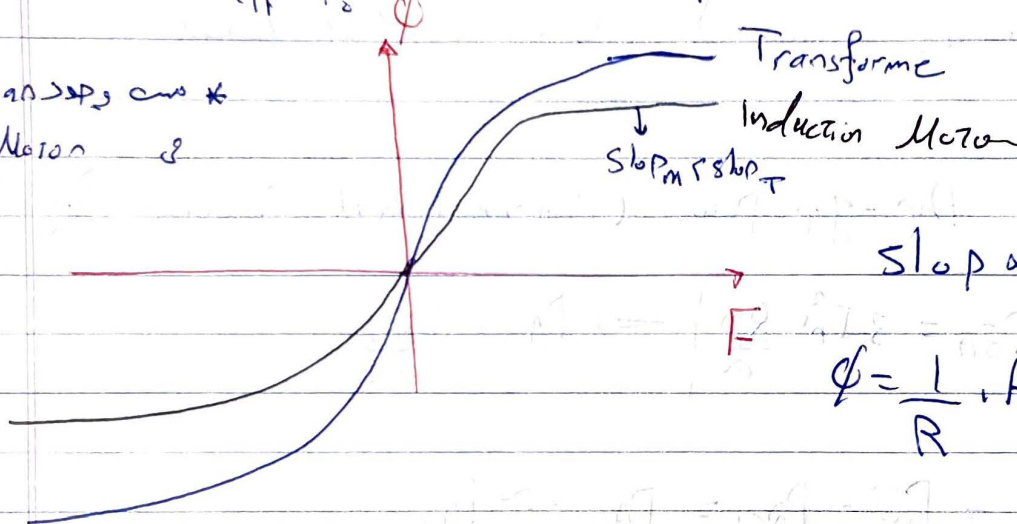


$$R_r' = a_{eff}^2 R_r, \quad X_r' = a_{eff}^2 X_r, \quad \vec{I}_r' = \frac{1}{a_{eff}} \vec{I}_r$$

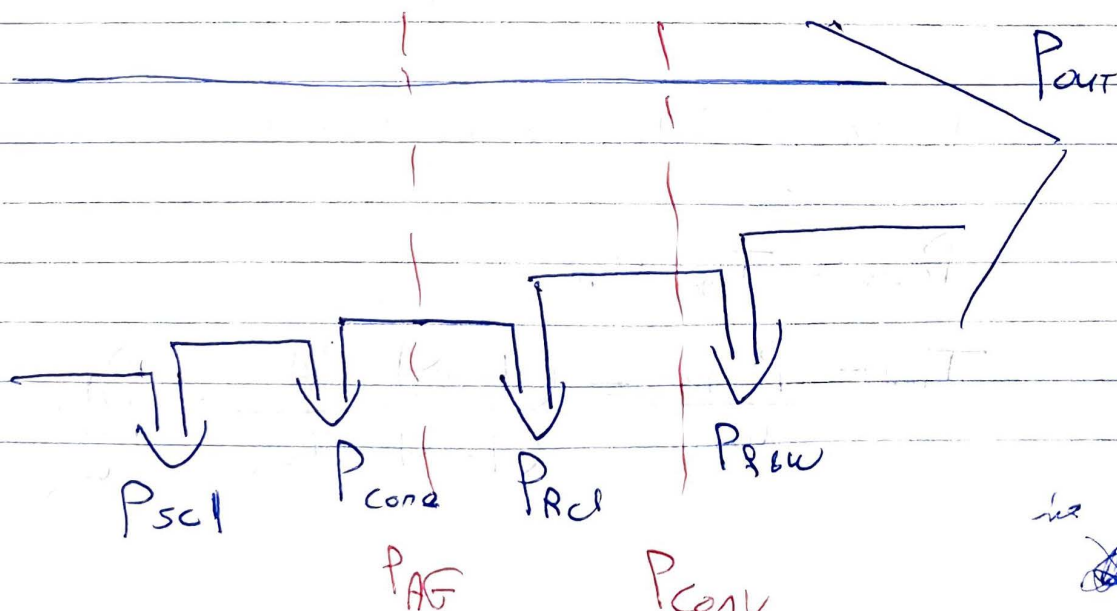
$$\vec{E}_1 = a_{eff} \vec{E}_r$$

$\omega$ , slip

Angle between  $\vec{V}_s$  and  $\vec{I}_s$   
Motor  $\phi$



## Power calculations



$P_{sc1}$ : Stator copper losses

$$P_{sc1} = 3 I_{sc}^2 R_s$$

$P_{core}$ : Core losses

$$P_{core} = \frac{3 E_1^2}{R_c}$$

$P_{rc1}$ : Rotor copper losses

$$P_{rc1} = 3 I_r'^2 R_r' = 3 I_r'^2 R_r$$

$P_{f&w}$ : Friction & windage losses

$P_{conv}$ : converted power to mechanical

$$P_{conv} = T_{ind} \omega_m$$

Induced Torque  $\leftarrow$   $\leftarrow$  Rotor (Mechanical speed)

$P_{AG}$ : Air-gap power (consumed in the Resistor  $\frac{R_r'}{s}$ )

$$P_{AG} = 3 I_r'^2 \frac{R_r'}{s} \Rightarrow P_{AG} = \frac{P_{rc1}}{s}$$

$$P_{conv} = P_{AG} - P_{rc1} = P_{AG} - s P_{AG}$$

$$P_{conv} = (1-s) P_{AG}$$

$$P_{out} = T_{load} \omega_m$$

$$P_{out} = P_{conv} - P_{f&w}$$

Induced Torque equation

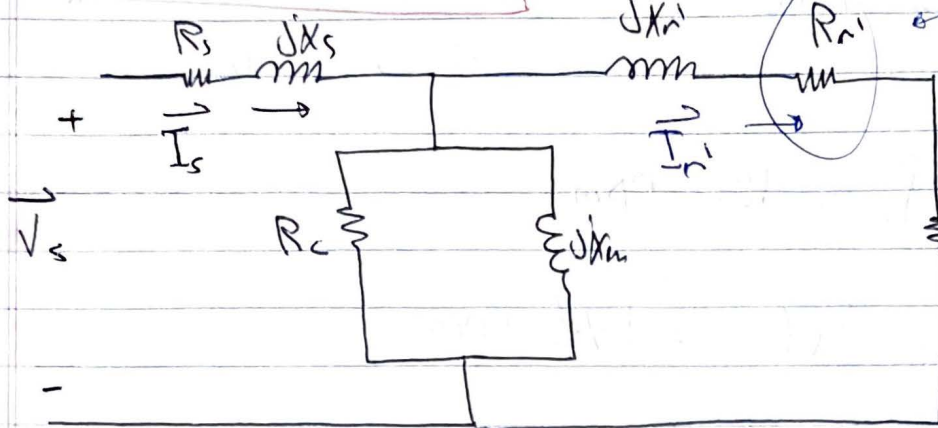
$$P_{conv} = T_{ind} \omega_m$$

$$T_{ind} = \frac{P_{conv}}{\omega_m} = \frac{(1-s) P_{AG}}{(1-s) \omega_s} = \frac{P_{AG}}{\omega_s}$$

