

First 636 / 141 جز 1 و 2

Physics
(L)

Mechanics

Kinematics (description of motion)

* chapters 1, 2, 3, 4

Dynamics (cause of motion)

* chapters 5, 6, 7, 8, 9

10, 11, 15

Chapter one
Measurements

(vector) vectors

scalar (مقياس)

Physical Quantities

Derived Quantities

* coulomb $\rightarrow$ Amp $\cdot$ s

* speed $\rightarrow$ m / s

* Force $\rightarrow$ N $\rightarrow$ Kg $\cdot$ m / s²

Basic Quantities

* mass $\rightarrow$ Kg

* time $\rightarrow$ s

* length $\rightarrow$ m

* amount of substance $\rightarrow$ mol

* light Intensity $\rightarrow$ candela

* current $\rightarrow$ Amp

* temperature $\rightarrow$ K



physics
(10)

ex.1

$$X(m) = at^2 + bt + c \quad t \text{ in Sec}$$

$$a = \frac{m}{s^2} \quad (at^2 = m \rightarrow a = \frac{m}{t^2})$$

$$b = \frac{m}{s} \quad (bt = m \rightarrow b = \frac{m}{t})$$

$$c = m \quad (c = m)$$

ex.2

$$24 \text{ km/h} = ? \text{ m/s}$$

$$24 \text{ km/h} = 24 \frac{\text{km}}{\text{h}} = 24 \frac{(1000) \text{ m}}{(3600) \text{ s}}$$

$$= 6.7 \text{ m/s}$$

ex.3

$$F = Rx$$

$$F = N$$

$$x = m$$

$$\therefore R = N/m$$

Q.3

(a) $1 \text{ km} = ? \text{ micron}$

$$* 1 \text{ Mm} = \text{micron} = 10^6 \text{ m}$$

$$\therefore 1 \text{ km} = 1000 \text{ m}$$

$$1 \text{ km} = 1 \times 10^3 \times \left(\frac{1}{10^6} \right) \text{ Mm}$$

$$\therefore 1 \text{ km} = 10^{-3} \text{ Mm}$$

⑥ $1 \text{ cm} = \frac{1}{100} \text{ m}$
 $1 \text{ cm} = 10^{-2} \text{ m}$
 $1 \text{ cm} = 10^2 \cdot \left(\frac{1}{10^4}\right) \text{ m}$

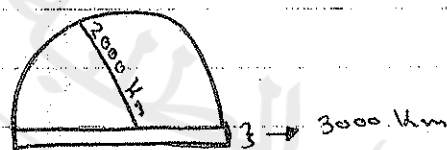
$\therefore 1 \text{ cm} = 10^2 \times 10^6$
 $1 \text{ cm} = 10^4 \text{ m}$

Q. 7 Water Volume = $(26 \text{ km}^2) \cdot (2 \text{ in}) = 2 \text{ acre-foot}$
↙ area ↘ length

* $1 \text{ ft} = 12 \text{ in}$
 * $1 \text{ acre} = 43560 \text{ ft}^2$
 * $1 \text{ m}^2 = 10.76 \text{ ft}^2$

$\therefore \text{Water Volume} = (26 \times 10^6) \text{ m}^2 \cdot \left(\frac{2}{12}\right) \text{ ft}$
 $= (26 \times 10^6 \times 10.76) \text{ ft}^2 \cdot \left(\frac{1}{6}\right) \text{ ft}$
 $= \left(26 \times 10^6 \times 10.76 \times \frac{1}{43560}\right) \text{ acre} \cdot \frac{1}{6} \text{ ft}$
 $= 1070 \text{ acre} \cdot \text{ft}$

Q. 9



Volume area $\times$ thickness
 $= \left(\frac{\pi r^2}{2}\right) \times (d)$
 $= \left(\frac{3.14}{2}\right) \cdot (2 \times 10^8) \times (3 \times 10^5)$
 $= 9.8 \times 10^{20} \text{ cm}^3$

* $1 \text{ m} = 10^2 \text{ cm}$
 * $\text{km} = 10^3 \text{ m}$
 * $r = 2000 \text{ km}$
 * $r = 2000 \times 10^3 \text{ m}$
 * $r = 2000 \times 10^3 \times 10^2 \text{ cm}$
 * $r = 2 \times 10^8 \text{ cm}$
 * $d = 3000 \text{ km}$
 * $d = 3000 \times 10^3 \text{ m}$
 * $d = 3 \times 10^5 \text{ cm}$

Physics (L)

ex.1 $36 \text{ m/s} = ? \text{ km/h}$

* $1 \text{ km} = 10^3 \text{ m} \rightarrow \frac{1 \text{ km}}{10^3 \text{ m}} = 1$ $1 = \frac{10^3 \text{ m}}{1 \text{ km}}$

* $1 \text{ h} = 3600 \text{ s} \rightarrow 1 = \frac{3600 \text{ s}}{1 \text{ h}}$ $1 = \frac{1 \text{ h}}{3600 \text{ s}}$

$\therefore 36 \text{ m/s} = 36 \text{ m} \div \text{s}$

$= 36 \text{ m} \div 1 \text{ s}$

$= 36 \times \left(\frac{1 \text{ km}}{10^3 \text{ m}} \right) \div 1 \times \left(\frac{1 \text{ h}}{3600 \text{ s}} \right)$

$= 36 \left(\frac{\text{km}}{10^3} \right) \div \frac{1 \text{ h}}{3600}$

$= 129.6 \text{ km/h}$

ex.2 $1 \text{ g/cm}^3 = ? \text{ kg/m}^3$

* $1 \text{ kg} = 10^3 \text{ g} \rightarrow \frac{1 \text{ kg}}{10^3 \text{ g}} = 1$ $1 = \frac{10^3 \text{ g}}{1 \text{ kg}}$

* $1 \text{ m} = 10^3 \text{ cm} \Rightarrow 1 \text{ m}^3 = 10^6 \text{ cm}^3 \rightarrow \frac{1 \text{ m}^3}{10^6 \text{ cm}^3} = 1$ $1 = \frac{10^6 \text{ cm}^3}{1 \text{ m}^3}$

$\therefore 1 \text{ g/cm}^3 = 1 \text{ g} \div 1 \text{ cm}^3$

$= 1 \times \left(\frac{1 \text{ kg}}{10^3 \text{ g}} \right) \div \text{cm}^3 \left(\frac{1 \text{ m}^3}{10^6 \text{ cm}^3} \right)$

$= 10^{-3} \text{ kg} \div 10^{-6} \text{ m}^3$

$= 10^3 \text{ kg/m}^3$

Units for constants in any formula:-

ex.1 $F = k \frac{q_1 q_2}{r^2} \Rightarrow k = \frac{N}{m}$

ex.2 $E_0 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \Rightarrow E_0 = \frac{N \cdot m^2}{C^2}$

$\therefore E_0 = \frac{q_1 q_2}{4\pi r^2 F} \rightarrow C^2 / N \cdot m^2$

* $q \rightarrow C$

* $r = m$

* $q = C$

* $F = N$

ex.3 $x(t) = A \cos(\omega t + Kx)$

* $A = m$ (amplitude)

* $x = m, t = s$

* $\omega, K, A = \frac{m}{s}$

* $\omega t = 1 \text{ (rad)}$

$\omega = \frac{\text{rad}}{t(s)} \rightarrow \omega = \text{rad/s}$

* $Kx = \text{rad}$

$K = \frac{\text{rad}}{x(m)} \rightarrow K = \text{rad/m}$

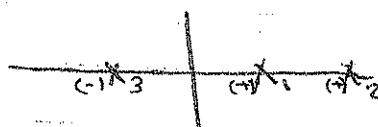
Chapter two

Motion Along A straight line

* Motion: relate change in position and time.

* Position:-
 → reference point
 → direction

* Position is $x(+)$ or $(-)$



- initial position (x_1)

- final position (x_2)

- Displacement = change in position

- Displacement = Δx

* Displacement = $x_2 - x_1$ (+) $\rightarrow$

* if the final position x_3

$\therefore$ Displacement = Δx

= $x_3 - x_1$ (-) $\leftarrow$

* average Velocity (vector) = $\frac{\text{Displacement}}{\text{time}}$

(vector) $V_{avg} = \frac{\text{distance}}{\Delta \text{time}}$

(متجه)

* note :-

(~~متجه~~) average speed = $\frac{\text{distance}}{\text{time}}$
scalar

$S_{avg} = \frac{\text{distance}}{t}$

example :- the distance between Jerusalem and Birzeit = 30 km

* the time " " " " " = 2 h

* $S_{avg} = 30/2 = 15 \text{ km/h}$

* Displacement (J-B) = 18 km

* $V_{avg} = 18/2 = 9 \text{ km/h}$

note:

physics (D)

Q.53

* AU = Astronomical unit = $(d_{E-S}) = 92.9 \times 10^6 \text{ mi}$



المسافة بين الشمس والأرض

$\therefore \text{AU} = 92.9 \times 10^6 \text{ mi}$ (mi = mile)

* Parsec (pc)

* $1'' \rightarrow$ زاوية قياس المسافة

* $1^\circ = 60'$

* $1' = 60''$

* $1' \rightarrow$ دقيقة

* $1'' \rightarrow$ ثانية

* $1^\circ = 3600''$

$\therefore$ يجب إيجاد العلاقة بين AU و pc

* $\theta \text{ rad} = \frac{\text{arc (المسافة)}}{\text{radius (الم)}} \rightarrow$

نقطة لاد (1') للراديا

* $1'' = \frac{1^\circ}{3600}$

* $1'' = \frac{1^\circ}{3600} \times \frac{\pi}{180} \text{ rad} \rightarrow 4.85 \times 10^{-6} \text{ rad}$

$\therefore 4.85 \times 10^{-6} \text{ rad} = \frac{\text{AU}}{\text{pc}} \rightarrow \text{AU} = 4.85 \times 10^{-6} \text{ pc}$

* light year (ly) = the distance travelled by light in 1 year.

$\therefore \text{ly} = c \cdot t$

(المسافة/السرعة الزمنية) = الزمن $\times$ سرعة الضوء

$= \left(186000 \frac{\text{mi}}{\text{s}} \right) \cdot (365 \times 24 \times 60 \times 60 \text{ s})$

$\therefore \text{ly} = 5.86 \times 10^{12} \text{ mi}$

المطلوب: حساب المسافة بين الأرض والشمس بال (pc) وال (ly)

(a) d_{E-S} in pc?

$$1 \text{ AU} = d_{E-S} \quad (\text{مسافة الشمس})$$

$$\therefore 1 \text{ AU} = 4.85 \times 10^{-6} \text{ pc}$$

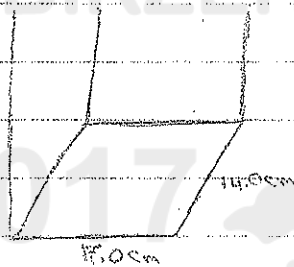
(b) d_{E-S} in ly?

$$= 92.9 \times 10^6 \text{ mi}$$

$$= 92.9 \times 10^6 \left(\frac{1.6}{5.86 \times 10^2} \right)$$

$$= 0.15 \times 10^{-4} \text{ ly}$$

Q. 31



$$m_{\text{candy}} = 0.0200 \text{ g}$$

$$V_{\text{candy}} = 50.0 \text{ mm}^3$$

$$\rho_{\text{candy}} = \frac{m}{V} \quad (\text{mass density})$$

$$= \frac{0.02 \times 10^{-3} \text{ kg}}{50 \times 10^{-3} \text{ cm}^3}$$

$$= 4 \times 10^{-4} \text{ cm}^3 \quad (\text{كثافة})$$

$$\left(\frac{\text{كمية المادة}}{\text{الوقت}} \right) \frac{dH}{dt} = 0.250 \text{ cm/s}$$

$$* 1 \text{ kg} = 1000 \text{ g}$$

$$* 1 \text{ cm} = 10 \text{ mm}$$

$$* 1 \text{ cm}^3 = 10^3 \text{ mm}^3$$

$$* \frac{dm}{dt} \rightarrow (\text{kg/s}) \text{ في المسألة}$$

$$* M = \rho \cdot A \cdot h \quad \left(R = \frac{M}{V} \right) \quad (\text{المساحة المقطعية})$$

$$* A = 17 \times 14 \text{ cm}^2$$

$$* \frac{dM}{dt} = \rho \cdot A \cdot \frac{dH}{dt}$$

$$= \left(\frac{4 \times 10^{-4} \text{ kg}}{\text{cm}^3} \right) \cdot (17 \times 14 \text{ cm}^2) \cdot \left(\frac{0.250 \text{ cm}}{\text{s}} \right)$$

$$= 238 \times 10^{-2} \text{ kg/s}$$

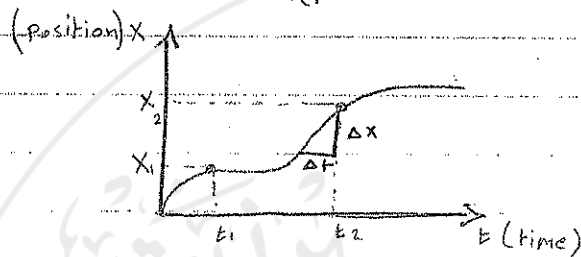
$$\Rightarrow \frac{238 \times 10^{-2} \text{ kg}}{\text{s}}$$

$$\rightarrow 1.028 \text{ kg/s}$$

Physics (1)

* $V_{\text{instantaneous}} \rightarrow V = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}$
 $V = \frac{dx}{dt}$ } instantaneous velocity

eg. 1



* لإيجاد السرعة اللحظية من الرسم نرسم مماساً ونأخذ ميله

* هذا الرسم البياني جسم يسير في خط مستقيم

∴ $V = \frac{dx}{dt}$

= the slope of the tangent (x-t) at a certain point.

