

Discussion 7.8

(8) Order the following functions from slowest to fastest growing as $x \rightarrow \infty$

a. 2^x b. x^2 c. $(\ln 2)^x$ d. e^x

$$\bullet \lim_{x \rightarrow \infty} \frac{x^2}{2^x} = \lim_{x \rightarrow \infty} \frac{2x}{2^x (\ln 2)} = \frac{2}{\ln 2} \lim_{x \rightarrow \infty} \frac{1}{2^x \ln 2} = 0$$

x^2 grows slower than 2^x ✓

$$\bullet \lim_{x \rightarrow \infty} \frac{2^x}{e^x} = \lim_{x \rightarrow \infty} \left(\frac{2}{e}\right)^x = 0$$

2^x grows slower than e^x ✓

$$\bullet \lim_{x \rightarrow \infty} \frac{(\ln 2)^x}{x^2} = \lim_{x \rightarrow \infty} \frac{(\ln 2)^x}{1} \lim_{x \rightarrow \infty} \frac{1}{x^2}$$

$$= (0) (0)$$

$$= 0$$

$$\begin{aligned} 1 &< 2 < e \\ \ln 1 &< \ln 2 < \ln e \\ 0 &< \ln 2 < 1 \end{aligned}$$

x^2 grows faster than $(\ln 2)^x$

That is,

$(\ln 2)^x$ grows slower than x^2

Hence, the order is

$$(\ln 2)^x, x^2, 2^x, e^x$$