$$T_{ind} = \frac{3 I_r'^2 R_r'}{\omega_s}$$

Lev AP  $\cos \phi$

$$\frac{R_r'}{a} = \frac{R_r'}{a} - R_r' \cdot \frac{R_r'}{a}$$



$$\left(\frac{1-a}{a}\right) R_r' = R_{conv}$$

Converted to Power

$$P_{conv} = (1-a) P_{AG} = (1-a) \frac{3 I_r'^2 R_r'}{a}$$

Motor efficiency

$$= 3 I_r'^2 R_{conv}, \quad R_{conv} = \frac{1-a}{a} R_r'$$

$$\eta = \frac{P_{out}}{P_{in}}$$

$$P_{in} = 3 V_s I_s \text{ PF} = \sqrt{3} V_L I_L \text{ PF}$$

Rotor efficiency

$$\eta_r = \frac{P_{conv}}{P_{AG}} = \frac{(1-a) P_{AG}}{P_{AG}} = 1-a$$

Example A 460V, 25 hp, 60 Hz, 4 poles, Y-connected induction motor has the following impedances in ohms, per phase Refered to the stator circuit.

$$R_s = 0.641 \, \Omega, \quad R_r' = 0.332 \, \Omega$$

$$X_s = 1.106 \, \Omega, \quad X_r' = 0.464 \, \Omega, \quad X_m = 26.3 \, \Omega$$

The Total Rotational losses are 1.1 kW and assumed to be constant, for a Rotor slip of 2.2% at the Rated Voltage & Rated frequency, find the Motor's

- a) speed b) current c) PF d)  $P_{conv}$  &  $P_{AO}$

a) Speed,  $\omega_m$

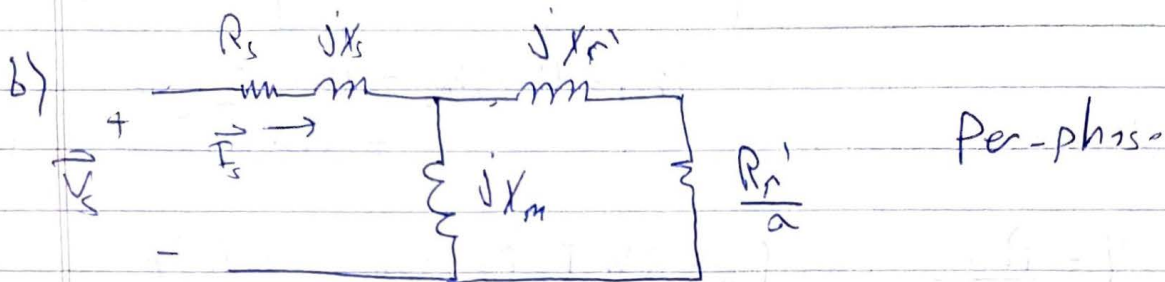
$$a = \frac{\omega_s - \omega_m}{\omega_s} = 1 - \frac{\omega_m}{\omega_s}$$

$$\omega_m = (1 - a)\omega_s$$

$$\omega_m = (1 - a)\omega_e$$

$$\omega_s = \frac{120}{P} f_e = 1800 \text{ rpm}$$

$$\omega_m = (1 - 0.022) \times 1800 = 1760 \text{ rpm}$$



$$\vec{I}_s = \frac{\vec{V}_s}{Z_{eq}}, \quad Z_{eq} = 14.07 \angle 33.6^\circ \Omega$$

$$\vec{I}_s = \frac{(460/\sqrt{3}) \angle 0^\circ}{14.07 \angle 33.6^\circ} \Rightarrow \vec{I}_s = 18.28 \angle -33.6^\circ \text{ A}$$

c)  $PF = \cos(\theta_{V_s} - \theta_{I_s}) = \cos(0 - (-33.6))$

$$= \cos(33.6) = 0.833 \text{ lagging}$$

d) we can use current divide to find  $I_r$  but there is easiest way.

$$P_{in} = 3 V_s I_s PF = 3 \left( \frac{460}{\sqrt{3}} \right) (18.28) (0.833) = 12.53 \text{ kW}$$

$$P_{sc1} = 3 I_s^2 R_s = 3 (18.28)^2 (0.641)$$

$$P_{sc1} = 0.685 \text{ kW}$$

$$P_{AG} = P_{in} - P_{core} - P_{sc1} = 12.53 - 0.681 \Rightarrow P_{AG} = 11.845 \text{ kW}$$

$$P_{conv} = (1 - a) \cdot P_{AG} = 11.525 \text{ kW}$$

e)

$$P_{conv} = T_{ind} \omega_m$$

$$T_{ind} = \frac{P_{conv}}{\omega_m} = \frac{11.525}{1760 \frac{\pi}{30}} = \frac{56.9}{62.8} \text{ N.m}$$

$$P_{out} = T_{Load} \omega_m$$

$$P_{out} = P_{conv} - P_{fe} = 11.525 - 1.1 = 10.425 \text{ kW}$$

$$T_{Load} = \frac{10.425}{1760 \left( \frac{\pi}{30} \right)} = 56.9 \text{ N.m}$$

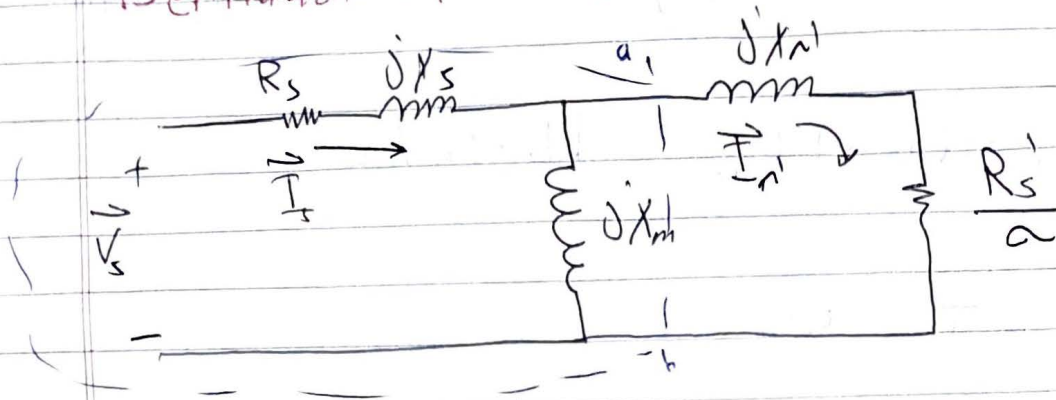
f)