* $a_{\text{avg}} = \frac{\Delta V}{\Delta t}$ (average acceleration) (متجه التسارع) (vector)

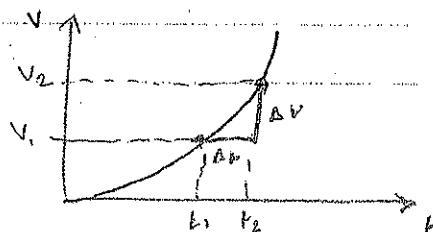
∴ $a_{\text{avg}} = \frac{m/s}{s} = m/s/s = m/s^2$

* $a = \lim_{\Delta t \rightarrow 0} \frac{\Delta V}{\Delta t}$

= $\frac{dv}{dt}$ (instantaneous acceleration) (التسارع اللحظي)

= the slope of the tangent (v-t) at a certain point.

eg. 2



ex. 1
Q. 5

$\frac{m/s}{\uparrow}$ $\frac{m/s^2}{\uparrow}$ $\frac{m/s^3}{\uparrow}$
 $x = 3t - 4t^2 + t^3$

$x = m$

$t = sec$

$$\textcircled{a} \quad x(1) = 3(1) - 4(1)^2 + 1(1)^3 \\ = 0 \text{ m}$$

$$\textcircled{b} \quad x(2) = 3(2) - 4(2)^2 + 1(2)^3 \\ = -2 \text{ m}$$

$$\textcircled{c} \quad x(3) = 3(3) - 4(3)^2 + 1(3)^3 \\ = 0 \text{ m}$$

$$\textcircled{d} \quad x(4) = 3(4) - 4(4)^2 + 1(4)^3 \\ = 12 \text{ m}$$

$$\textcircled{e} \quad \Delta x = x(4) - x(0) \\ = 12 - 0 \\ = 12 \text{ m}$$

$$\textcircled{f} \quad V_{avg} \text{ between } (t_1 = 2s, t_2 = 4s)$$

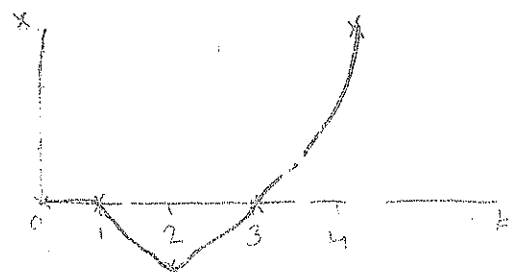
$$= \frac{\Delta x}{\Delta t}$$

$$= \frac{x(4) - x(2)}{4 - 2}$$

$$= \frac{12 - (-2)}{2}$$

$$= 7 \text{ m/s}$$

g



① (addition)

$v(2) = ?$, $v(4)$

لما قوت x و t مكان

(السرعة v عند $t=2$ و $t=4$)

(السرعة v عند $t=2$ و $t=4$) $v = \frac{dx}{dt} = 3 - 8t + 3t^2$

$v(2) = 3 - (8 \times 2) + 3(2)^2 \rightarrow -1 \text{ m/s}$

$v(4) = 3 - (8 \times 4) + 3(4)^2 \rightarrow 19 \text{ m/s}$

② Find a_{avg} from $(t_1 = 2s, t_2 = 4s)$

$a_{avg} = \frac{\Delta v}{\Delta t}$

$= \frac{v(4) - v(2)}{4 - 2} \rightarrow \frac{19 - (-1)}{2} = 10 \text{ m/s}^2$

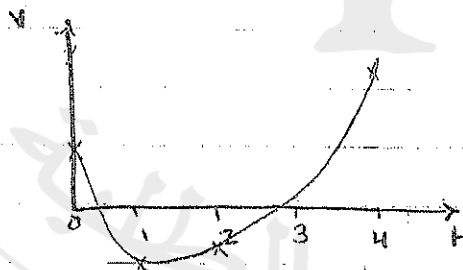
③ Find a at $(t = 2s, t = 4s)$

$a = \frac{dv}{dt} = -8 + 6t$ (التسارع a عند $t=2$ و $t=4$)

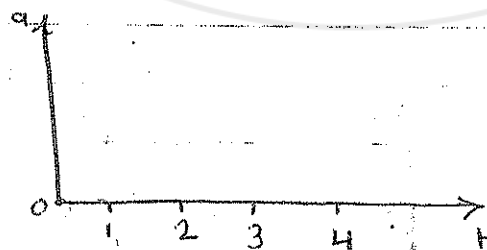
$a(2) = -8 + 12 = 4 \text{ m/s}^2$

$a(4) = -8 + 24 = 16 \text{ m/s}^2$

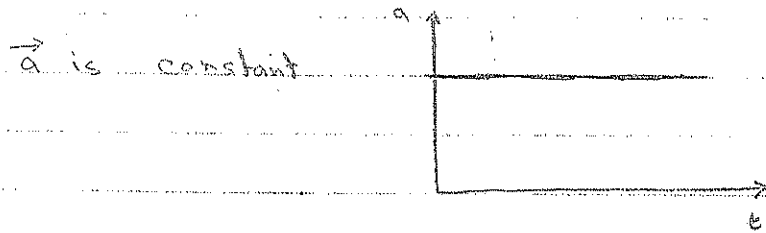
④



⑤



Motion with constant acceleration :-



مختصات و اوقات
پیش و درستی

$t=0$
 x_0
 v_0

① $a = \frac{\Delta v}{\Delta t}$

$a = \frac{v_2 - v_1}{t_2 - t_1} \rightarrow a = \frac{v - v_0}{t - 0}$ (مختصات و اوقات)

$v - v_0 = at \rightarrow v = v_0 + at$ ③

② $v_{avg} = \frac{\Delta x}{\Delta t}$

$v_{avg} = \frac{x - x_0}{t} \rightarrow x - x_0 = v_{avg} t$ ① (لکه این فرمول)

also $v_{avg} = \frac{v_0 + v}{2}$ only for constant a: ②

⑤ $x - x_0 = \left(\frac{v_0 + v}{2} \right) \times t$ ② in ①

③ $x - x_0 = \left(\frac{v_0 + v_0 + at}{2} \right) \times t$

II
 $x - x_0 = v_0 t + \frac{1}{2} a t^2$

③ $x - x_0 = \left(\frac{v_0 + v}{2} \right) \left(\frac{v - v_0}{a} \right) \rightarrow v^2 = v_0^2 + 2a(x - x_0)$ III
 $x - x_0 = vt - \frac{1}{2} at^2$ ③ in ③

Physics

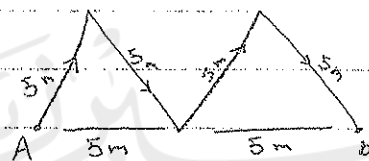
(D)

1.2

$$\vec{V}_{avg} = \frac{\text{displacement}}{\Delta t}$$

$$S_{avg} = \frac{\text{distance}}{\Delta t}$$

example →

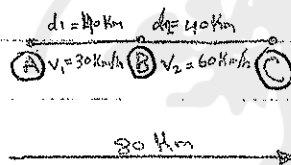


total = 20s

$$(qib; l; l) S_{avg} = \frac{20m}{30s} = \frac{2}{3} m/s$$

$$(qib; l; l) \vec{V}_{avg} = \frac{10m}{30s} = \frac{1}{3} m/s$$

Q.3



$$* V = \frac{\text{displacement}}{t} \rightarrow t_1 = \frac{40 km}{30 km/h} \rightarrow \frac{4}{3} h$$

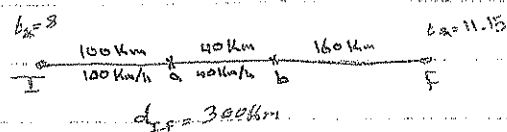
$$* t_2 = \frac{40 km}{60 km/h} \rightarrow \frac{2}{3} h$$

$$* t_{a-c} = \frac{4}{3} h + \frac{2}{3} h = 2 h$$

$$* V_{avg} = \frac{80 km}{2 h} \rightarrow 40 km/h$$

$$* S_{avg} = 40 km/h \quad (qib; l; l) \quad (qib; l; l) \quad (qib; l; l)$$

Q.11



$$t_1(I-A) = \frac{100 \text{ km}}{100 \text{ km/h}} \rightarrow 1 \text{ h}$$

$$t_2(A-B) = \frac{40 \text{ km}}{40 \text{ km/h}} \rightarrow 1 \text{ h}$$

$$t_3(B-F) = 3.25 - 2 = 1.25 \text{ h}$$

$$\therefore d_{(B-F)} = 160 \text{ km}$$

$$V_{B-F} = \frac{160 \text{ km}}{1.25 \text{ h}} = 128 \text{ km/h}$$

المعدل المتوسط $\left\{ \begin{array}{l} V_{\text{avg}} = \text{Savg for the total trip} \\ V_{\text{avg } I-F} = \frac{300 \text{ km}}{1+1+1.25 \text{ h}} \rightarrow 92.3 \text{ km/h} \end{array} \right.$

Q.12

$$x = 9.75 + 150t^3$$

$$x = \text{cm}, t = \text{sec}$$



(a) $V_{\text{avg}} = \frac{\Delta x}{\Delta t}$ (متوسط السرعة)

$$= \frac{x(3) - x(2)}{3 - 2} \rightarrow 28.5 \text{ cm/s}$$

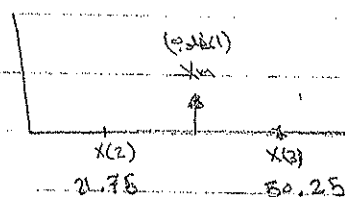
(b) $V = \frac{dx}{dt} \rightarrow 4.5t^2$

$$V(2) = 18 \text{ cm/s}$$

(c) $V(3) = 40.5 \text{ cm/s}$

(d) $V(2.5) = 28.125 \text{ cm/s}$

© V_{inst} at middle way between $t=2s \rightarrow t=3s$



$$X_{\text{mid}} = \frac{X(3) - X(2)}{2} + X(2) = \frac{X(3) - X(2)}{2} \rightarrow 36 \text{ cm}$$

$$X = 9.75 + 1.5t^3$$

$$\therefore 36 = 9.75 + 1.5t^3$$

$$t_{\text{mid}} = 2.6 \text{ s}$$

$$\therefore V = 4.2t^2$$

$$V_{\text{mid}} = 4.2(2.6)^2 \\ = 30.4 \text{ cm/s}$$

Q. 22 $X = ct^2 - bt^3$ $X = m, t = s$

① $X = ct^2 \rightarrow c = m/s^2$

② $bt^3 = X \rightarrow b = m/s^3$

③ $\text{find } \frac{dx}{dt}, \text{ solve for } t$

$$V = \frac{dx}{dt} \rightarrow 8t - 6t^2 \quad (b, c \text{ dy } t \text{ dy } s)$$

$$V = 0 = 8t - 6t^2 \rightarrow t = 0, \frac{4}{3}$$

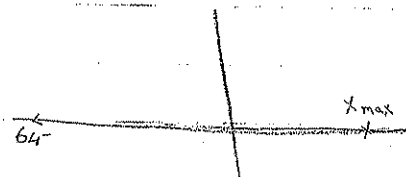
$$X(0) = 0 \quad / \quad X\left(\frac{4}{3}\right) = \max X = 2.37 \text{ cm}$$

④ + ⑤ from $t=0$ to $t=4s$

$$X(0) = 0 \quad / \quad X(4) = -64 \text{ m}$$

* displacement = $X(4) - X(0) \rightarrow -64 - 0 = -64 \text{ m}$

* distance = $2.37 + 2.37 + 64 = 68.74 \text{ m}$ (2.37 + 2.37 + 64)



physics

(L)

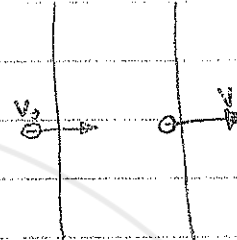
1.2

ex. 1

Q23

$$U_0 = 1.5 \times 10^5 \text{ m/s}$$

$$V = 5.7 \times 10^6 \text{ m/s}$$



$$L = 1 \text{ cm}$$

* $v^2 = v_0^2 + 2a(x - x_0)$

$$v^2 = v_0^2 + 2a(x - x_0)$$

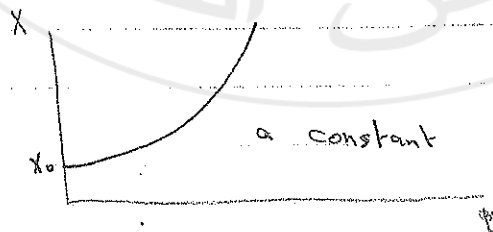
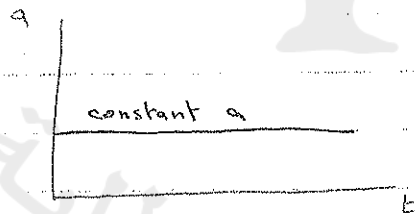
$$* L = x - x_0 = 1 \times 10^{-2} \text{ m}$$

$$\therefore (5.7 \times 10^6)^2 = (1.5 \times 10^5)^2 + 2a(1 \times 10^{-2})$$

(المعادلة) (a) نوجد

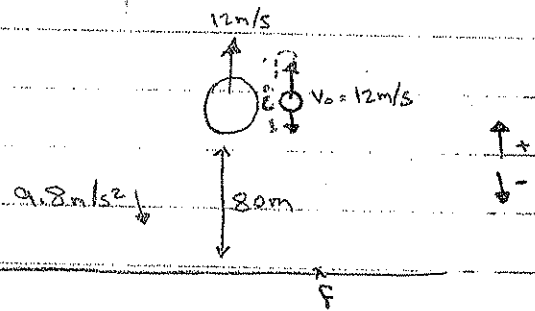
* The best example for moving with constant $\vec{a}$ is:
Free Fall :-

a is always downward and equal to 9.8 m/s^2 .



ex. 2
Q. 49

ascend → يصعد
ascends with constant V
 $V = 12 \text{ m/s}$



سورة القعدة وضعت البالون الا يتد اتيه وسورة البالون

(a) $t(i-F)$

$$y - y_0 = V_0 t + \frac{1}{2} a t^2$$

$$-80 = +12t + \frac{1}{2} \times -9.8 \times t^2$$

أما انزل من (F ← t) ما على ما على
الا، لا يتد اتيه؛ ما على ما على (سورة البالون)

(b) V on the ground &

$$V = V_0 + at$$

$$V = 12 + -9.8 \times t \quad (\text{سورة البالون})$$

Graphical integration in motion :-

$$* \quad V = \frac{dx}{dt} \quad (\text{سورة البالون})$$

$$\int_{x_0}^x dx = \int_{t_1}^{t_2} V dt \quad \longrightarrow \quad X - X_0 = \int_{t_1}^{t_2} V dt$$

= Area under the curve of $(v-t)$

$$* \quad a = \frac{dv}{dt} \quad (\text{سورة البالون})$$

$$\int_{v_0}^v dv = \int_{t_1}^{t_2} a dt \quad \longrightarrow \quad v - v_0 = \int_{t_1}^{t_2} a dt$$

= Area under the curve of $(a-t)$

ex.3

Q.81

$$a = 5t$$

$$a = \text{m/s}^2$$

$$\text{at } t = 2s \rightarrow v = 17 \text{ m/s}$$

$$t = 5$$

$$\text{Find } v \text{ at } t = 4s$$

$$a = \frac{dv}{dt} \rightarrow \int dv = \int a \cdot dt$$

$$v = \int (5t) \cdot dt \rightarrow v = \frac{5t^2}{2} + C$$

for $t = 2s$

$$17 = \frac{5(2)^2}{2} + C \rightarrow C = 7$$

$$\therefore v = \frac{5t^2}{2} + 7$$

$$\therefore v(4) = \frac{5(4)^2}{2} + 7 \rightarrow 47 \text{ m/s}$$

ex.4

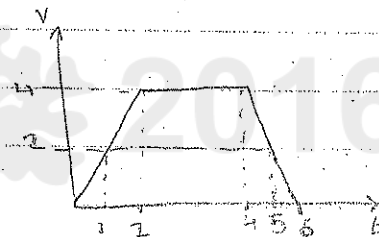
Q.90

Important

@ coordinate at $t = 5s$

$$x = \text{area (under the curve)}$$

$$(x_0, y_0) \quad x - x_0 = \frac{1}{2} \times 2 \times 4 + 2 \times 4$$



(b) v at $t = 5s$

$$v = 2 \text{ m/s}$$

(c) a at $t = 5s$

$$a = \text{slope} \rightarrow \frac{dx}{dt} \rightarrow \frac{-4}{2} \text{ m/s}^2 \quad (\text{area under the curve})$$

(d) V_{avg} between $(1-5)s$

area (1-5) is $\frac{1}{2} \times (5-1) \times 2 = 4$

$$V_{\text{avg}} = \frac{\text{area}}{4}$$

(e) a_{avg} $(1-5)s$

$$\frac{\Delta x}{\Delta t} \rightarrow \frac{2-2}{4} = \text{zero}$$

Physics

(D)

Chapter (2)

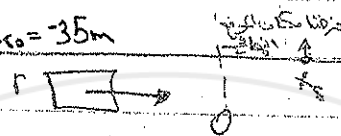
Q.35

$$v_c = 0$$

$$x_{c0} = -35 \text{ m}$$

$$v_g = 20 \text{ m/s}$$

$$x_{g0} = 270 \text{ m}$$



$$a_g = -g$$

* for green car:

$$\Delta x = v_{x0} t$$

$$(x_f - x_i) = v_{x0} t$$

$$(x_f - 270) = -20(12)$$

$$x_f = 30 \text{ m}$$

المسافة بين
والشارع = 30 م

* for red car:

$$(x_f - x_i) = v_{x0} t + \frac{1}{2} a_x t^2$$

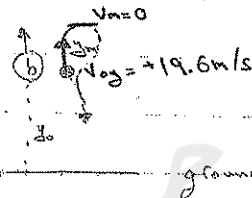
$$(30 - 35) = 0 + \frac{1}{2} a_x (12)^2$$

$$a = 0.9 \text{ m/s}^2$$

الارتفاع في
الوقت = 9.8 م

Q.51

$$v_0 = 9.6 \text{ m/s}$$



المخلوطة آتية من ارتفاع (25 م)

$$h_0 - m = 25$$

$$h_0 - g = 65$$

$$a_g = -9.8 \text{ m/s}^2$$

أفندق في البداية عند 10 م والارتفاع عند 25 م والارتفاع

في المرة الأولى وسنحل المسألة

$$\Delta y = v_{oy} \cdot t + \frac{1}{2} a_y t^2$$

$$y_m = (+19.6 \times 2) + \left(\frac{1}{2} \times -9.8 \times (2)^2 \right)$$

$$y_m = 19.6 \text{ m}$$

(الارتفاع القصوى) y_m هي أقصى ارتفاع يصل إليه الجسم أثناء حركته.

