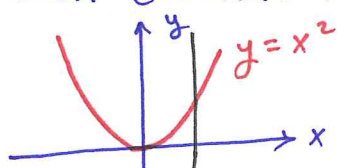


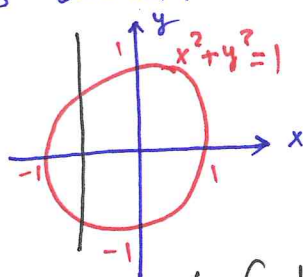
7.1 Inverse Functions and Their Derivatives

①

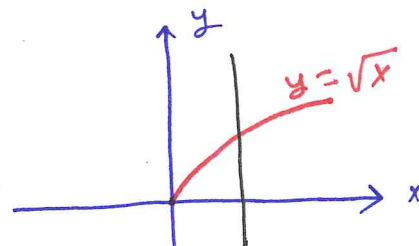
* A function is a rule that assigns a value from its range to each element in its domain.



This is function

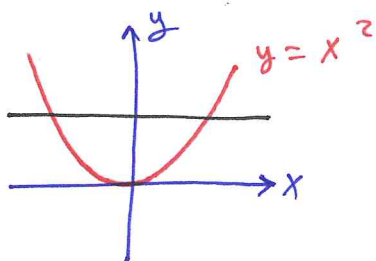


This is not a function

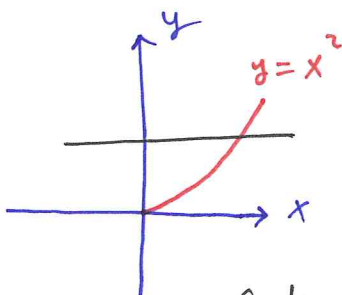


This is a function

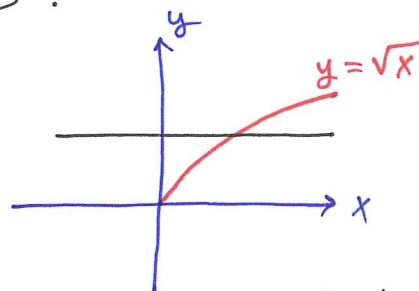
* A function $f(x)$ is one-to-one on a domain D if $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$ in D .



Not one-to-one function

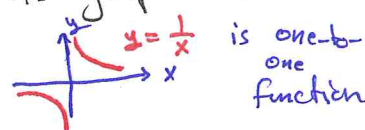


One-to-one function on $D = [0, \infty)$



one-to-one function for any domain

* A function $f(x)$ is one-to-one if and only if its graph intersects each horizontal line at most once.



* Def: Let $f: D \rightarrow R$ be one-to-one. The inverse function

$f^{-1}: R \rightarrow D$ is defined by $f^{-1}(b) = a$ if $f(a) = b$.

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* The domain of f^{-1} is R and the range is D

* $f^{-1}(x) \neq \frac{1}{f(x)}$ but $[f(x)]^{-1} = \frac{1}{f(x)}$

* Only one-to-one functions have inverse.

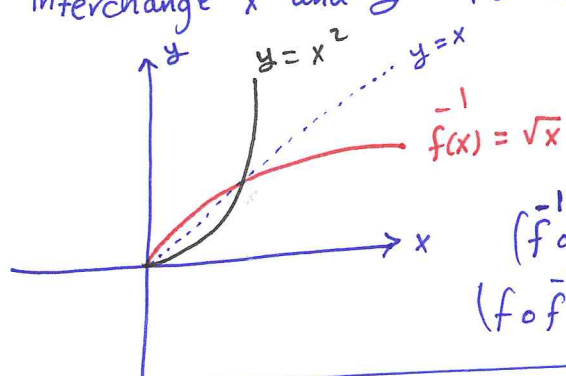
* $(f^{-1} \circ f)(x) = x$ for all $x \in D(f)$