$$\eta = \frac{P_{out}}{P_{in}} = \frac{10.425}{12.53} = 83.7\%$$

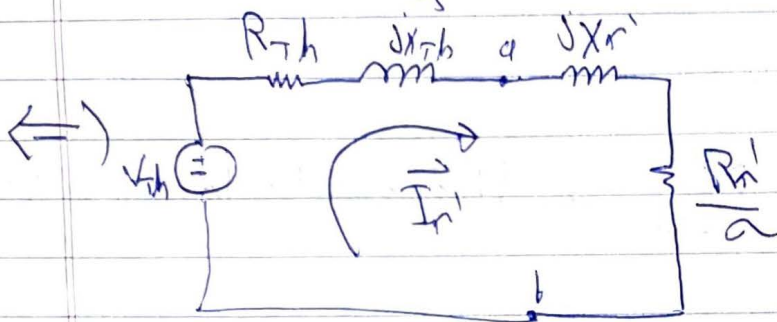
$$\eta_r = 1 - a = 1 - 0.022$$

درد اینجانب را در مورد Torque و الی و الی Parameters (موجوده باله افه)

## Derivation of Induced Torque equation



$$T_{ind} = \frac{3 I_n'^2 R_n'}{a \omega_s} = \frac{P_{AG}}{\omega_s}$$



$$\vec{V}_{th} = \vec{V}_\infty = \frac{jX_m}{R_s + j(X_s + X_m)} \cdot \vec{V}_s$$

$$X_m + X_s \gg R_s \Rightarrow \vec{V}_{th} \approx \frac{X_m}{X_m + X_s} \cdot \vec{V}_s$$

$$Z_{th} = (R_s + jX_s) \parallel jX_m$$

$$Z_{th} = \frac{(R_s + jX_s)(jX_m)}{R_s + j(X_s + X_m)}$$

$$X_s + X_m \gg R_s \Rightarrow Z_{th} \approx \frac{(R_s + jX_s)(jX_m)}{j(X_s + X_m)}$$

$$Z_{Th} = R_s \left( \frac{X_m}{X_m + X_s} \right) + j X_s \left( \frac{X_m}{X_s + X_m} \right) = R_{Th} + j X_{Th}$$

$$R_{Th} = R_s \left( \frac{X_m}{X_m + X_s} \right) \approx R_s, \quad X_{Th} = X_s \left( \frac{X_m}{X_m + X_s} \right) \approx X_s$$

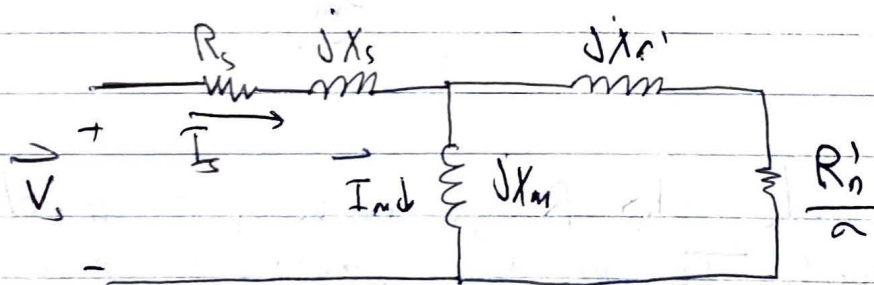
$$\vec{I}_r' = \frac{\vec{V}_{Th}}{(R_{Th} + \frac{R_r'}{a}) + j(X_{Th} + X_r')}$$

$$I_{r'} = \frac{V_{Th}}{\sqrt{(R_{Th} + \frac{R_r'}{a})^2 + (X_{Th} + X_r')^2}}$$

done

$$T_{ind} = \frac{(3) V_{Th}^2}{\omega_s \left[ \left( R_{Th} + \frac{R_r'}{a} \right)^2 + (X_{Th} + X_r')^2 \right]} \cdot \frac{R_r'}{a}$$

Induced Torque equation



Per-Phase equivalent circuit

$$T_{ind} = \frac{P_{AG}}{\omega_s}; \quad P_{AG} = 3 I_{r'}^2 \frac{R_r'}{a}$$

$$T_{ind} = \frac{3 I_{r'}^2 R_r'}{a \omega_s}$$

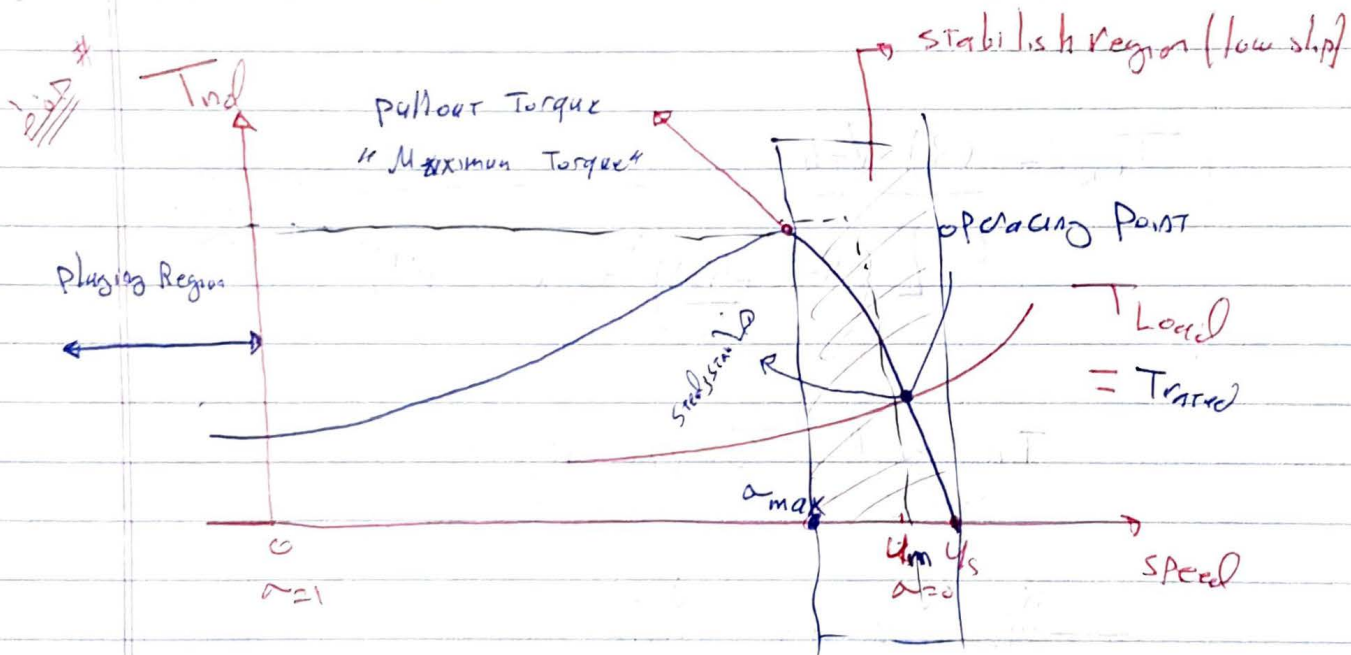
$$\vec{V}_{Th} = \frac{jX_m}{R_s + j(X_m + X_l)} \cdot \vec{V}_s$$