(b) $\Delta y = v_{oy} \cdot t + \frac{1}{2} a_y t^2$ المسافة التي يقطعها الجسم من البداية إلى الأرض

$$\Delta y = v_{oy} \cdot t + \frac{1}{2} a_y t^2$$

$$y_o = (+19.6 \times 6) + \left(\frac{1}{2} \times -9.8 \times (6)^2 \right)$$

$$y_o = -58.8 \text{ m}$$

(c) additional

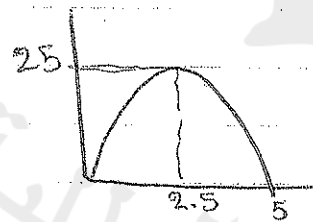
find the impact velocity with the ground

$$v_g = v_{oy} + a_y \cdot t$$

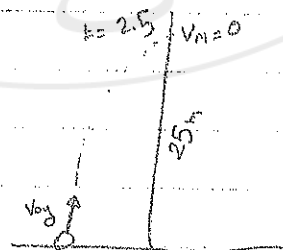
$$v_g = +19.6 + (-9.8 \times 6)$$

$$v_g = -39 \text{ m/s}$$

Q.64



$$a_y, v_{oy} = ?$$



$$\textcircled{a} \therefore \Delta y = \left(\frac{V_{0y} + V_y}{2} \right) \cdot t \quad \left(\text{نستخدم هنا متوسط السرعة} \right)$$

$$+ 25 = \left(\frac{V_{0y} + 0}{2} \right) \cdot 2.5$$

$$\therefore V_{0y} = 20 \text{ m/s}$$

$\therefore$ سرعة الكرة عند انطلاقها

$$V_y = V_{0y} + a_y t$$

$$0 = +20 + a_y (2.5)$$

$$a_y = -8 \text{ m/s}^2$$

جامعة بيرزيت
BIRZEIT UNIVERSITY

2017 2016

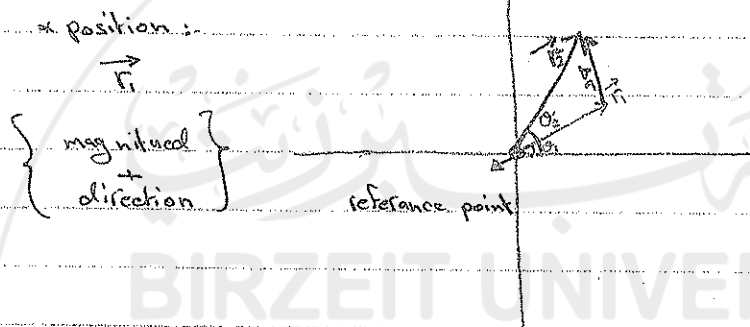
مجلس الطلبة

physics

(L)

1.3. (vectors)

physical quantities $\left\{ \begin{array}{l} \rightarrow \text{scalar (determined by magnitude only)} \\ \rightarrow \text{Vector (determined by magnitude and direction)} \end{array} \right.$
eg. (position, displacement, velocity, acceleration, force)



$\vec{r}_1$: first position

$\vec{r}_2$: second position

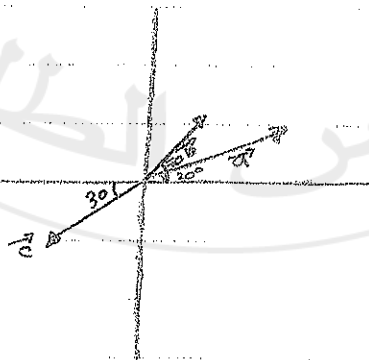
$\Delta \vec{r}$: displacement = change in position (تغير في الموضع)

$$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$$

Adding vectors Geometrically = (جمع المتجهات هندسياً)

$$\vec{r}_2 = \Delta \vec{r} + \vec{r}_1$$

example:

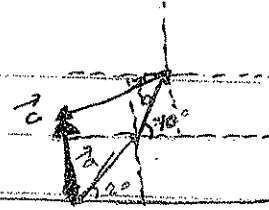


$$a = |\vec{a}| = 5, 20^\circ \text{ with } +x$$

$$b = |\vec{b}| = 3, 70^\circ \text{ with } +x$$

$$c = |\vec{c}| = 4, 30^\circ \text{ with } -x$$

$$\text{find } \vec{a} + \vec{b} + \vec{c}$$



وبالنسبة لنقل فاصلياً ذيل الأول

فأشياء أخرى

في حالة عدم استخدام المنقلة والمسطرة = $\vec{a} + \vec{b} + \vec{c} = \vec{0}$
 نقيس طول المتجهات ونحسب الزاوية بين المتجهات

Example 3:

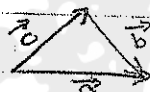


$$\vec{a} + \vec{b} + \vec{c} = \vec{0}$$

$$\vec{a} + \vec{b} = -\vec{c}$$

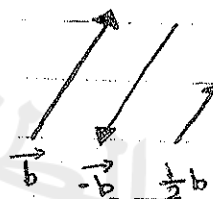
أو

Example 3:

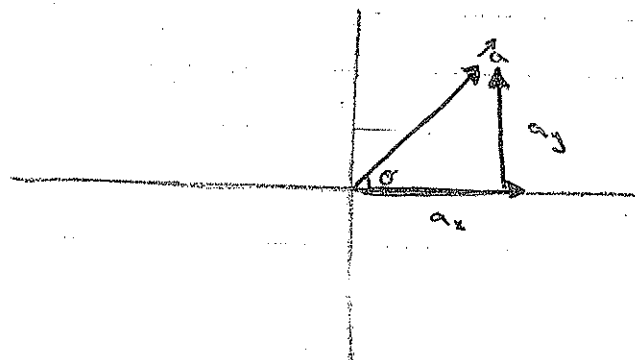


$$\vec{c} + \vec{b} = \vec{a}$$

المثلث كمثلث قائم الزاوية



Component of vectors : (مكونات المتجهات)



$\therefore a$ and θ are given:

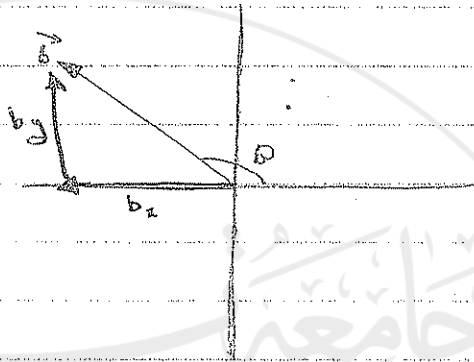
$$a_x = a \cos \theta$$

$$a_y = a \sin \theta$$

another example:

$$b_x = b \cos B \text{ (or } \theta \text{)}$$

$$b_y = b \sin B \text{ (or } \theta \text{)}$$



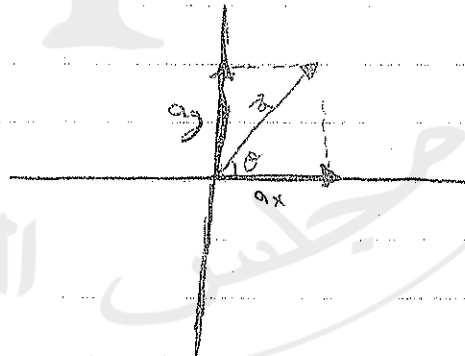
$\therefore$ angle: with x , counter clock wise.

note:

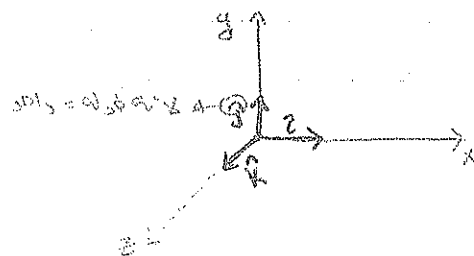
if you are given $\vec{a}_x + \vec{a}_y$, find a and θ .

$$\therefore \vec{a} = \sqrt{a_x^2 + a_y^2}$$

$$\theta = \tan^{-1} \left(\frac{a_y}{a_x} \right)$$



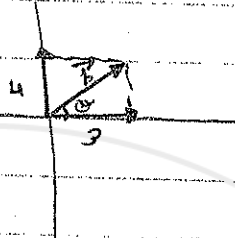
Adding vectors by component:



$$\therefore \vec{a} = a_x \hat{i} + a_y \hat{j}$$

example

$$\vec{b} = 3\hat{i} + 4\hat{j}$$



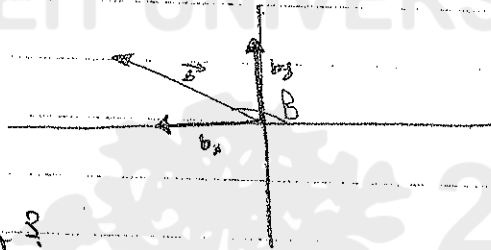
$$\therefore |\vec{b}| = \sqrt{3^2 + 4^2}$$

$$|\vec{b}| = 5$$

$$\textcircled{2} \tan \theta = \frac{4}{3} = 1.3333$$

$$\therefore \theta = 53^\circ$$

example



Find b_x, b_y ?

$$b_x = b \cdot \cos B$$

$$= 15 \cdot \cos 120$$

$$= -7.5$$

$$b_y = 15 \cdot \sin 120$$

$$= 13$$

$$\therefore \vec{b} = -7.5\hat{i} + 13\hat{j}$$

Physics (D)

1.2

Q.70

particle 1, $x = 6t^2 + 3t + 2$

x in m, t in s

particle 2, $a = -8t$, at $t=0$, $v=15$ m/s

when $v_{p1} = v_{p2}$

for p1 $v_{p1} = \frac{dx}{dt} = 12t + 3$

for p2 $a = \frac{dv}{dt}$

$\int dv = \int a dt$

$\therefore v_{p2} = -4t^2 + C$

at $t=0$, $v=15$ m/s

$15 = -4(0) + C \Rightarrow C = 15$

$\therefore v_{p2} = -4t^2 + 15$

$v_{p1} = v_{p2}$

$12t + 3 = -4t^2 + 15$

$4t^2 + 12t - 12 = 0$

$t^2 + 3t - 3 = 0 \Rightarrow t = 0.85$

at $t = 0.85$ s, $v_{p1} = v_{p2} = 12(0.85) + 3 = 13.8$ m/s

1.3

Q.5

$$\vec{b} + \vec{c} = \vec{a}$$

$$\vec{c} = \vec{a} - \vec{b}$$

$$|\vec{c}| = \sqrt{(a)^2 + (b)^2}$$

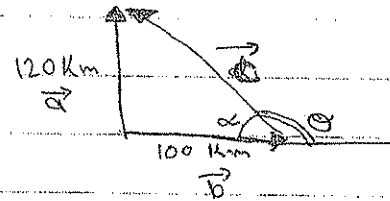
$$= \sqrt{(120)^2 + (100)^2}$$

$$= 156 \text{ km}$$

$$\tan \theta = \frac{100}{120} \Rightarrow \theta = 40^\circ \quad / \quad \tan \theta = \frac{120}{100} \Rightarrow \theta = 50^\circ$$

$\therefore d_1 = 156 \text{ km}$, 40° west of north

in vector notation $\vec{c} = \vec{a} - \vec{b}$
 $\vec{c} = 120\hat{j} - 100\hat{i}$



Q.12

$$\vec{d} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3$$

$$\vec{d} = 40\hat{i} + 30\hat{j} + (25 \cos 60^\circ \hat{i} + 25 \sin 60^\circ \hat{j})$$

$$\vec{d} = 40\hat{i} + 30\hat{j} + 12.5\hat{i} + 21.65\hat{j}$$

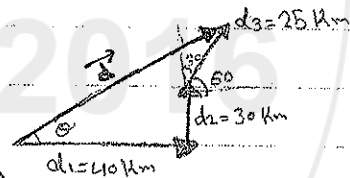
$$\vec{d} = 52.5\hat{i} + 51.65\hat{j}$$

$$|\vec{d}| = \sqrt{(52.5)^2 + (51.65)^2}$$

$$= 73.6 \text{ km}$$

$$\tan \theta = \frac{y}{x} = \frac{51.65}{52.5} \Rightarrow \theta = 44.5^\circ$$

$\therefore \vec{d} = 73.6 \text{ km}$ in 44.5° North of east.



Q. 20

$$\vec{d}_2 + \vec{d}_3 = \vec{d}_1$$

$$\vec{d}_3 = \vec{d}_1 - \vec{d}_2$$

$$\vec{d}_3 = 4.8\hat{j} - (7.8\cos 50^\circ + 7.8\sin 50^\circ\hat{j})$$

$$\vec{d}_3 = 4.8\hat{j} - 5.01\hat{j} - 5.97\hat{j}$$

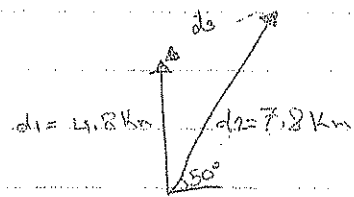
$$\vec{d}_3 = -5.2 - 1.2\hat{j} \text{ km}$$

$$|\vec{d}_3| = \sqrt{(-5)^2 + (-1.2)^2}$$

$$= 5.14 \text{ km}$$

$$\therefore \tan \theta = \frac{1.2}{5} \Rightarrow \theta = 13.5^\circ$$

$$\therefore \vec{d}_3 = 5.14 \text{ km in } 13.5^\circ \text{ south due west}$$



Physics

(L)

Chapter 3

Q. 26

$$\vec{A} = 2\hat{i} + 3\hat{j}$$

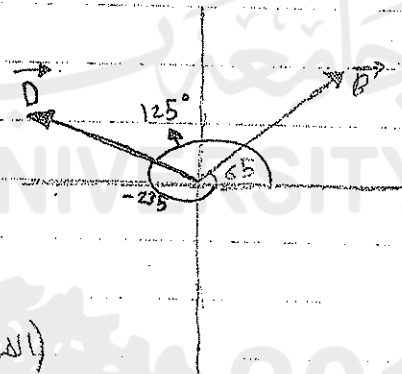
$$\vec{B} = 4 \text{ at } +65^\circ$$

$$\vec{C} = -4\hat{i} - 6\hat{j}$$

$$\vec{D} = 5 \text{ at } -235^\circ$$

$$\vec{S} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

$$\vec{S} = (2\hat{i} + 3\hat{j}) + (4 \cos 65^\circ \hat{i} + 4 \sin 65^\circ \hat{j}) + (-4\hat{i} - 6\hat{j}) + (5 \cos 125^\circ \hat{i} + 5 \sin 125^\circ \hat{j})$$



$$\vec{S} = -3.2\hat{i} + 4.7\hat{j} \text{ (النتيجة النهائية)}$$

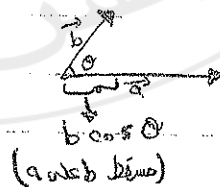
$$* |\vec{S}| = \sqrt{(-3.2)^2 + (4.7)^2} \Rightarrow 5.68$$

$$* \theta = \tan^{-1}\left(\frac{4.7}{-3.2}\right) = -55.7^\circ \approx -56^\circ \text{ (الزاوية بين } \vec{S} \text{ و } \hat{i})$$

Vector Multiplications:

① Dot product (Scalar Product):

$$\vec{a} \cdot \vec{b} = a(b \cos \theta)$$



② if a and b are given by their components then:

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k} \text{ (نفسه القوس)}$$

$$\vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\hat{i} \cdot \hat{j} = \hat{i} \cdot \hat{k} = \hat{j} \cdot \hat{k} = 0$$

example:

* Note: $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} = |\vec{a}| |\vec{b}| \cos \theta$

$\therefore \vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{B} = 2\hat{i} + 6\hat{j} + 7\hat{k}$

Find the angle (θ) between $\vec{A}$ & $\vec{B}$?