* $(f \circ f^{-1})(y) = y$ for all $y \in D(f^{-1}) = R(f)$

Example Find the inverse of the function $y = x^2$, $x \geq 0$ (2)

$$\sqrt{y} = \sqrt{x^2} = |x| = x \text{ since } x \geq 0$$

interchange x and y to obtain $y = \sqrt{x} \Rightarrow f^{-1}(x) = \sqrt{x}$



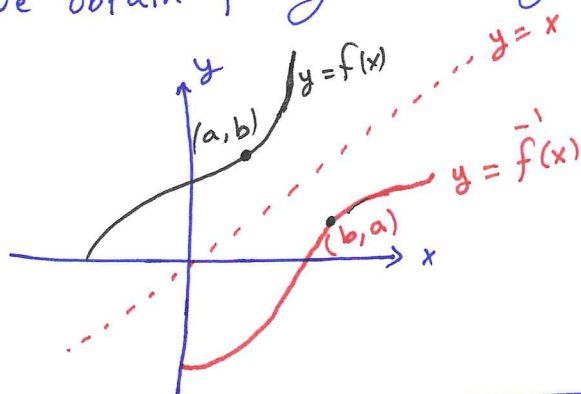
$$y: D = [0, \infty), R = [0, \infty)$$

$$f^{-1}: D = [0, \infty), R = [0, \infty)$$

$$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}(x^2) = \sqrt{x^2} = x$$

$$(f \circ f^{-1})(x) = f(f^{-1}(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x$$

* We obtain f^{-1} by reflecting the graph of f about the line $y = x$.



x	-2	0	1	5
$y = f(x)$	4	3	2	6

y	4	3	2	6
$f^{-1}(y)$	-2	0	1	5

Example Find the inverse of the function $f(x) = x^2 - 2x$, $x \leq 1$

$$f(x) = (x-1)^2 - 1 \Leftrightarrow y = (x-1)^2 - 1 \Leftrightarrow (x-1)^2 = y+1$$

$$|x-1| = \sqrt{y+1} \Leftrightarrow 1-x = \sqrt{y+1} \Leftrightarrow x = 1 - \sqrt{y+1}$$

$$y = 1 - \sqrt{x+1} \Leftrightarrow f^{-1}(x) = 1 - \sqrt{x+1} \text{ with } D = [-1, \infty)$$

$$R = (-\infty, 1]$$

Theorem If $f: D \rightarrow R$ is one-to-one with f' exists and never zero on D

then $f^{-1}: R \rightarrow D$ is differentiable on R and

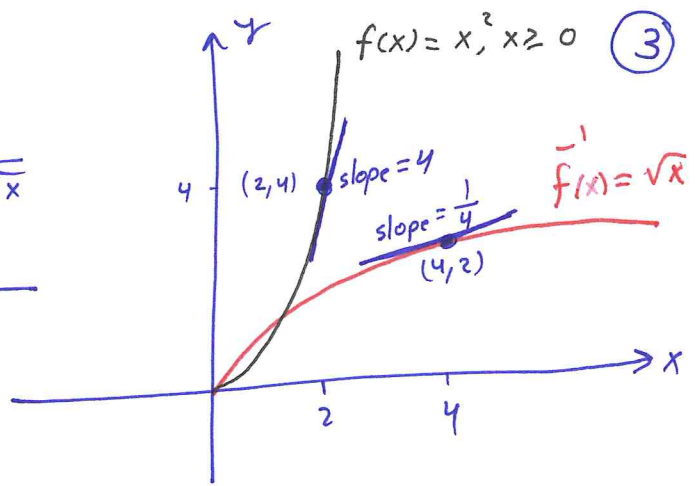
$$(f^{-1})'(b) = \frac{1}{f'(f^{-1}(b))} \quad \text{or} \quad \left. \frac{df^{-1}}{dx} \right|_{x=b} = \frac{1}{\left. \frac{df}{dx} \right|_{x=f^{-1}(b)}}$$

$$f(x) = x^2, x \geq 0 \quad \left| \quad f^{-1}(x) = \sqrt{x}\right.$$

$$f'(x) = 2x$$

$$(f^{-1})'(x) = \frac{1}{2\sqrt{x}}$$

$$f(2) = 2(2) = 4$$



$$(f^{-1})'(4) = \frac{1}{f'(\bar{f}^{-1}(4))}$$

$$= \frac{1}{f'(\sqrt{4})} = \frac{1}{f'(2)} = \frac{1}{2(2)} = \frac{1}{4}$$

Example Let $f(x) = 3x^2$. Find $\frac{df^{-1}}{dx}$ at $x = f(\sqrt{2})$

$$\frac{df^{-1}}{dx}(f(\sqrt{2})) = \frac{1}{f'(\bar{f}^{-1}(f(\sqrt{2})))} = \frac{1}{f'(\sqrt{2})} = \frac{1}{3(2)} = \frac{1}{6}$$