$$V_{Th} = X_m$$

$$T_{ind} = \frac{3 V_{Th}^2}{\omega_s \left[ R_{Th} + \frac{R_r'}{a} \right]^2 + (X_{Th} + X_r')^2} \cdot \frac{R_r'}{a}$$

$$T_{ind} = f(\alpha) = g(\gamma_m)$$

$$\gamma_m = (1 - \alpha) K$$



Note: when the slip Range is very small, the Induced Torque is approximately proportional to the slip (linear relationship)

$$T_{ind} \approx \frac{3 V_{Th}^2}{\omega_s \left[ \left( \frac{R_r'}{a} \right)^2 \right]} \cdot \frac{R_r'}{a} = \frac{3 V_{Th}^2}{\omega_s R_r'} \cdot a = K a$$

$$T_{ind} - T_{load} = J \frac{d\omega}{dt}$$

under steady state (speed = constant)

$$\Rightarrow T_{ind} = T_{load}$$

## Plugging Region

its used to stop the motor rapidly by switching any two of 3 phases.

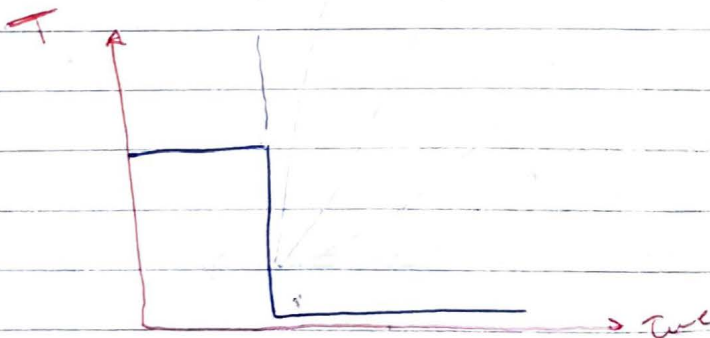
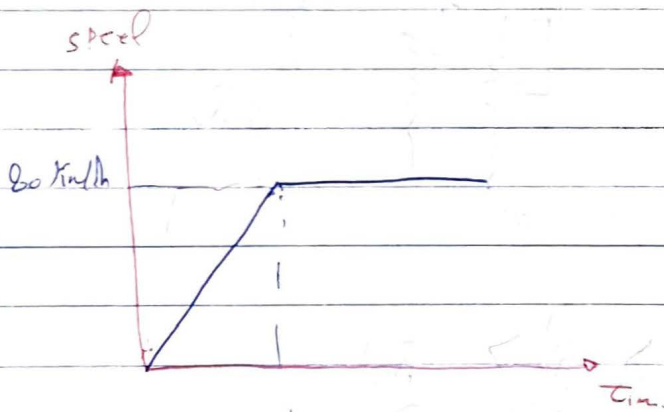
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$T_{start}$  Starting Torque

$T_{start}$  is slightly higher than the motor rated Torque

$T_{max}$  = pullout or maximum Torque.

$$T_{max} = (2-3) T_{rated}$$



$$\frac{dT_{ind}}{da} = 0$$

$$a_{max} = \frac{R_r'}{\sqrt{R_{th}^2 + (X_{th} + X_r')^2}}$$

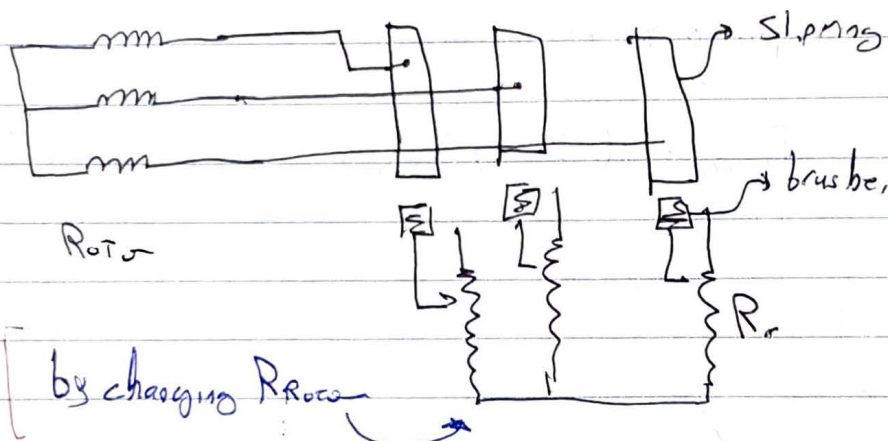
①

عبر

$$T_{max} = \frac{3 V_{th}^2}{2 \omega_s \left[ R_{th} + \sqrt{R_{th}^2 + (X_{th} + X_r')^2} \right]}$$

