

Problem

Define $F: \mathbf{Z} \times \mathbf{Z} \rightarrow \mathbf{Z} \times \mathbf{Z}$ as follows: For all ordered pairs (a, b) of integers, $F(a, b) = (2a + 1, 3b - 2)$.

Find the following:

a. $F(4, 4)$

b. $F(2, 1)$

c. $F(3, 2)$

d. $F(1, 5)$

Step-by-step solution

Step 1 of 5

Consider a function $f: \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z} \times \mathbb{Z}$.

For all ordered pairs of integers (a, b) , the function defined as:

$$F(a, b) = (2a + 1, 3b - 2).$$

Step 2 of 5

(a)

The objective is to find the function value at $(a, b) = (4, 4)$.

$$\begin{aligned} F(4, 4) &= (2 \cdot 4 + 1, 3 \cdot 4 - 2) \\ &= (8 + 1, 12 - 2) \\ &= (9, 10) \end{aligned}$$

Hence, the function value at $(a, b) = (4, 4)$ is $F(4, 4) = \boxed{(9, 10)}$.

Step 3 of 5

(b)

The objective is to find the function value at $(a, b) = (2, 1)$.

$$\begin{aligned} F(2, 1) &= (2 \cdot 2 + 1, 3 \cdot 1 - 2) \\ &= (4 + 1, 3 - 2) \\ &= (5, 1) \end{aligned}$$

Hence, the function value at $(a, b) = (2, 1)$ is $F(2, 1) = \boxed{(5, 1)}$.

Step 4 of 5

(c)

The objective is to find the function value at $(a, b) = (3, 2)$.

$$\begin{aligned} F(3, 2) &= (2 \cdot 3 + 1, 3 \cdot 2 - 2) \\ &= (6 + 1, 6 - 2) \\ &= (7, 4) \end{aligned}$$

Hence, the function value at $(a, b) = (3, 2)$ is $F(3, 2) = \boxed{(7, 4)}$.

Step 5 of 5

(d)

The objective is to find the function value at $(a, b) = (1, 5)$.

$$\begin{aligned} F(1, 5) &= (2 \cdot 1 + 1, 3 \cdot 5 - 2) \\ &= (2 + 1, 15 - 2) \\ &= (3, 13) \end{aligned}$$

Hence, the function value at $(a, b) = (1, 5)$ is $F(1, 5) = \boxed{(3, 13)}$.