$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$

* $|\vec{A}| = \sqrt{(3)^2 + (4)^2 + (5)^2} \Rightarrow 7.1$

* $|\vec{B}| = \sqrt{(2)^2 + (6)^2 + (7)^2} \Rightarrow 9.4$

* $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

$\vec{A} \cdot \vec{B} = 6 + 24 + 35 \Rightarrow -53$

$\therefore$ we have

$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$

$= \frac{-53}{(7.1)(9.4)} \Rightarrow -0.79$

$\therefore \theta = 142^\circ$

* Examples of Dot product:-

- Work = $\vec{F} \cdot d$ joule

- $\phi_E = \vec{E} \cdot \vec{A}$ (electric flux)

- $\phi_B = \vec{B} \cdot \vec{A}$ (magnetic flux)

- $\phi_{\text{fluid}} = \vec{A} \cdot \vec{v}$ (flow rate)

- Power = $\vec{F} \cdot \vec{v}$ watt

2 cross product :- (vector product) :-

$$\therefore \vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta$$

$$\sin \theta \perp \vec{a} \text{ and } \vec{b}$$

$$\vec{b} \times \vec{a} = |\vec{b}| |\vec{a}| \sin \theta$$



$$\vec{a} \times \vec{b}$$

outward (خارج)

$$\vec{b} \times \vec{a}$$

inward (داخل)

$$a \times b = - (b \times a)$$

② if $\vec{a}$ and $\vec{b}$ are given by components then-

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$\vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$$

$$\therefore \vec{a} \times \vec{b} = a_x b_y \hat{k} + a_x b_z (-\hat{j})$$

$$a_y b_x(-k) + a_z b_z(z)$$

$$a_2 b_x \hat{j} + a_2 b_y (-\hat{i})$$

$$a(\hat{i}) + b(\hat{j}) + c(\hat{k})$$

$$2\pi = 8 \times 8 = 64 \times 1 = 64$$

$\hat{L} \times \hat{J} = \hat{K}$ (مصفوفة التباديل)

$$\hat{c} \times \hat{k} = -\hat{z}$$

$$\hat{A} \hat{X} \hat{A} = -\hat{A}$$

$$\hat{f} \times \hat{k} = \hat{c}$$

$$\hat{K} \times \hat{L} = \hat{S}$$

$$\hat{K} \hat{K}^\dagger = -\hat{I}$$



موقوفہ

12/17/20

22

John W. Weeks

Physics

(1)

1.3

Q. 36. $\vec{a}_1 = 3\hat{i} - 2\hat{j} + 4\hat{k}$
 $\vec{a}_2 = 5\hat{i} + 2\hat{j} - \hat{k}$

$(\vec{a}_1 + \vec{a}_2) \cdot (\vec{a}_1 \times \vec{a}_2)$
 $4 (\vec{a}_1 + \vec{a}_2) \cdot (\vec{a}_1 \times \vec{a}_2)$

* $\vec{S} = \vec{a}_1 + \vec{a}_2$

$\vec{S} = 2\hat{i} + 3\hat{k}$

* $\vec{a}_1 \times \vec{a}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 4 \\ 5 & 2 & -1 \end{vmatrix}$

$\vec{C} = \hat{i}(2-8) - \hat{j}(-3-20) + \hat{k}(6-10)$
 $= -6\hat{i} - 17\hat{j} - 4\hat{k}$

then ✓

ii. $4 \vec{S} \cdot \vec{C}$

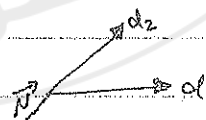
$\vec{S} \cdot \vec{C} = 12 + 12 \Rightarrow \text{zero}$

Note the following:-

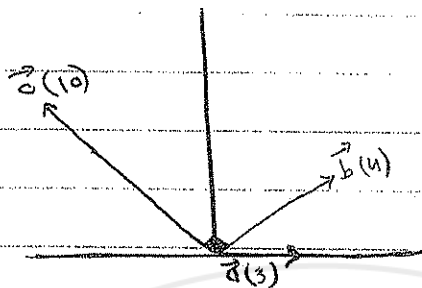
$\vec{a}_1 \cdot (\vec{a}_1 \times \vec{a}_2) = 0$

$\vec{a}_1 \cdot \vec{a}_2 = 1 \perp \vec{a}_1, \vec{a}_2$

$\vec{a}_1 \cdot (\vec{a}_1 \times \vec{a}_2) (\cos 90^\circ) = 0$



Q. 43



$$a_x = 3, a_y = 0$$

$$b_x = 4 \cos 30^\circ, b_y = 4 \sin 30^\circ$$

$$c_x = 10 \cos 120^\circ, c_y = 10 \sin 120^\circ$$

$$\vec{a} = 3\hat{i}$$

$$\vec{b} = 2\sqrt{3}\hat{i} + 2\hat{j}$$

$$\vec{c} = -5\hat{i} + 5\sqrt{3}\hat{j}$$

$$\vec{c} = p\vec{a} + q\vec{b}$$

$$-5\hat{i} + 5\sqrt{3}\hat{j} = 3p\hat{i} + 2\sqrt{3}q\hat{i} + 2q\hat{j}$$

$$\therefore -5 = 3p + 2\sqrt{3}q \quad (1)$$

$$\therefore 5\sqrt{3} = 2q \quad (2)$$

$$\therefore q = 5.6$$

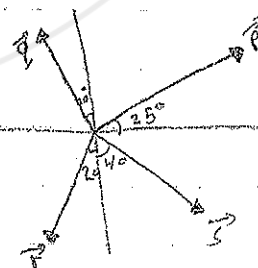
Q. 56

$\vec{p}$: 13 m, at 25° counter clock $y \rightarrow +x$

$\vec{q}$: 12 m, at 10° counter clock $y \rightarrow +y$

$\vec{r}$: 8 m, at 20° clockwise $(-y)$

$\vec{s}$: 9 m, at 40° clock wise $(-y)$



$$\vec{u} = \vec{p} + \vec{q} + \vec{r} + \vec{s}$$

$$\vec{u} = (13 \cos 25^\circ \hat{i} + 13 \sin 25^\circ \hat{j}) + (12 \cos 100^\circ \hat{i} + 12 \sin 100^\circ \hat{j}) + (8 \cos 250^\circ \hat{i} + 8 \sin 250^\circ \hat{j}) + (9 \cos 310^\circ \hat{i} + 9 \sin 310^\circ \hat{j})$$

Q. 64

$$\vec{d}_1 = 4\text{m}$$
$$\vec{d}_2 = 3\text{m}$$

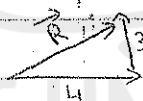
$$* \quad \vec{d}_1 + \vec{d}_2 = \vec{R}$$

$$\begin{array}{c} 4 \rightarrow 3 \\ \hline 7 \rightarrow \vec{R} \end{array} \quad \therefore \text{X}$$

$$* \quad \vec{d}_1 + \vec{d}_2 = \vec{R} \quad (\text{displacement})$$

$$\begin{array}{c} 4 \rightarrow \\ \hline 1 \leftarrow 2 \end{array} \quad \therefore \checkmark$$

$$* \quad \vec{d}_1 + \vec{d}_2 = \vec{R}$$



$$\vec{R} = \sqrt{3^2 + 4^2}$$
$$= 5$$

2017 2016

physics
(L)
chapter 3

ex $\vec{A} = 3\hat{i} - 2\hat{j} + 5\hat{k}$
 $\vec{B} = 6\hat{i} + 4\hat{j} - 7\hat{k}$

① Find the angle between $\vec{A}$ & $\vec{B}$

* $\vec{A} \cdot \vec{B} = AB \cos \phi$

* $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

$\vec{A} \cdot \vec{B} = 18 + (-8) + (-35)$

$\vec{A} \cdot \vec{B} = -25$

نقطة A و B

$A = \sqrt{3^2 + (-2)^2 + 5^2} \Rightarrow 6.17$

$B = \sqrt{6^2 + 4^2 + (-7)^2} \Rightarrow 10$

نقطة $\cos \phi$

$\cos \phi = \frac{\vec{A} \cdot \vec{B}}{AB} \Rightarrow \frac{-25}{6.17 \times 10}$

$\therefore \phi = 114^\circ$

② Find $\hat{B}$ ($\vec{B}$ is a vector)

$\hat{B} = \frac{\vec{B}}{B}$

$= \frac{6\hat{i} + 4\hat{j} - 7\hat{k}}{10}$

$\therefore \hat{B} = 0.6\hat{i} + 0.4\hat{j} - 0.7\hat{k}$

© Find the angle between $\vec{A}$ + x axis.

$$\vec{A} \cdot \hat{i} = A(\hat{i}) \cos \theta \Rightarrow A \cos \theta$$

$$\vec{A} \cdot \hat{i} = 3$$

$$\therefore \cos \theta = \frac{\vec{A} \cdot \hat{i}}{A} = \frac{3}{6.17}$$

$$\therefore \theta = 60^\circ$$

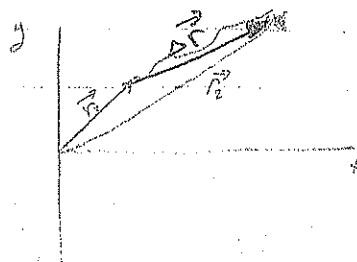
© Find the angle between $\vec{A}$ + y axis

© Find the angle between $\vec{A}$ + z axis

chapter (4)

Motion 2+3 dimensions

Position and displacement :



$$\vec{r}_1 = x_1 \hat{i} + y_1 \hat{j} + z_1 \hat{k} \quad (\text{initial position})$$

$$\vec{r}_2 = x_2 \hat{i} + y_2 \hat{j} + z_2 \hat{k} \quad (\text{final position})$$

$$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$$

(Displacement)

$$\therefore \Delta \vec{r} = (x_2 - x_1) \hat{i} + (y_2 - y_1) \hat{j} + (z_2 - z_1) \hat{k}$$

$$\oplus \Delta \vec{r} = \Delta x \hat{i} + \Delta y \hat{j} + \Delta z \hat{k} \quad (\text{we'll use this})$$

$\therefore$ average Velocity :-

$$\vec{V}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t}$$

$$\therefore \vec{V}_{\text{avg}} = \frac{\Delta x}{\Delta t} \hat{i} + \frac{\Delta y}{\Delta t} \hat{j} + \frac{\Delta z}{\Delta t} \hat{k}$$

$$(\text{السرعة اللحظية}) \quad \vec{V}_{\text{instantaneous}} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t}$$

$$\therefore \vec{V}_{\text{inst}} = \frac{d\vec{r}}{dt}$$

$$\oplus \vec{V}_{\text{inst}} = \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} + \frac{dz}{dt} \hat{k}$$

$$\oplus \vec{a}_{\text{inst}} = \frac{d\vec{V}}{dt} \Rightarrow \frac{d^2x}{dt^2} \hat{i} + \frac{d^2y}{dt^2} \hat{j} + \frac{d^2z}{dt^2} \hat{k}$$

$$= \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j} + \frac{dv_z}{dt} \hat{k}$$

✱ Solve the sample problems p.g (59+63)

$$\boxed{\text{Q.15}} \quad \vec{r}_0 = 0$$

$$\vec{a}_{\text{inst}} = -12\hat{i} - \frac{1}{2}\hat{j}$$

$$\vec{v}_0 = 32\hat{j}$$

$$\text{at } x_{\text{max}}, \text{ find } \vec{v} + \vec{r} \text{ :}$$

(y-motion + x-motion) $\rightarrow$ $\vec{v} + \vec{r}$: $\rightarrow$ $\vec{v} + \vec{r}$: $\rightarrow$ $\vec{v} + \vec{r}$:

- X-motion :-

$$V_{0x} = 3 \text{ m/s}$$

$$x_0 = 0$$

$$a_x = -1 \text{ m/s}^2$$

* At x_{max} $V_x = 0$

$$V_x = V_{0x} + a_x t$$

$$0 = 3 - 1 \times t$$

$$\therefore t = 3 \text{ s to reach } x_{\text{max}}$$

$$\therefore x_{\text{max}} = ?$$

$$x = x_0 + \left(\frac{V_{0x} + V}{2} \right) \cdot t$$

$$x_{\text{max}} = 0 + \left(\frac{3 + 0}{2} \right) \times 3$$
$$= 4.5 \text{ m (x-axis)}$$

- y-motion :-

$$y_0 = 0$$

$$V_{0y} = 0$$

$$a_y = -0.5 \text{ m/s}^2$$

$$* V_y = V_{0y} + a_y t \quad \left(\frac{3.33 \text{ m/s}}{1.1} \right)$$

$$\therefore V_y = 0 + -0.5 \times 3$$

$$\therefore V_y = -1.5 \text{ m/s}$$

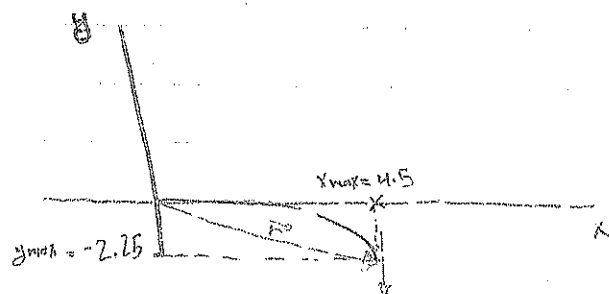
$$\therefore y_{\text{max}} = ?$$

$$y - y_0 = \left(\frac{V_{0y} + V_y}{2} \right) \times t$$

$$y - 0 = \left(\frac{0 + -1.5}{2} \right) \times 3$$

$$= -2.25 \text{ m (y-axis)}$$

$$\therefore \vec{r} = 4.5\hat{i} - 2.25\hat{j} \text{ m}$$



* the direction of $\vec{v}$ is always at the tangent of the path.

physics
(1)

ch. 3

problem

$$\vec{A} = 4\hat{i} + 5\hat{j} + 6\hat{k}$$

$$\vec{B} = -1\hat{i} + 2\hat{j} + 3\hat{k}$$

(a) Find the angle between $\vec{A}$ & $\vec{B}$:

$$* \vec{A} \cdot \vec{B} = AB \cos \theta$$

$$* \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$= -4 + 10 - 18 \Rightarrow -12$$

$$* |\vec{A}| = \sqrt{4^2 + 5^2 + 6^2}$$

$$= 8.8$$

$$* |\vec{B}| = \sqrt{(-1)^2 + 2^2 + 3^2}$$

$$= 3.75$$

$$\therefore -12 = (8.8)(3.75) \cos \theta$$

$$\therefore \theta = 111^\circ$$

(b) Find the angle between $\vec{A}$ & x-axis :

$$\vec{A} \cdot \hat{i} = A \cos \theta = 8.8 \cos \theta$$

$$\vec{A} \cdot \hat{i} = 4$$

$$\cos \theta = \frac{4}{8.8}$$

$$\therefore \theta = 63^\circ$$

③ Find the angle between $\vec{A}$ and y-axis?

$$\vec{A} \cdot \hat{j} = 8.8 \cos \beta$$

$$\vec{A} \cdot \hat{j} = 5$$

$$\therefore \cos \beta = \frac{5}{8.8}$$

$$\therefore \beta = 55.4^\circ$$

* additional :-

$$\cos \alpha = \frac{A_z}{A} \Rightarrow \frac{6}{8.8}$$

$$\therefore \alpha = 13.2^\circ$$

④ Find $\hat{B} = \frac{\vec{B}}{B}$

$$= \frac{-1\hat{i} + 2\hat{j} + 3\hat{k}}{3.45}$$

$$= -0.29\hat{i} + 0.53\hat{j} + 0.8\hat{k}$$

⑤ Find a vector perpendicular to $\vec{A} + \vec{B}$?