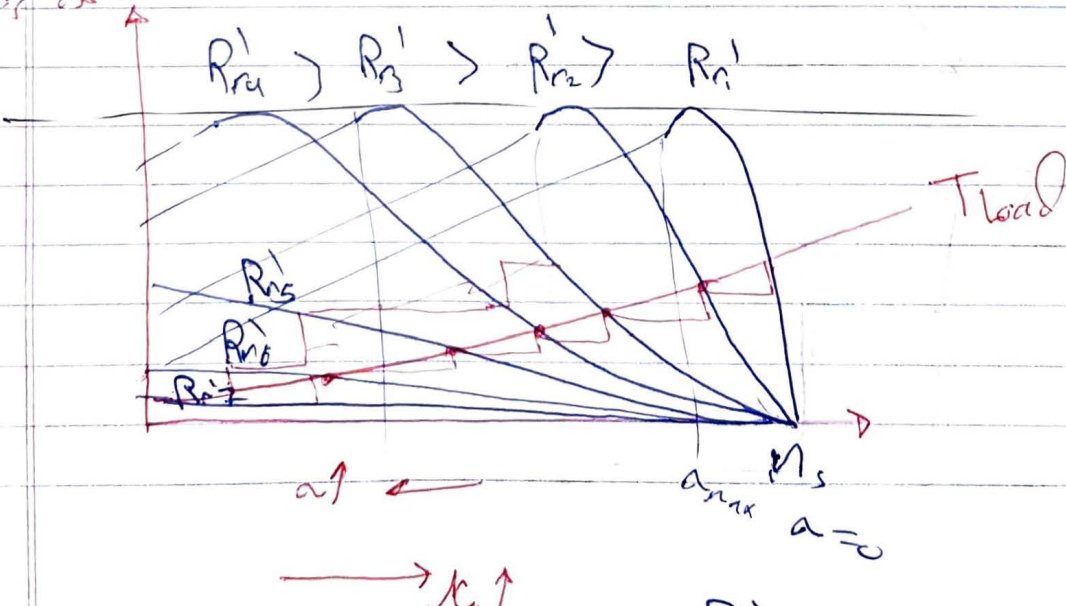


Wound Rotor Induction Motor مع هذا النوع دة للانسجة ان اعبر  $R_r$  فيه



امثلة على  
التيار المتغير  
في المحرك  
عند تغيير  
مقاومة  
الدور

by changing  $R_{rotor}$



$$① \quad R_{r1}' < R_{r1}' < R_{r2}' \Rightarrow R_{r1}' \uparrow \Rightarrow a_{max} \uparrow$$

$$a \uparrow \Rightarrow \text{speed} \downarrow \Rightarrow (1 - a) \downarrow$$

$$\text{efficiency} \Rightarrow \eta \downarrow \Rightarrow T_{start} \uparrow$$

$$② \quad R_{r1}' > R_{r2}' \Rightarrow T_{start} \downarrow \Rightarrow \eta \downarrow$$

Example A 460 V, 25 hp, 4 pole, Y-connected, wound Rotor IM has the following impedances

$$R_s = 0.641 \, \Omega, \quad R_{r1}' = 0.332 \, \Omega$$

$$X_m = 1.106 \, \Omega, \quad X_{r1}' = 0.464 \, \Omega, \quad X_m = 26.1 \, \Omega$$

- What is  $T_{max}$ ? At what speed does it occur?
- What is  $T_{start}$ ?
- If  $R_{r1}'$  is doubled, what is the speed at which  $T_{max}$  occurs? What is the new  $T_{start}$ ?

Solution

$$a) \quad T_{max} = \frac{3 \sqrt{1.5}}{2 \omega_s [R_{th} + \sqrt{R_{th}^2 + (X_{th} + X_m)^2}]}$$

$$R_{th} = \left( \frac{Y_m}{X_m + X_s} \right) R_s \approx R_s, \quad X_{th} \approx X_s$$

$$V_{th} = \frac{X_m}{\sqrt{R_s^2 + (X_{st} + X_m)^2}} \cdot V_s$$

$$f = 60 \text{ Hz}$$

$$R_{Th} = 0.59 \Omega \quad X_{Th} = X_s = 1.06 \Omega \quad V_{Th} = \frac{4.8 \angle 0^\circ}{\sqrt{3}}$$

$$\omega_s = \left(\frac{2}{p}\right) \omega_e = 188.5 \text{ rad/s} = 255.2 \text{ Volt}$$

$$T_{max} = 229 \text{ N.m}$$

$$a_{max} = \frac{R_n'}{\sqrt{R_{Th}^2 + (X_{Th} + X_n')^2}} = \frac{0.332}{\sqrt{\quad}} = 0.198$$

$$Y_m = (1 - a_{max}) \quad Y_s = 1444 \text{ rpm}$$

b) max when  $\boxed{a=1}$   $T = \frac{g \sqrt{h}}{\text{Start}} \cdot \frac{R_n'}{a}$   
 $w_s \left[ \frac{R_n'}{a} + R_n h + (K_n + K_n')^2 \right]$

$$T_{\text{start}} = 164 \text{ N.m}$$

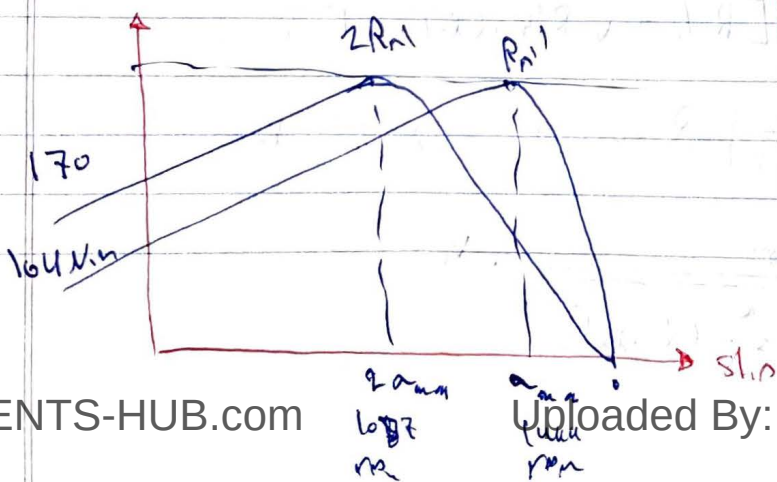
c)  $a_{\max} = \underline{(2)(0.141)} = 0.282$  Since  $R_{n2}' = 2R_1'$

$$y_m = (1 - a_{u_{4x}}) y_s = 10.87 \text{ rpm}$$

only  $R_{n1} = 2R_{n1}$

$$T_{\text{start}} = \frac{(3)(255.2)^2}{\dots} = 170 \text{ N.m}$$

$$186.5 [(0.332 \times 2) + (0.591)^2] + \dots$$



Matlab

$$K = 0,000000001, 1$$

$$R_{TH} = 1 \Omega$$

$$x_{1/2} = 9'$$

$$R_p' = 2 \times 10^{-2}$$

$\theta = -$

$y = \text{---}$  plot  $(x, y)$

## Calculation of starting current

$$I_{L\text{START}} = \frac{S_{\text{START}}}{\sqrt{3} V_L}$$

$$S_{\text{START}} = \text{Rated power} \times \text{code letter factor} \\ (\text{hp})$$

| Nominal code letter | Locked Rotor [KVA/hp] |
|---------------------|-----------------------|
| A                   | 0 - 3.15              |
| B                   | 3.15 - 3.55           |
| C                   | 3.55 - 4.00           |
| D                   | 4.00 - 4.50           |
| E                   | 4.50 - 5              |
| F                   | 5 - 5.6               |
| :                   | :                     |

Ex What is the starting current of a 15 hp, 208V, code letter 3φ induction motor?