$$\vec{C} = \vec{A} \times \vec{B}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -5 & 8 \\ 4 & 5 & -6 \end{vmatrix}$$

$$\vec{C} = (15 - 20)\hat{i} - (12 - 6)\hat{j} + (8 - 5)\hat{k}$$

$$\vec{C} = -5\hat{i} - 6\hat{j} + 3\hat{k}$$

$$\vec{C} \perp \vec{A} + \vec{B}$$

chapter 4



$$\vec{r} = 36\hat{i} - 4t^2\hat{j} + 2\hat{k}$$

Q.6

$$(a) \vec{v}(t) = \frac{d\vec{r}}{dt}$$

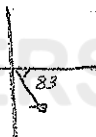
$$v(t) = 32 - 8t\hat{j}$$

$$(b) \vec{v}(3) = 32 - 24\hat{j} \text{ m/s}$$

$$(c) v(3) = \sqrt{(3)^2 + (-24)^2} = 24.2 \text{ m/s}$$

$$(d) \tan \phi = \frac{-24}{3}$$

$$\phi = -83^\circ$$



$\therefore v(3) = 24.2 \text{ m/s}$ at 83° clockwise with $+x$ -axis

additional:

(e) Find the average velocity from $t_1 = 3s \rightarrow t_2 = 10s$.

$$\therefore v_{avg} = \frac{\Delta \vec{r}}{\Delta t} \Rightarrow \frac{\vec{r}(10) - \vec{r}(3)}{7}$$

$$= \frac{(302 - 400\hat{j} + 2\hat{k}) - (92 - 36\hat{j} + 2\hat{k})}{7}$$

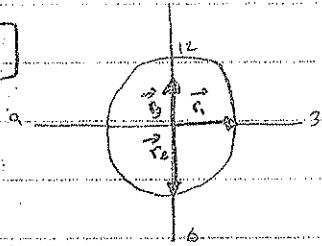
$$= \frac{212 - 364\hat{j}}{7}$$

$$= 32 - 52\hat{j}$$

$$(f) \vec{v}(10) = 32 - 80\hat{j}$$

$$(g) \vec{a}(t) = \frac{d\vec{v}}{dt} = -8\hat{j} \text{ m/s}^2$$

Q.4



$$r = 12 \text{ cm}$$

$$\textcircled{a} \vec{r}_1 = 12\hat{i}$$

$$\vec{r}_2 = -12\hat{j}$$

$$\vec{DC} = \vec{r}_2 - \vec{r}_1$$

$$\vec{DC} = -12\hat{i} - 12\hat{j}$$

$$DC = \sqrt{(12)^2 + (12)^2} = 17 \text{ cm}$$



$\therefore \vec{DC}$ is 17 cm at 225° counter clockwise

with +X-axis

2017 $\textcircled{b}$ Find $\vec{DC}$ after $\frac{1}{2} h$, from $\vec{r}_1$

$$\vec{r}_1 = 12\hat{j}$$

$$\therefore \vec{DC} = \vec{r}_1 - \vec{r}_2$$

$$= 12\hat{j} - 12\hat{j}$$

$$= 24\hat{j}$$

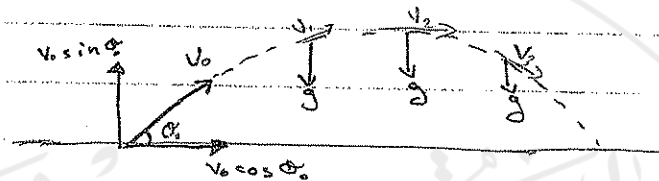
physics (L)

Ch. 4

القذوفات

Projectile Motion :-

it is a motion in two dimensions



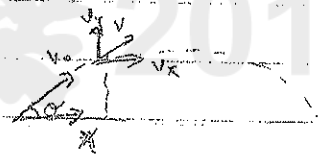
X-motion is independent from y-motion, could be consider to be 2 linear motion

X-motion :-

V_x is constant

$$\therefore V_x = V_0 \cos \theta_0$$

$$* X - X_0 = (V_0 \cos \theta_0) \times t \quad (1)$$



Y-motion :-

$$V_{0y} = V_0 \sin \theta_0$$

$$a_y = -g$$

$$V_y = V_{0y} + a_y t$$

$$\therefore V_y(t) = V_0 \sin \theta_0 - g t$$

$$y(t) - y_0 = V_{0y} t + \frac{1}{2} a_y t^2$$

$$\therefore y(t) - y_0 = (V_0 \sin \theta_0) t - \frac{1}{2} g t^2 \quad (2)$$

$$* \text{ from (1) } t = \frac{x}{V_0 \cos \theta_0} \Rightarrow (2) \Rightarrow y = x \tan \theta_0 - \frac{g x^2}{2 V_0^2 \cos^2 \theta_0}$$

$$\therefore y(t) - y_0 = (V_0 \sin \theta_0) \frac{x}{V_0 \cos \theta_0} - \frac{1}{2} g \frac{x^2}{V_0^2 \cos^2 \theta_0}$$

$$\therefore y = (\tan \theta) x - \frac{g \cdot x^2}{2 \cdot V_0^2 \cdot \cos^2 \theta_0}$$

trajectory path

Find the time of fly :-

up = $\frac{1}{2} g t^2$; $\sin \theta$; $\frac{1}{2} g t^2$

$$y = V_{0y} t + \frac{1}{2} g t^2$$

$$0 = V_0 \sin \theta t - \frac{1}{2} g t^2$$

$$t = \frac{2 V_0 \sin \theta}{g} \quad (\text{time of fly})$$

* horizontal ^{cos θ} range = $(V_0 \cos \theta) t_{\text{flight}} = R$

$$\therefore R = V_0^2 \left(\frac{2 \sin \theta \cos \theta}{g} \right)$$

$$R = \frac{V_0^2}{g} \times \sin 2\theta$$

* time to reach $y_{\text{max}} = \frac{1}{2} \text{ time of fly}$

$$\therefore = \frac{V_0 \sin \theta}{g}$$

* at y_{max} $V_y = 0$, find y_{max} ?

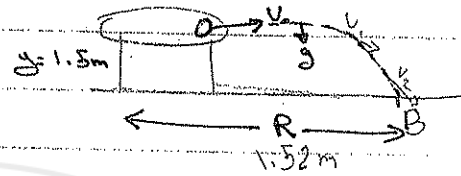
$$V_y^2 = V_{0y}^2 + 2 g y$$

$$0 = (V_0 \sin \theta)^2 + 2(-g) y_{\text{max}}$$

$$\therefore y_{\text{max}} = \frac{V_0^2 \sin^2 \theta}{2g}$$

ex - Q.22

$$V_{0x} t = R$$



② y motion :-

$$V_{0y} = 0$$

$$a_y = -g$$

$$y - y_0 = -1.5\text{m}$$

$$y - y_0 = V_{0y} t + \frac{1}{2} a t^2$$

$$-1.5 = 0 + \frac{1}{2} (-9.8) t^2$$

$$t = \sqrt{\frac{2 \times 1.5}{9.8}} = 0.55\text{s}$$

③ $x = V_{0x} \times t$ (given)

$$1.52 = V_{0x} \times 0.55$$

$$V_{0x} = 2.75 \text{ m/s}$$

④ additional :-

Find V, V_x, V_y at $t = 0.55\text{s}$

$$V_x = 2.75 \text{ m/s (const)}$$

$$V_y = V_{0y} + a_y t$$

$$V_y = 0 - 9.8 \times 0.55$$

$$\therefore V_y = -5.39 \text{ m/s}$$

$$V = \sqrt{(V_x)^2 + (V_y)^2}$$

$$V = \sqrt{(2.75)^2 + (-5.39)^2}$$

physics

(1)

ch. 4

Q. 11

$$a_x = 4 \text{ m/s}^2, \quad v_{0x} = 8 \text{ m/s}$$

$$a_y = -2 \text{ m/s}^2, \quad v_{0y} = 12 \text{ m/s}$$

① $\vec{v}$? when y is max

at y_{max} , $v_y = 0$

$$v_y = v_{0y} + a_y t$$

$$0 = 12 + (-2)t$$

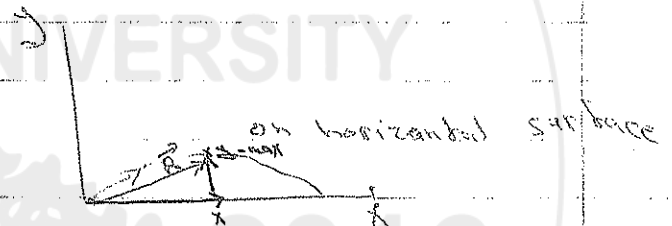
$$\therefore t = 6 \text{ s [to reach } y_{\text{max}}]$$

$$\therefore v_x = v_{0x} + a_x t$$

$$v_x = 8 + 4 \times 6$$

$$v_x = 32 \text{ m/s}$$

$$\vec{v} = 32 \hat{i} \text{ m/s}$$



② additional :-

Find $x+y$ coordinate at y_{max} ?

$$y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$$

$$y_{\text{max}} = 12 \times 6 + \frac{1}{2} (-2) (6)^2$$

$$y_{\text{max}} = 36 \text{ m}$$

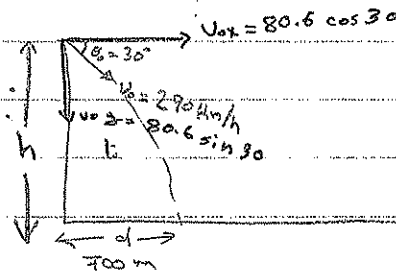
$$x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2$$

$$x = 8 \times 6 + \frac{1}{2} \times 4 \times 36$$

$$x = 120 \text{ m}$$

$$\vec{R} = 120 \hat{i} + 36 \hat{j}$$

Q. 24



$$v_0 = 290 \text{ km/h}$$

$$v_0 = 80.6 \text{ m/s}$$

$$t, h = ?$$

(a) $a_y = -9.8 \text{ m/s}^2$

$$a_x = 0$$

$$x - x_0 = v_{0x} t \quad (\text{horizontal displacement})$$

$$700 = 80.6 \cos 30 \cdot t$$

$$t = 10 \text{ s}$$

$$y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2 \quad (\text{vertical displacement})$$

$$h = -(80.6 \sin 30)(10) + \frac{1}{2} (-9.8)(10)^2$$

$$h = -893 \text{ m}$$

(b) additional:

Find $v_x + v_y$ at point B

① $v_x = v_{0x} = 80.6 \cos 30 = 70$

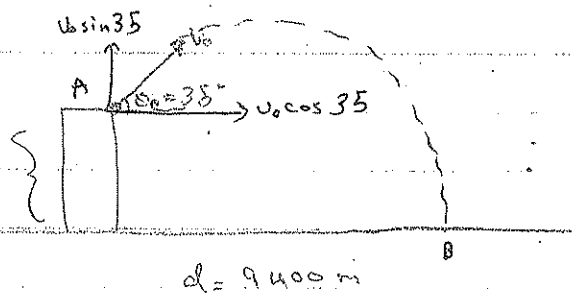
② $v_y = v_{0y} + a_y t$
 $= -80.6 \sin 30 + (-9.8)(10)$

$$= -138.3 \text{ m/s}$$

$$\therefore \vec{v} = 70\hat{i} - 138.3\hat{j}$$

Q. 91

$h = 3300 \text{ m}$



$$u \sin 35 = 0$$

$$x - x_0 = v_{0x} t$$

$$9400 = u \cos 35 \cdot t$$

$$u t = \frac{9400}{\cos 35} \quad \text{--- (1)}$$

$$y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$$

$$-3300 = (u \sin 35) t + \frac{1}{2} (-9.8) t^2$$

$$-3300 = (u t) \sin 35 - 4.9 t^2 \quad \text{--- (2)}$$

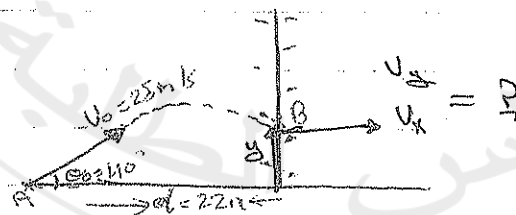
Substituting (1) into (2)

$$-3300 = \frac{9400}{\cos 35} \sin 35 - 4.9 t^2$$

$$t = 45 \text{ s}$$

$$u = 255 \text{ m/s}$$

Q. 32



المسألة: إذا تم إطلاق جسم بسرعة $u_0 = 25 \text{ m/s}$ بزاوية 40° مع الأفق، فاحسب المسافة الأفقية التي يقطعها الجسم قبل أن يسقط.

$$\textcircled{a} \quad x_0 = v_{0x} t$$

$$d = v_0 \sin 40^\circ t$$

$$22 = 19.15 t$$

$$t = 1.15 \text{ s} \quad (B \leftarrow A \text{ و } \sin 40^\circ)$$

$$\therefore y = y_0 = v_{0y} t + \frac{1}{2} a_y t^2$$

$$y = (25 \sin 40^\circ)(1.15) + \frac{1}{2} (-9.8)(1.15)^2$$

$$y = +12 \text{ m}$$

$$\begin{aligned} * \text{ at } B \quad v_x &= v_{0x} = 25 \cos 40^\circ \\ &= 19.15 \text{ m/s} \end{aligned}$$

$$\therefore v_y = v_{0y} + a_y t$$

$$v_y = 25 \sin 40^\circ + (-9.8)(1.15)$$

$$v_y = +4.43 \text{ m/s}$$

في الحيز رأسي قبل و بعد في الحيز الأفقي

physics

(L)

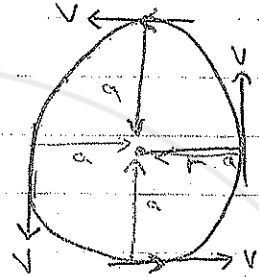
ch. 4

Uniform Circular :-

v is constant in magnitude

$$V = \frac{2\pi r}{T} \quad (T = \text{periodic time})$$

the direction of v is at the tangent of the circle.



example :-

Q. 56



* s is at 750 km above the earth's surface

* T : 98 m

① $V = ?$

② $a = ?$

① $V = \frac{2\pi r}{T}$

(r = earth's radius = 6400 km)

$\therefore r = r_{\text{earth}} + r_s = 6400 + 750 = 7150 \text{ km}$

$\therefore V = \frac{2\pi (7150 \times 10^3)}{98 \times 60}$

$V = 7640 \text{ m/s}$

② $a = \frac{V^2}{r}$

$a = \frac{(7640)^2}{7150 \times 10^3} \text{ m/s}^2$

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

$$a = \frac{V^2}{r} \quad (\text{centripetal acceleration})$$

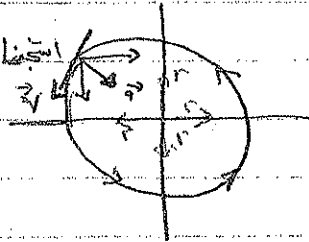
* a is always towards the center

Q. 60

$$T = 2s$$

$$r = 3.5m$$

$$\text{at } t=1, \vec{a} = 6\vec{i} - 4\vec{j} \text{ m/s}^2$$



$$\text{a) } \vec{V} \cdot \vec{a} = ?$$

$$\vec{V} \cdot \vec{a} = zero \quad (\cos 90)$$

$$\text{b) } \vec{r} \times \vec{a} = ?$$

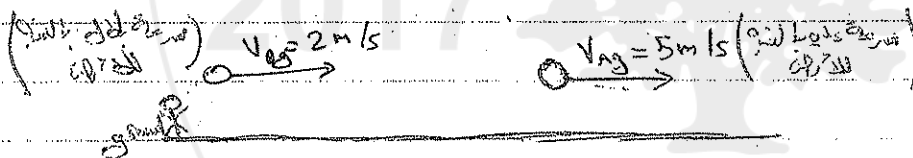
$$\vec{r} \times \vec{a} = |\vec{r}| |\vec{a}| \sin 180$$

$$= zero$$

$$\begin{aligned} \vec{V} &\perp \vec{a} \\ \vec{V} &\perp \vec{r} \\ \vec{a} &\parallel \vec{r} \end{aligned}$$

Relative Motion :- (الحركة النسبية)

example in one dimension :-

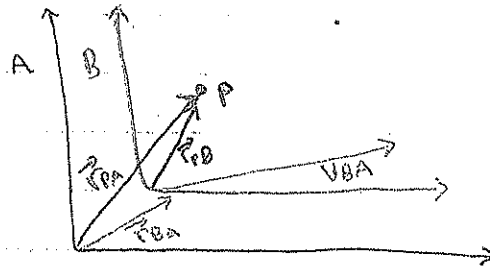


$$\begin{aligned} \vec{V}_{AB} &= \vec{V}_{Ag} - \vec{V}_{Bg} \\ &= 5 - 2 \Rightarrow 3 \text{ m/s} \end{aligned}$$

$$\begin{aligned} \vec{V}_{BA} &= \vec{V}_{Bg} - \vec{V}_{Ag} \\ &= 2 - 5 \Rightarrow -3 \text{ m/s} \end{aligned}$$

Two dimensions 2-

- frame: A at rest.
- frame: B is moving at constant $\vec{V}_{BA}$.



- They are observing point P

$\therefore$ from the graph 2-

$$\vec{r}_{PA} = \vec{r}_{BA} + \vec{r}_{PB}$$

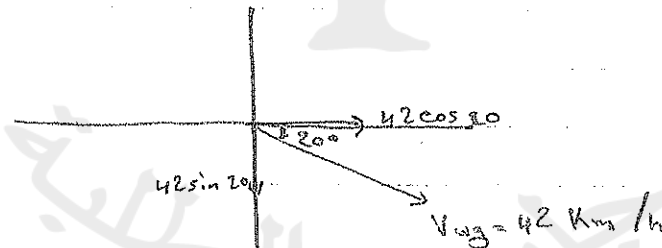
$$\vec{V}_{PA} = \vec{V}_{BA} + \vec{V}_{PB}$$

$$\vec{Q}_{PA} = \vec{Q}_{BA} + \vec{Q}_{PB} \quad \left(\begin{array}{l} \vec{Q}_{BA} = 0 \\ \vec{Q}_{PB} = 0 \end{array} \right)$$

$$\vec{Q}_{PA} = \vec{Q}_{PB}$$

example 2

Q. 74



$$V_{Pg} = \frac{55 \text{ km}}{(18/60) \text{ h}} \Rightarrow 183 \text{ km/h}$$



$$\vec{V}_{Pg} = \vec{V}_{Pw} + \vec{V}_{Bg}$$

$$\vec{V}_{Pw} = \vec{V}_{Pg} - \vec{V}_{Bg}$$

$$\vec{V}_{Pw} = 183\hat{j} - [42 \cos 20\hat{i} - 42 \sin 20\hat{j}]$$

$$\vec{V}_{Pw} = 183\hat{j} - 39.5\hat{i} + 14.4\hat{j}$$

$$= -39.5\hat{i} + 197.4\hat{j}$$

$$|V_{Pw}| = \sqrt{(-39.5)^2 + (197.4)^2} \Rightarrow 200 \text{ km/h}$$

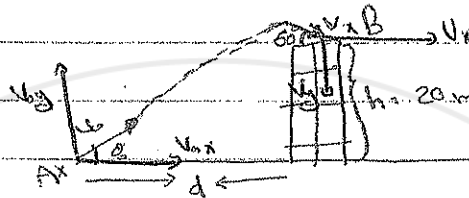
$$\theta = \tan^{-1} \left(\frac{197.4}{-39.5} \right)$$

physics

(D)

ch. 4

Q. 48



$$t_{AB} = 4.5 \text{ s}$$

$$d = ?$$

$$V_0 = ?$$

$$\theta_0 = ?$$

$$\begin{aligned} \textcircled{1} \quad y - y_0 &= V_{0y}t + \frac{1}{2} a_y t^2 \\ + 20 &= V_{0y}(4.5) + \frac{1}{2} (-9.8) (4.5)^2 \\ V_{0y} &= 26.5 \text{ m/s} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad V_y &= V_{0y} + a_y t \\ V_y &= 26.5 - (9.8 \times 4.5) \\ V_y &= -18.5 \text{ m/s} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad \text{in } 18.5 &= V \sin 60 \\ \therefore V &= 20.3 \text{ m/s} \end{aligned}$$

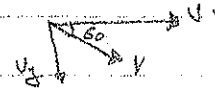
$$\begin{aligned} \textcircled{4} \quad V_x &= V \cos 60 \\ V_x &= 20.3 \times \cos 60 \\ V_x &= 10.15 \text{ m/s} \end{aligned}$$

$$\textcircled{5} \quad V_x = V_{0x} = 10.15$$

$$\therefore V_0 = \sqrt{(V_{0x})^2 + (V_{0y})^2}$$

$$V_0 = 28.4 \text{ m/s}$$

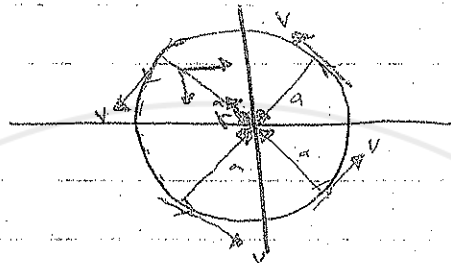
$$\textcircled{6} \quad \theta_0 = 69^\circ$$



$$(0 = a_x \cos 60) \quad (\text{initial } V_{0x} \cos 60)$$

$$\begin{aligned} \textcircled{5} \quad d &= v_0 \times t_{AB} \\ &= 10.15 \times 4.5 \\ &= 45.7 \text{ m} \end{aligned}$$

Q. 68



$$T = 2s / r = 3.5 \text{ m}$$



$$\textcircled{a} \quad \vec{v} \cdot \vec{a} = 0 = |\vec{v}| |\vec{a}| \cos 90^\circ$$

$$\textcircled{b} \quad \vec{r} \times \vec{a} = |\vec{r}| |\vec{a}| \sin 180^\circ = 0$$

Q. 68 at $t_1 = 2s$, $\vec{v}_1 = 3\hat{i} + 4\hat{j}$
at $t_2 = 5s$, $\vec{v}_2 = -3\hat{i} - 4\hat{j}$

$$\textcircled{a} \quad \vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

$$\therefore \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} \Rightarrow \frac{-6\hat{i} - 8\hat{j}}{3}$$

$$\therefore \vec{a}_{avg} = -2\hat{i} - 2.67\hat{j} \text{ m/s}^2$$

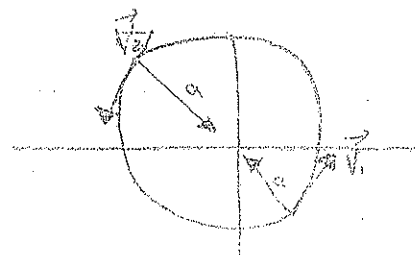
ب) بعد التغير في السرعة $\vec{v}_1$ في اتجاه السرعة $\vec{v}_2$ في الاتجاه الثاني
لا تسمى السرعة بالمتجه $\vec{v}_1$ و $\vec{v}_2$ في الاتجاه الثاني

$$62 - 61 = 3s \left(\frac{T}{2} \right)$$

$$T = 6 \text{ sec}$$

$$\left(\frac{\text{السرعة}}{\text{الزمن}} = \frac{\text{المسافة}}{T} \right) \therefore v = \frac{2\pi r}{T} \Rightarrow r = \frac{5(6)}{2(3)}$$

$$\therefore a = \frac{v^2}{r} = \frac{5^2}{4.8}$$



Q. 76

$$\vec{V}_{PA} = \vec{V}_{PB} + \vec{V}_{BA}$$

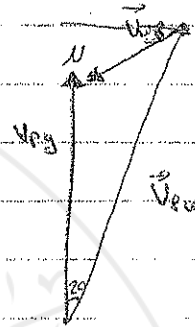
P: object

A: rest frame

B: moving frame (V_{BA})

$$\therefore V_{BW} = 500 \text{ km/h (going west)} \quad \text{(في اتجاه الغرب)}$$

$$V_{Pg} = \frac{900}{2} = 450 \text{ km/h}$$



$$\vec{V}_{Pg} = \vec{V}_{BW} + \vec{V}_{Pg}$$

(في اتجاه الغرب) (في اتجاه الغرب) (في اتجاه الغرب)

$$\vec{V}_{Pg} = \vec{V}_{BW} - \vec{V}_{Pg}$$

$$\vec{V}_{Pg} = 450 \hat{j} - (500 \sin 20^\circ \hat{i} + 500 \cos 20^\circ \hat{j})$$

$$= -171 \hat{i} - 20 \hat{j} \text{ km/h}$$

$$|\vec{V}_{Pg}| = \sqrt{(-171)^2 + (-20)^2} = 172 \text{ km/h}$$

$$\theta = \tan^{-1} \left(\frac{20}{171} \right) \text{ South of west}$$

Physics

(L)

ch. 5

Force and Motion :-

* chapters (1+2+3 +4) description of motion (Kinematics)

* chapters (5+6) Cause of motion (Dynamics)

Newton's Laws of motion :-

① Newton's first law :-

If no force acts on a body, the body's Velocity cannot change, that is the body has no acceleration.

From this law, we understand the following :-

(مسألة) [a] $\vec{F}_{net} = 0$, on the body, $\vec{v}$ of the body = constant / zero

[b] $\vec{F}$ is an external agent, try to change the state of the body
(مقاومة التغير في الحالة، مقاومة التسارع، التسارع)

[c] Inertial law (قانون القصور الذاتي) :- mass cannot change it's state alone

② Newton's second law :-

$$\vec{F}_{net} = m \cdot \vec{a}$$

$$\vec{F}_{net x} = m \cdot \vec{a}_x$$

$$\vec{F}_{net y} = m \cdot \vec{a}_y$$

From this law, we understand the following :-

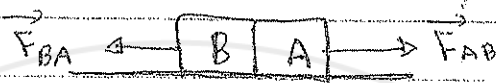
① $F = \text{kg} \cdot \text{m/s}^2 = 1 \text{ Newton}$

$\frac{1 \text{ m}}{1 \text{ s}^2} \rightarrow 1 \text{ N}$

② $F = m \cdot a \Rightarrow 5 = (1) a_1 / 5 = (2) a_2 / 5 = (3) a_3$, i.e. the mass try to resist the force (مقاومة التسارع)، so the mass is called (inertial mass).

③ Newton's third law:-

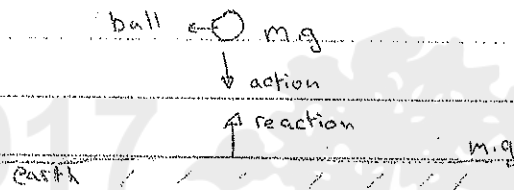
When two bodies interact, each body exerts a force on the other body, the two forces are equal in magnitude and opposite in direction.



$$\therefore \vec{F}_{AB} = (-) \vec{F}_{BA} \quad , \quad F_{AB} = F_{BA}$$

From this law, we understand the following:-

- Ⓐ There is no single force.
- Ⓑ The action acts on body (A), the reaction acts on body (B).
- Ⓒ The action and the reaction acts at the same moment.



falling ball is in the state of free fall.

* Application of Newton's law:-

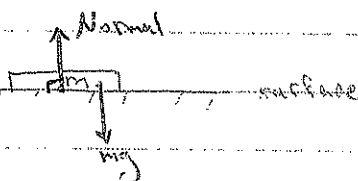
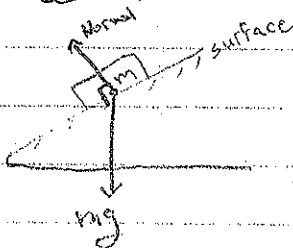
Ⓐ weight = the gravitational force acts on the body.

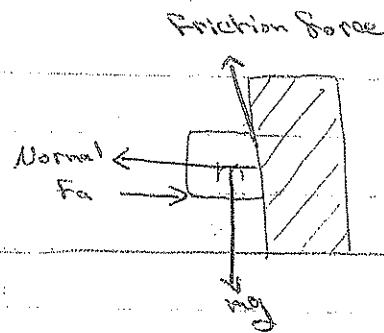
$$\vec{F}_{net} = m\vec{a}$$

$$\vec{F}_{net} = m\vec{g}$$

$$\vec{a} = \vec{g} = 9.8 \text{ m/s}^2$$

Ⓑ Normal force = it's the force from the supporting surface on the body.





Physics

(1)

Q.70

Sam (A) at rest ——— P

Frame (B) $\vec{v}$ constant

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$$



$$\vec{v}_{bw} = 142 \text{ km/h}$$

$$\vec{v}_{wg} = 8.2(-\hat{i}) \text{ km/h}$$

(a+b) $\vec{v}_{bg} = ?$

$$\vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg}$$

$$\vec{v}_{bg} = 142 - 8.2\hat{i}$$

$$= 5.82 \text{ km/h}$$

(c) $\vec{v}_g = ?$

$$\vec{v}_g = \vec{v}_{bg} + \vec{v}_{bw}$$

$$= -6.8 + 5.82$$

$$= -0.2\hat{i} \text{ km/h}$$

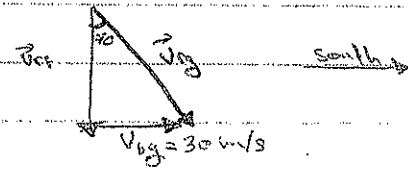


Q.75 $\vec{V}_{bg} = 30 \text{ m/s}$ (south)

① $\sin 70^\circ = \frac{V_{bg}}{V_{rg}}$

$\sin 70^\circ = \frac{30}{V_{rg}}$

$\therefore V_{rg} = \frac{30}{\sin 70^\circ} \Rightarrow 32 \text{ m/s}$



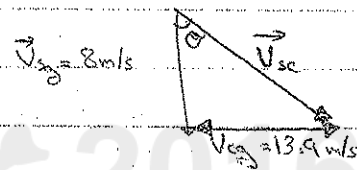
② $\cos 70^\circ = \frac{V_{rt}}{V_{rg}}$

$V_{rt} = \frac{\cos 70^\circ}{32} \Rightarrow 10.9 \text{ m/s}$

Q.77

① $\tan \theta = \frac{13.9}{8}$

$\theta = 60^\circ$



$\vec{V}_{sg} = \vec{V}_{sc} + \vec{V}_{cg}$

Q.20 $\vec{a} = 0.4 \text{ m/s}^2$ (left)

$\theta = 2$

$A \rightarrow C = x$

* For A $\Rightarrow x = 3t$ --- ①

* For B $\Rightarrow \vec{d}_{B \rightarrow C} = \vec{V}_0 t + \frac{1}{2} \vec{a} t^2$

$\vec{d}_{B \rightarrow C} = 0 + \frac{1}{2} \times 0.4 (t)^2$

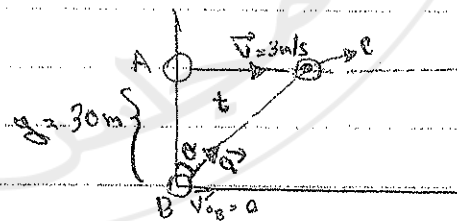
$\vec{d}_{B \rightarrow C} = 0.2 t^2$ --- ②

* $\vec{d}_{B \rightarrow C}$ lies $\rightarrow d^2 = x^2 + y^2$ --- ③

$d^2 = (3t)^2 + (30)^2$

$(0.2t^2)^2 = 9t^2 + 900 \Rightarrow 0.04t^4 - 9t^2 - 900 = 0$

$t^4 - 225t^2 - 22500 = 0 \Rightarrow t^2 = (-) \pm 225 \pm \sqrt{(225)^2 - 4(1)(-22500)}$



Physics
(L)
ch.5

normal force :-

Q.49 $M = 5 \text{ kg}$

Find the acceleration?