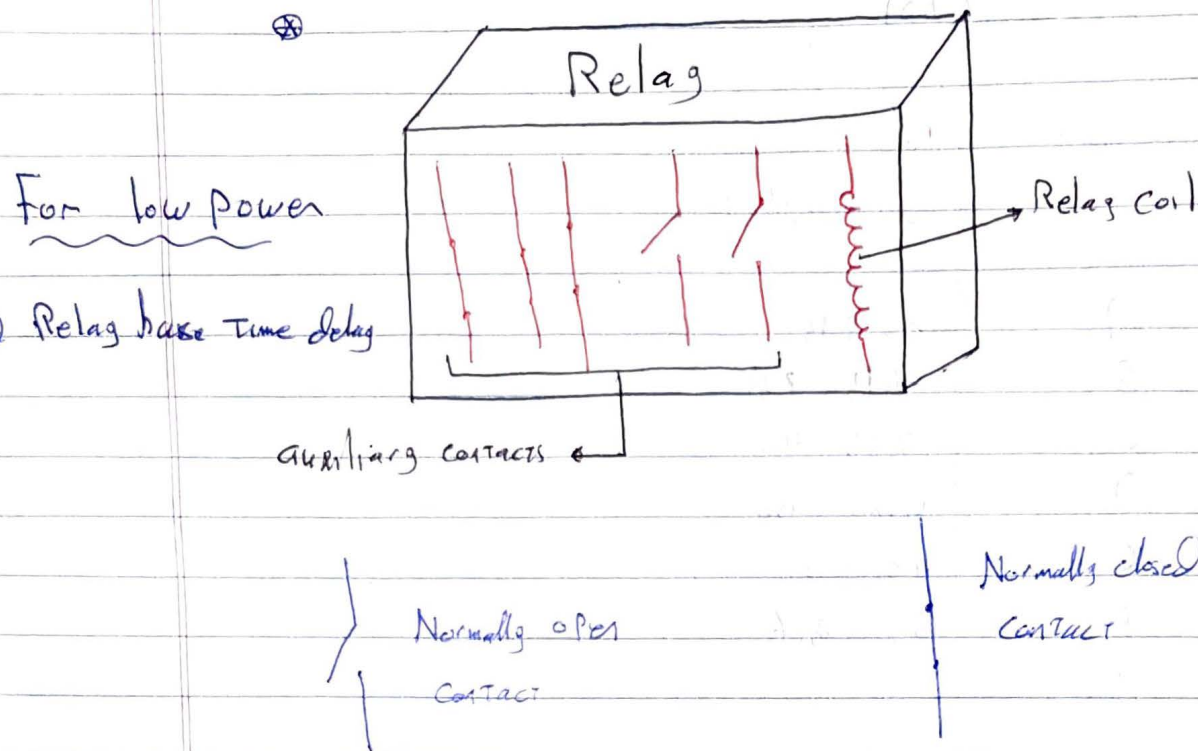
$$S_{\text{START}} = (15)(5.6) = 84 \text{ KVA}$$

$$I_{L\text{START}} = \frac{S_{\text{START}}}{\sqrt{3} V_L} = 233 \text{ A}$$

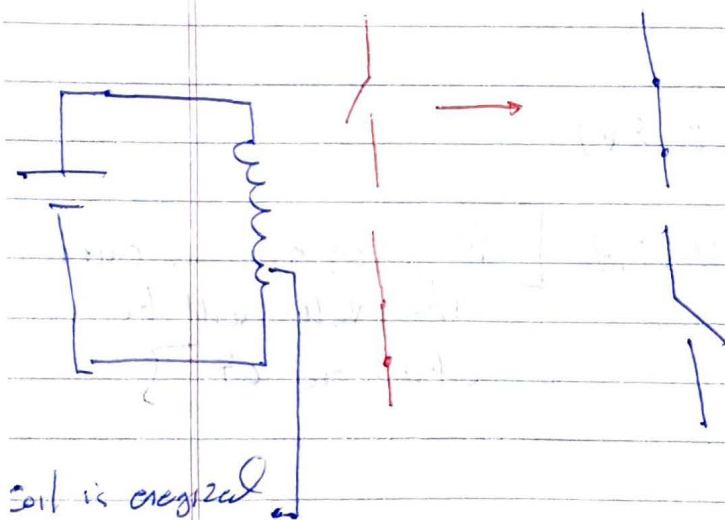
[ Range of starting current:  
The value will be  
closed to it ]

# Relay logic control of 3- $\phi$ induction motor (ON-off control)

**Relay** is an electro-magnet switch that has a coil and a set of auxiliary contacts. The contacts can be either normally open or normally closed.

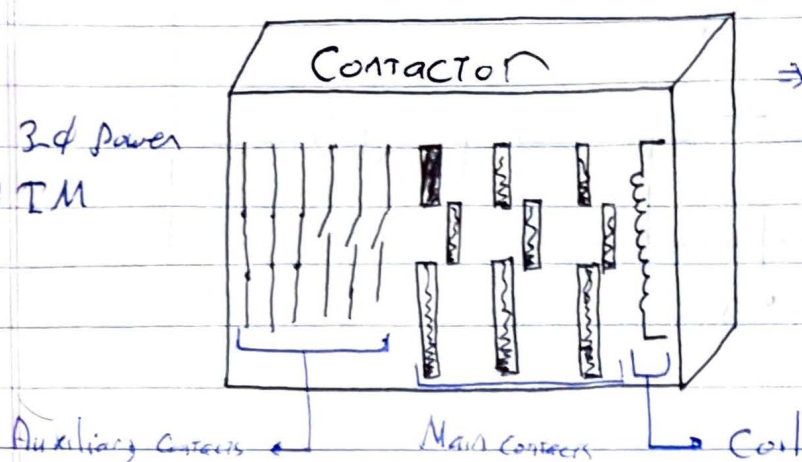


When the coil is energized, the normally open contact will close and the normally close contact will open.