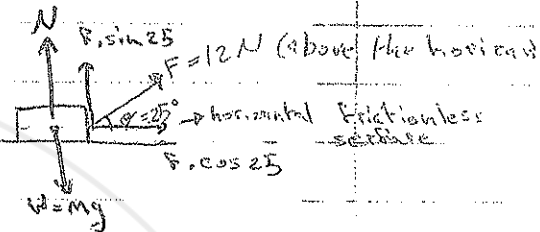
a) $\vec{F}_{\text{net}} = m\vec{a}_x$

$(\vec{F}_{\text{net}})_x = m \cdot a_x$

$F \cdot \cos 25^\circ = m \cdot a_x$

$a_x = \frac{F \cdot \cos 25^\circ}{m}$

$= \frac{12 \cdot \cos 25^\circ}{5}$



b) Find Normal force

$(\vec{F}_{\text{net}})_y = m a_y$ (sub = 0 because it's not moving vertically)

$(\vec{F}_{\text{net}})_y = 0$

$N + F \sin 25^\circ - mg = 0$

$N = mg - F \sin 25^\circ$

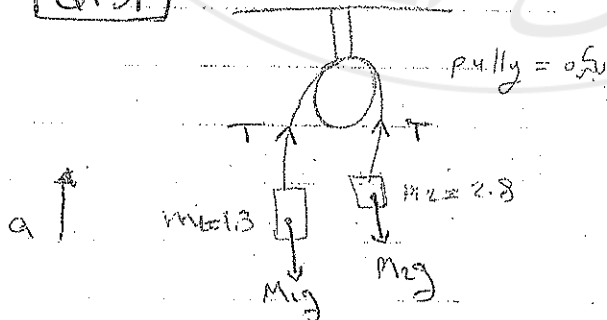
$N = 43.9 \text{ N}$

c) $\vec{F} = 0$ just before m is lifted

→ still at 0

Tension Force :- (T)

Q.51



$(a = 0.31 \text{ or } 0.32)$

$$\vec{a}, T = ?$$

The equation of motion for m_1 :-

$$m_1 a = (F_{\text{net}})_1$$

$$m_1 a = T - M_1 g \quad \text{--- (1)}$$

The equation of motion for m_2 :-

$$(-m_2 a = T - m_2 g \quad \text{--- (2)}) \times (-1)$$

$$m_2 a = m_2 g - T \quad \text{--- (2')}$$

$$\text{is } (1) + (2')$$

$$(m_1 + m_2) a = m_2 g - m_1 g$$

$$a = \frac{(m_2 - m_1) g}{(m_1 + m_2)}$$

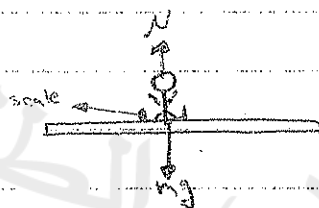
$$a = \frac{1.5 \times 9.8}{4.1} \Rightarrow a = 3.6 \text{ m/s}^2$$

$$\text{From (1)} \quad T = m_1 a + m_1 g$$

$$T = m_1 (a + g)$$

$$T = 11.42 \text{ N}$$

Elevator motion:-



(a) elevator at rest or moving with constant $\vec{v}$:-

$$m a = N - m g \Rightarrow 0 = N - m g \Rightarrow N = m g$$

(b) elevator moves upward with $\vec{a}$:-

$$(\text{you feel heavy}) \quad m a = N - m g \Rightarrow N = m a + m g$$

(c) elevator moves downward with $\vec{a}$:-

$$(\text{you feel light}) \quad -m a = N - m g \Rightarrow m a = m g - N \Rightarrow N = m g - m a$$

(d) elevator moves downward with decelerations:-

$$\therefore N = m a + m g$$

Physics

(D)

ch. 3

Q. 10

$$m = 0.150 \text{ kg}$$

$$x(t) = -13 + 2t + 4t^2 - 3t^3$$

$$\vec{F}_{\text{net}} \text{ at } t = 2.6$$

$$\therefore \vec{v}(t) = \frac{dx}{dt} = 2 + 8t - 9t^2$$

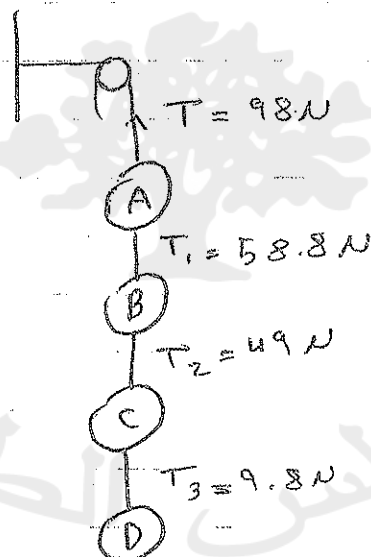
$$\vec{a}(t) = \frac{dv}{dt} = 8 - 18t$$

$$\therefore \vec{a}(2.6) = 8 - 18(2.6) = -38.8 \text{ m/s}^2$$

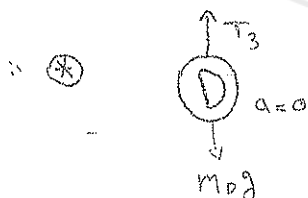
$$\therefore \vec{F}_{\text{net}} = m\vec{a} \Rightarrow -38.8 \times 0.15 = -5.82 \text{ N}$$

Q. 13

$$m_A = m_B = m_C = m_D = 1 \text{ kg}$$

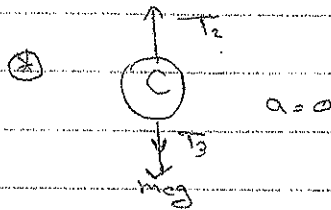


المسألة 13



$$\therefore \text{for } D: T_3 = m_D g$$

$$\therefore m_D = 1 \text{ kg}$$

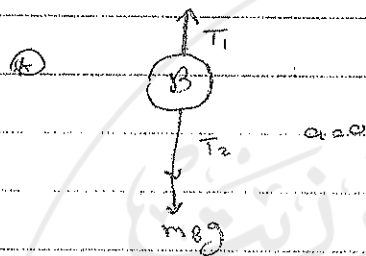


∴ For C :-

$$T_2 = T_3 + mg$$

$$\frac{T_2 - T_3}{g} = m_c$$

$$∴ m_c = 4 \text{ Kg}$$



For B :-

$$T_1 = T_2 + mg$$

$$m_B = \frac{T_1 - T_2}{g}$$

$$∴ m_B = 1 \text{ Kg}$$



For A :-

$$T = T_1 + mg$$

$$m_a = \frac{T - T_1}{g}$$

$$∴ m_a = 4 \text{ Kg}$$

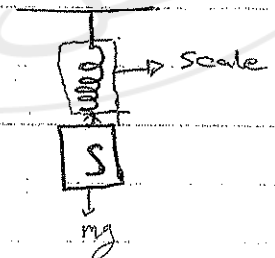
Q.15

$$m_s = 11 \text{ Kg}$$

(a) scale reading = mg

$$= (11)(9.8)$$

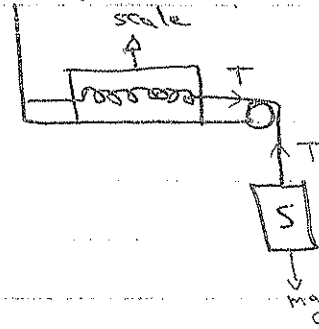
$$= 170.8 \text{ N}$$



$$S \Rightarrow T = mg \text{ (at rest)}$$

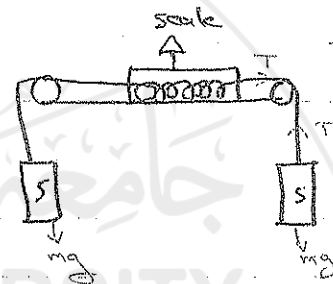
(b)

$$\therefore \text{scale reading} = mg \\ = 107.8 \text{ N}$$



(c)

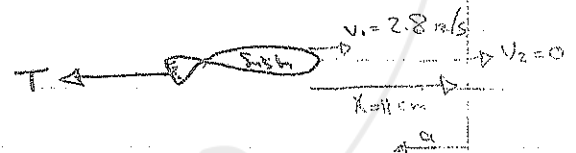
$$\text{Scale reading} = 107.8 \text{ N}$$



(الميزان يقياس لشيء أيضاً، فيه (a) تبقوا السقف، وفي (b) تبقوا الطاولة، وفي (c) تبقوا السكة الأرضية، لا تبقوا إذا كان السقف مع السكة الأرضية يكون قراءته = 0)

Q.26

$$N = mg \Rightarrow \text{الحركة أفقياً}$$



motion is horizontally

$$v_2^2 = v_1^2 + 2a \Delta x$$

$$0 = (2.8)^2 + 2a \cdot 0.1$$

$$a = -35.6 \text{ m/s}^2$$

$$\textcircled{a} T = ma$$

$$W = 90 \Rightarrow m = 9.2 \text{ kg}$$

$$\therefore T = 9.2 \times -35.6 \Rightarrow -326 \text{ N}$$

Q.34

$$T, F = ?$$

$$\sum \vec{F} = 0 \text{ (at rest)}$$

$$\textcircled{1} \sum F_x = 0$$

$$F - T \sin 37^\circ = 0$$

$$F = T \sin 37^\circ \quad \textcircled{1}$$

$$\textcircled{2} \sum F_y = 0$$

$$T \cos 37^\circ - mg = 0$$

$$T \cos 37^\circ = mg \quad \textcircled{2}$$

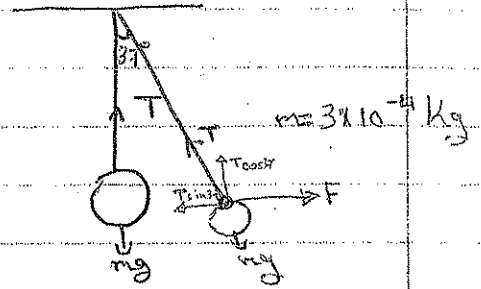
$$\textcircled{2} \text{ vs } \textcircled{1} \text{ gives}$$

$$\frac{F}{mg} = \tan 37^\circ$$

$$\therefore F = mg \tan 37^\circ$$

$$F = 2.2 \times 10^{-3} \text{ N}$$

$$\therefore T = 3.7 \times 10^{-3} \text{ N}$$



Q.92

$$\vec{V} = 2\hat{i} - 7\hat{j} \text{ m/s (unchanging } \vec{V})$$

$$\vec{F}_1 = 2\hat{i} + 3\hat{j} - 2\hat{k} \text{ N}$$

$$\vec{F}_2 = -5\hat{i} + 8\hat{j} - 2\hat{k} \text{ N}$$

$$\vec{F}_3 = ?$$

$$\therefore \text{unchanging } \vec{V} \text{ means } \vec{a} = 0$$

$$\vec{F}_{\text{net}} = 0$$

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

$$-3\hat{i} + 11\hat{j} - 4\hat{k} + \vec{F}_3 = 0$$

$$\vec{F}_3 = 3\hat{i} - 11\hat{j} + 4\hat{k}$$

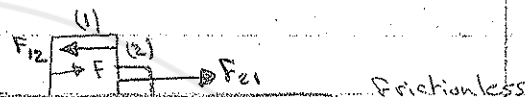
physics

(L)

Ch. 5

contact force between two bodies :-

Q. 35



$$F = (m_1 + m_2) \cdot a \quad \text{--- ① ---}$$

$$F = 3.2 \text{ N}$$

Find the force on (2) from (1)

$$m_1 = 2.3$$

∴ equation of motion for (m₂)

$$m_2 = 1.2$$

$$m_2 \cdot a = F_{21}$$

$$\text{From ①} \therefore a = \frac{3.2}{2.3 + 1.2} = 0.91 \text{ m/s}^2$$

$$\text{From ②} \therefore F_{21} = (1.2) (0.91) = 1.1 \text{ N}$$

* Try to find F_{12} from the equation of motion for (m₁)

$$m_1 a = F - F_{12}$$

Inclined Frictionless Plane :-

m on Frictionless surface

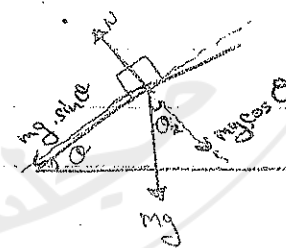
Find $N, a = ?$

$$(y\text{-axis}) \quad N - mg \cos \theta = 0$$

$$N = mg \cos \theta \quad \text{--- ① ---}$$

$$(x\text{-axis}) \quad ma = mg \sin \theta$$

$$a = g \sin \theta \quad \text{--- ② ---}$$



$$\theta_1 + \theta = 90 \quad \text{--- ③ ---}$$

$$\theta_2 + \theta = 90 \quad \text{--- ④ ---}$$

$$\theta_1 = \theta_2$$

* Try to solve Q. 34

ch. 6

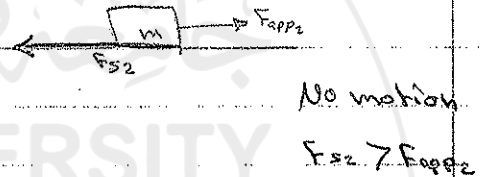
Force and motion - II

Force of friction:-

Friction force always
opposite to the direction
of sliding

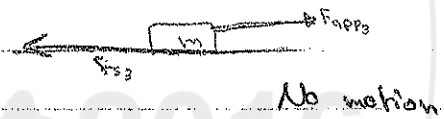


- F_s = static friction force
acts when m at rest.