CONTACTOR is an electro-magnet switch that has three main contacts with set of auxiliary contacts

For 3 $\phi$  power  
3-4 TM



⇒ The auxiliary contact can be either normally open or normally closed

Symbols:

—○— or —( — Coil of relay or contactor

—| — Normally open

—| — Push-button

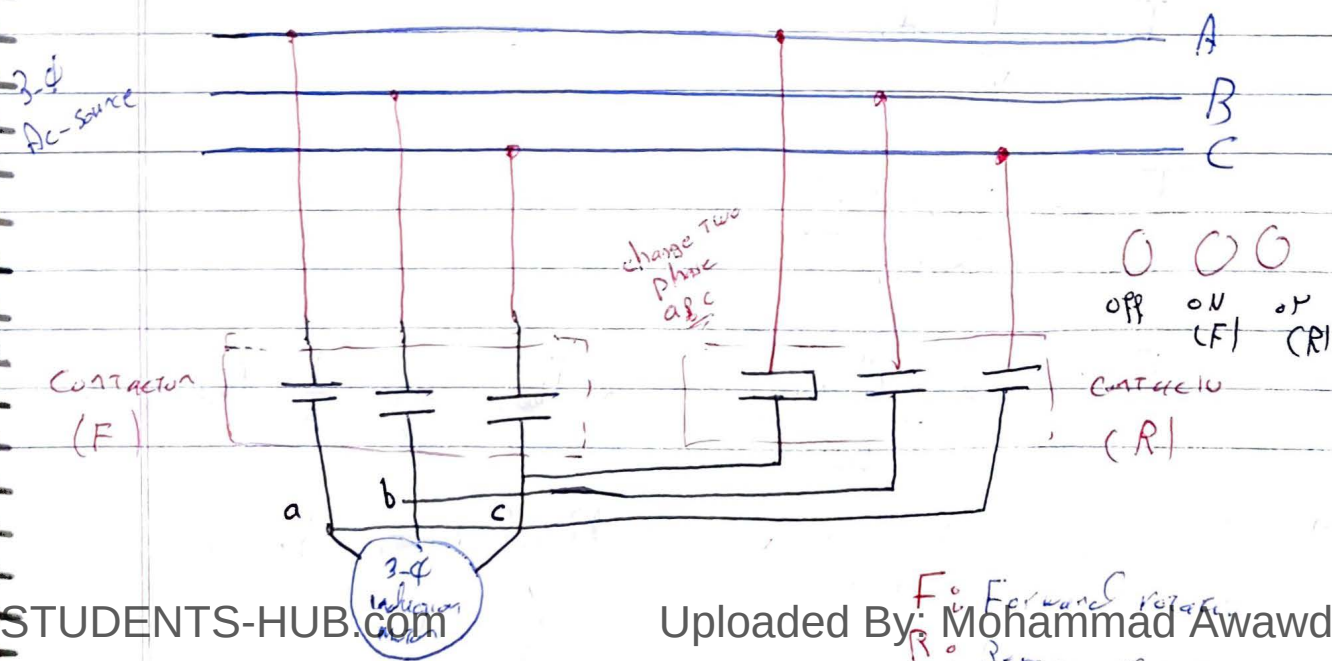
—○ — Push to close

—| — Normally closed

—○ — Push-button

Push to open

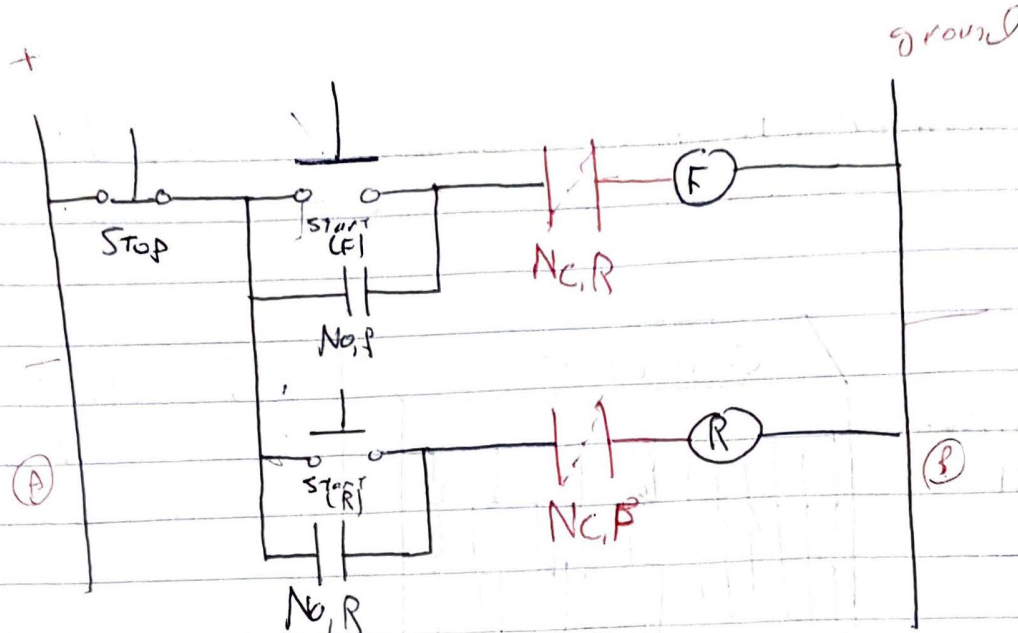
Example Control & power circuit to reverse the direction of motor



Uploaded By: Mohammad Awawdeh  
F: Forward relay  
R: Reverse relay

Ladder  
Diagram

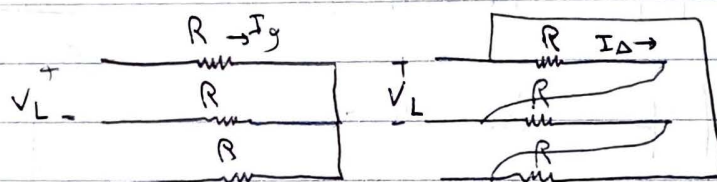
Interlocking



## Control Circuit

① Method to reduce the starting current of 3 $\phi$  induction motor.

① Y- $\Delta$  starter



$$I_{\Delta} = \frac{V_L}{R} \quad , \quad I_Y = \frac{V_L/\sqrt{3}}{R} = \frac{1}{\sqrt{3}} \frac{V_L}{R}$$

$$I_Y = \frac{1}{\sqrt{3}} I_{\Delta}$$

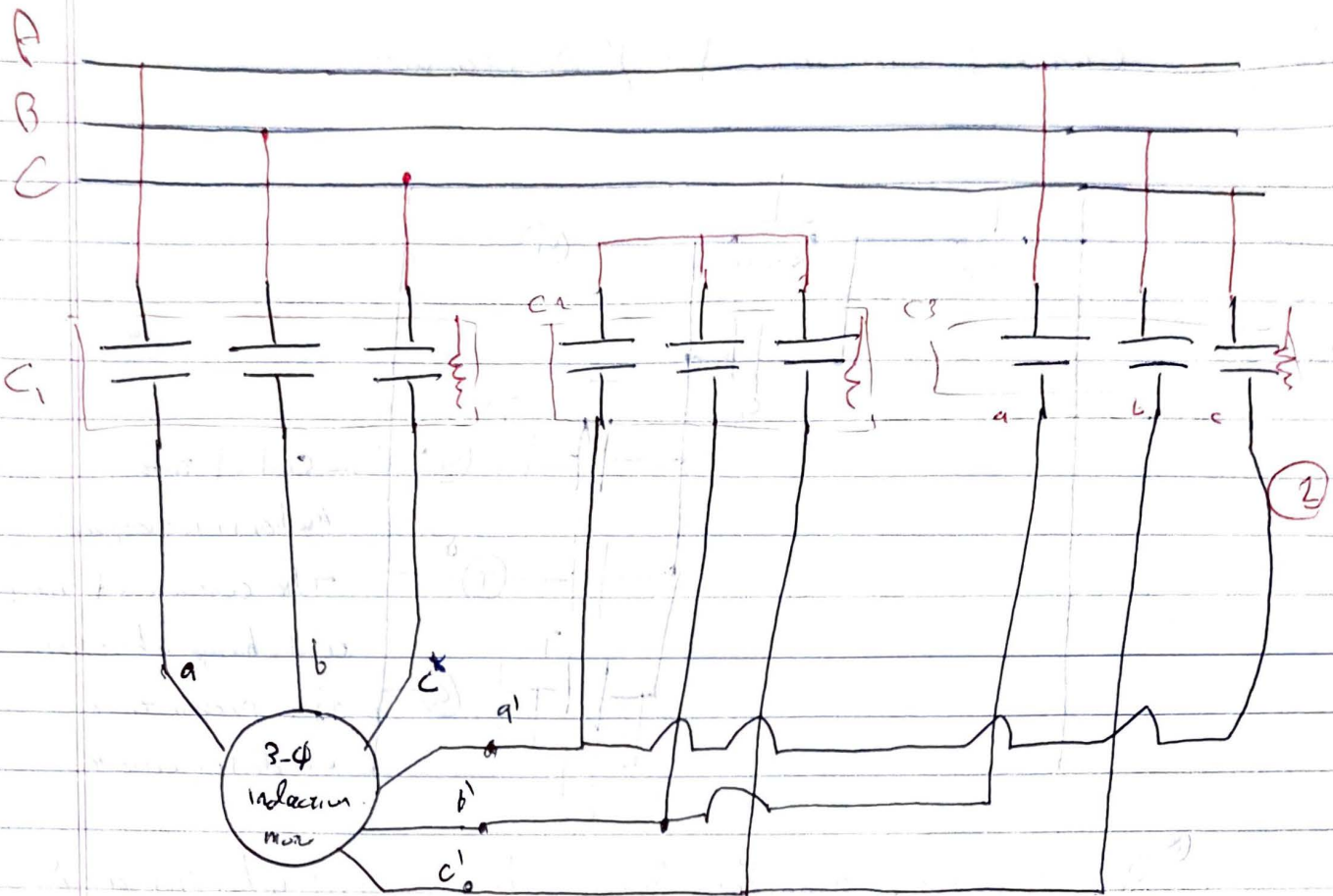
$$T_{ind} = \frac{3 I_a^2 R_a}{\omega_r}$$

$$T_{ind, start} = \frac{3 I_{a, start}^2 R_a}{\omega_s}$$

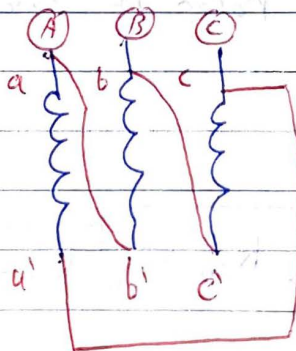
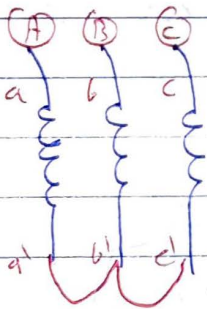
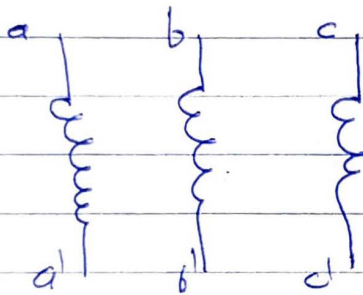
$$I_{a, start, Y} = \frac{1}{3} T_{ind, start, \Delta}$$

g & d's starting current of motor

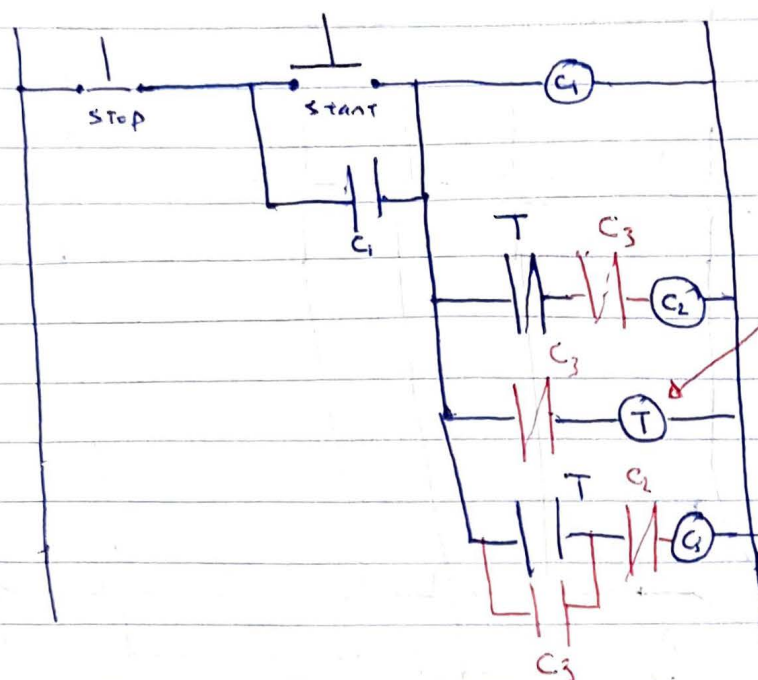
Rated voltage of motor



Stator windings (Must be before)



# Control circuit for $\star/\Delta$ starter



Coil of timer  
"when it is energized  
the contacts of timer  
will change their state  
after preset time  
or delay time"

\* Red Parameter using for Remove the Timer coil when it is at  $\Delta$

\* Try to add Reversed switch

② changing the rotor resistance "wound rotor induction machine"

$$T_{max} = \frac{3 V_{th}^2}{2 \omega_s [R_{th} + \sqrt{R_{th}^2 + (X_{th} + X_r')^2}]}$$

$$\cos \phi_{max} = \frac{R_r}{\sqrt{R_{th}^2 + (X_{th} + X_r')^2}}$$