- $\vec{F}_s = -\vec{F}_{app}$

- $F_{s, max} = \mu_s \cdot m$
 μ_s (coefficient of static friction)

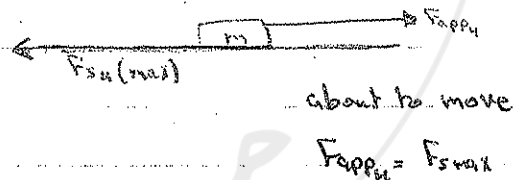


- when m moves, the
friction force is called
(Kinetic friction force) = $\mu_k \cdot N$

μ_k (coefficient of kinetic friction)

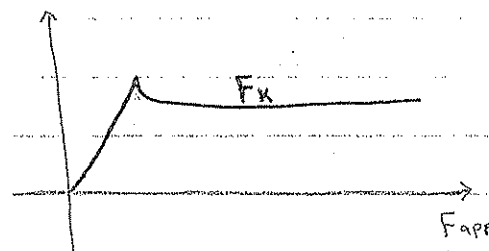
$\mu_k < \mu_s$

$F_k < F_{s, m}$



Friction force

$F_{s, m} = \mu_s \cdot N$



Q.11

$$M = 65$$

$$\theta = 15^\circ$$

a) F to start m , moving = ?

m about to move

$$F_{\text{net } x} = 0$$

$$F \cos 15 - M_s N = 0$$

$$F \cos 15 = M_s N \quad \text{--- (1)}$$

$$F_{\text{net } y} = 0$$

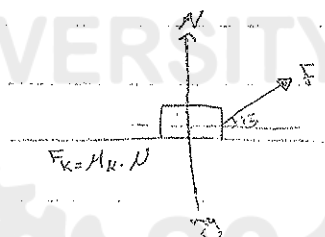
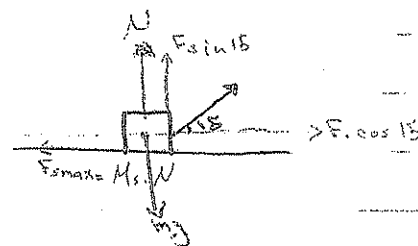
$$N + F \sin 15 - mg = 0 \quad \text{--- (2)}$$

$$F \leftarrow N \rightarrow$$

b) $M_k = 0.35$, find $a = ?$

$$F_{\text{net } x} = ma$$

$$ma = F \cos 15 - M_k N$$



2017 2016

physics

(D)

ch.5

Q. 31

$$N - mg \cos \theta = 0$$

$$N = mg \cos \theta$$

$$ma = mg \sin \theta$$

$$a = g \sin \theta$$

$$a = 5.2 \text{ m/s}^2$$

$$\textcircled{a} - v_2^2 = v_1^2 + 2a \Delta x$$

$$0 = (3.5)^2 + 2(-5.2)d$$

$$d = 1.18 \text{ m}$$

$$\textcircled{b} - v_2 = v_1 + at$$

$$0 = 3.5 + (-5.2)t$$

$$t = 0.7 \text{ s}$$

⑩ -



القوة المؤثرة على الكتلة هي القوة المحركة

على الكتلة على طول السطح المائل

$$\vec{v} = 3.5 \text{ m/s}$$

Q. 42

$F_{\text{from the tower}} + F_{\text{from the water}}$

ARE HORIZONTAL

9500 kg

mg, N (1.9 m/s²)

$$\vec{F} + \vec{F}_{bw} = m\vec{a}$$

$$8600 \cos 18^\circ \hat{i} + 8600 \sin 18^\circ \hat{j} + \vec{F}_{bw} = (9500)(0.12) \hat{i}$$

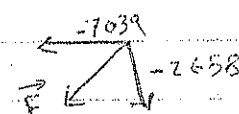
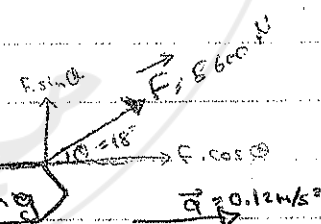
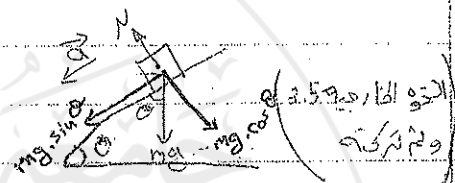
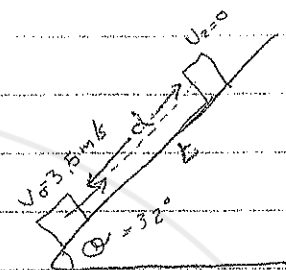
$$8179 \hat{i} + 2658 \hat{j} + \vec{F}_{bw} = 1140 \hat{i}$$

$$\vec{F}_{bw} = -7039 \hat{i} - 2658 \hat{j}$$

$$|\vec{F}| = 7524 \text{ N}$$

$$\theta = 20.7^\circ$$

$$|\vec{F}| = 7524 \text{ N at } \theta = (20.7^\circ + 180^\circ) \text{ with } (+x)$$

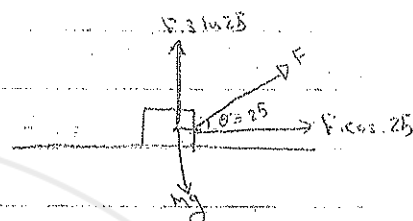


4.9. الجاذبية

c) $F = P$ just before lifting off the floor

$$F \sin 25^\circ = mg$$

$$F = \frac{mg}{\sin 25^\circ} = 117 \text{ N}$$



d) $\vec{a} = ?$

$$F \cos 25^\circ = m a_x$$

$$a_x = \frac{117 \cdot \cos 25^\circ}{5}$$

$$a_x = 21 \text{ m/s}^2$$

Q.53

$$m_1 = 12 \text{ kg}$$

$$m_2 = 24 \text{ kg}$$

$$m_3 = 8 \text{ kg}$$

$$a, T_1, T_2 = ?$$

$$(m_1 + m_2 + m_3) a = T_3$$

$$a = \frac{T_3}{\Sigma m} = \frac{64}{67} = 0.97 \text{ m/s}^2$$

$$m_1 a = T_1$$

$$T_1 = 11.6 \text{ N}$$

$$m_2 a = T_2 - T_1$$

$$T_2 = 34.8 \text{ N}$$

$$(m_3 a = T_3 - T_2)$$



physics
ch. 6

Q.19

$$\mu_s = 0.6$$

$$\mu_k = 0.4$$

a) the block will move or not?

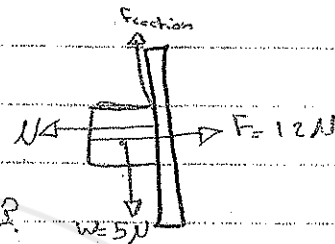
$$\sum F_x = 0$$

$$12 - N = 0 \Rightarrow N = 12 \text{ N}$$

$$F_{s, \max} = \mu_s \cdot N = 0.6 \times 12 = 7.2 \text{ N}$$

$$W < F_{s, \max}$$

it will not move



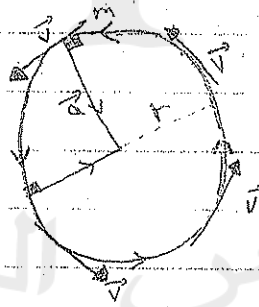
b) لا يتحرك، قوة الاحتكاك = 12 N

$$\sum F_y = 0$$

$$F_s - 5 = 0 \Rightarrow F_s = 5 \text{ N}$$

$$\vec{F}_{\text{net}} = 12\hat{i} + 5\hat{j} \text{ (من أجل أن البلوك)}$$

Ⓚ Uniform Circular Motion



$$v = \frac{2\pi r}{T}$$

$$a = \frac{v^2}{r} \text{ centripetal acceleration}$$

$$F = \frac{mv^2}{r} \text{ centripetal force } \left\{ \begin{array}{l} \text{القوة التي تقود الجسم في المسار الدائري} \\ \text{القوة التي تقود الجسم في المسار الدائري} \end{array} \right\}$$

Q. 82

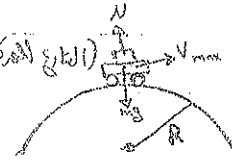
R = 250 m

$$\therefore \frac{mV^2}{R} = N - mg$$

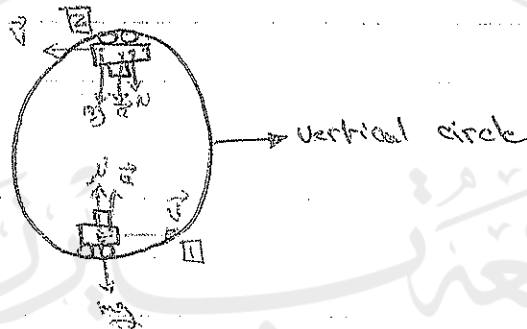
$\therefore V_{\max}$ is at $N \rightarrow 0$

$$\therefore \frac{mV_{\max}^2}{R} = mg$$

$$V_{\max}^2 = Rg \Rightarrow V_{\max} = 50 \text{ m/s}$$



example 3-



Position I $\Rightarrow \frac{mV_1^2}{R} = N - mg$

Position II $\Rightarrow -\left(\frac{mV_2^2}{R}\right) = -(mg + N)$

④ Car in flat circular turn (Unbanked Roadway):

$$F_{\text{sin}} = \frac{mV^2}{R}$$

$$\mu_s N = \frac{mV_{\max}^2}{R} \quad \text{--- ①}$$

$$N = mg \quad \text{--- ②}$$

$$\therefore \mu_s mg = \frac{mV_{\max}^2}{R}$$

$$V_{\max} = \sqrt{\mu_s R g}$$

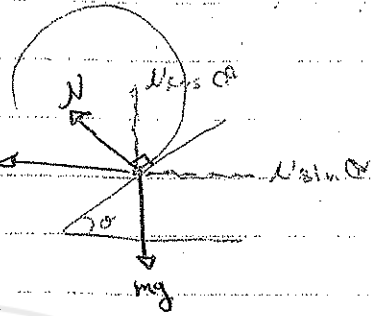


مركبة على
(دائرة)

$V_{\max}$ depends on μ_s

④ Banked Roadway :-

$$N \sin \theta = \frac{m v_{\max}^2}{R} \quad \text{--- (1)}$$



(گولڈ اسٹریٹ) $N \cos \theta = mg$ --- (2)

$$\frac{(1)}{(2)} \rightarrow \tan \theta = \frac{v_{\max}^2}{Rg}$$

$$\therefore v_{\max} = \sqrt{Rg \tan \theta}$$

جامعہ برزت
BIRZEIT UNIVERSITY

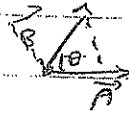
2017 2016

مجلس الطلبة

Chapter 6: Work, energy and power

Dot Product $\Rightarrow$ "scalar product"

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$



$$\vec{A} \cdot \vec{B} = 0 \quad \vec{A} \perp \vec{B}$$

$\vec{A} \cdot \vec{B}$ could be negative if $\theta > 90^\circ$

in Cartesian coordinates

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\hat{i} \cdot \hat{i} = (1)(1) \cos 0 = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\hat{i} \cdot \hat{j} = (1)(1) \cos 90 = \hat{i} \cdot \hat{k} = \hat{j} \cdot \hat{k} = 0$$

$$\vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

example:-

$$\vec{A} = 3\hat{i} - 4\hat{j} + 5\hat{k}$$

$$\vec{B} = 6\hat{i} + 7\hat{j} - 8\hat{k}$$

$$1) |\vec{A}| = \sqrt{3^2 + (-4)^2 + 5^2} = 7.1$$

$$2) |\vec{B}| = \sqrt{6^2 + 7^2 + 8^2} = 12.2$$

$$\vec{A} \cdot \vec{B} = 3(6) + (-4)(7) + (5)(-8) = 18 - 28 - 40$$

find the angle between vector A and vector B



9) $\vec{A} \cdot \vec{B}$

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$-50 = (7.1)(12.2) \cos \theta$$

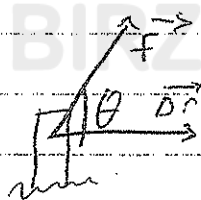
$$\cos \theta = \frac{-50}{(7.1)(12.2)}$$

$$\theta = 125$$

5) $\hat{A} = \frac{\vec{A}}{A} = \frac{3\hat{i} - \hat{j} + 5\hat{k}}{7.1} = 0.42\hat{i} - 0.14\hat{j} + 0.7\hat{k}$

6) $\hat{B} = \frac{\vec{B}}{B}$

Work done by a constant force = $\vec{F} \cdot \Delta \vec{r}$



$$= F(\Delta r) \cos \theta$$

$$(\Delta r) F \cos \theta$$

where Δr is displacement

Note the following:-

1) New ton $W = Nm = J$

2) New $W = 0$ in $\theta = 90$
 or $\Delta r = \underline{\underline{0}}$

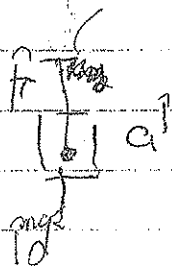
3) W is negative when

when $0^\circ > \theta > 90$

work done by gravity

case 1:

moving up & down



$T > mg$
 when 0°

1) find a

$$ma = T - mg$$

$$a = \frac{T - mg}{m}$$

$$W_g = (mg)(\cos\theta)/a$$

work done to create rest
 $d(-) (11 \text{ g/y})$

c) work done by T

$$W_T = T_y \cos\theta = \pi$$

$$T = T_y$$

where $\cos\theta$ by work

$N \rightarrow \text{Max}$

when $0^\circ > \theta > 90^\circ$

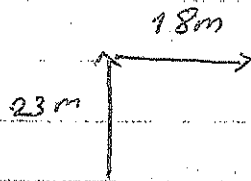
or when $\theta > 90^\circ$

$$W_{\text{net force}} = mgy + T_y$$

13) 1.2) 3)

$$13/m = 650 \text{ kg}$$

$$\alpha = 23 \text{ m}$$



$$23\mathbf{j} + 18\mathbf{i}$$

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$\Delta r = 18\mathbf{i} + 23\mathbf{j}$$

To find w

$$W = F_x \Delta x + F_y \Delta y$$

By

Lifting the beam at constant speed
the crane exerts a constant force
vertically upward equals in
Magnitude to the weight of the
beam. During the horizontal swing
The force is the same but is
perpendicular to the displacement

$$F \cdot \Delta r = (mg\mathbf{j}) (\Delta y\mathbf{j} + \Delta x\mathbf{i})$$

$$= (650 \text{ kg})(9.8 \text{ m/s}^2)(23 \text{ m})$$

$$= 147 \text{ kJ}$$

$$18/F = 1.8\mathbf{i} + 2.2\mathbf{j} \text{ N}$$

$$r = 56\mathbf{i} + 37\mathbf{j}$$

The force is constant

$$4 \text{ km} - x$$

3

$$\text{force} \Rightarrow W = F \cdot r = (1.8 \text{ N})(56 \text{ m}) + (2.2 \text{ N})(37 \text{ m}) = 169 \text{ J}$$

$$20) \quad x = 0 \text{ km} \quad x = 3 \text{ km}$$

$$x = 3 \text{ km} \quad \text{to} \quad x = 4 \text{ km}$$

$$W = \int_0^{3 \text{ km}} (F_x) dx$$

$$= \int_0^3 \left(\frac{40 \text{ N}}{3 \text{ km}} \right) x = \frac{40 \text{ N}}{3 \text{ km}} \left(\frac{3 \text{ km}}{2} \right)^2 = 60 \text{ kJ}$$

$$b) W = \int_{3\text{ km}}^{4\text{ km}} \left(\frac{40\text{ N}}{3\text{ km}} \right) (4\text{ km} - x) dx$$

$$= \frac{40\text{ N}}{3\text{ km}} \left[4x - \frac{x^2}{2} \right]_3^4 = 20\text{ kJ}$$

21) $k = 200\text{ N/m}$

a) 10 cm

$$W = \frac{1}{2} k x^2 = \frac{1}{2} (200\text{ N/m}) (0.1)^2 = 1\text{ J}$$

$$b) W = \frac{1}{2} k (x_2^2 - x_1^2) = \frac{1}{2} (200\text{ N/m}) (0.2^2 - 0.1^2) = 3\text{ J}$$

$$0 \rightarrow 20\text{ cm} = 4\text{ J}$$

$$0 \rightarrow 10\text{ cm} = 1\text{ J}$$

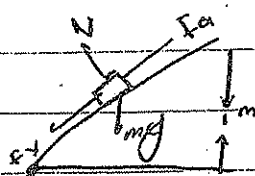
$$(10 \rightarrow 20\text{ cm} = 4\text{ J} - 1\text{ J} = 3\text{ J})$$

24) $k = 70\text{ N/m}$

$x = 4.6\text{ cm}$

$$F = W = \frac{1}{2} k x^2 = \frac{1}{2} (70 \times 10^{-3}) (0.046)^2 =$$

41)



$$F_a \cdot \Delta r = (200\text{ N}) (2\text{ m}) = 400\text{ J}$$

$$\Delta r = (1\text{ m}) / \sin 30 = 2\text{ m}$$

$$m = F_a / g (\sin \theta + \mu \cos \theta)$$

$$m = 200$$

$$(0.18) \left(\frac{\sqrt{3}}{2} \right) + 0.5$$

$$m = \frac{200}{1.8 + 0.5} = \frac{200}{2.3}$$

$$= 87.1\text{ kg}$$

Work-Kinetic Energy theorem

$$1 \text{ kW/m}^2 \times 75 \text{ m}^2$$

$$75 \text{ kW}$$

$$\text{Time} = \frac{40 \text{ kWh}}{75 \text{ kW}}$$

$$W_{\text{net}} = \Delta K$$

$$\Rightarrow \text{Power} = \frac{W}{\Delta t} \left[\frac{\text{J}}{\text{s}} \right] \text{ watt } [\text{W}] \quad \underline{2.67 \text{ h}}$$

$$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

$$W = \int F_x dx$$

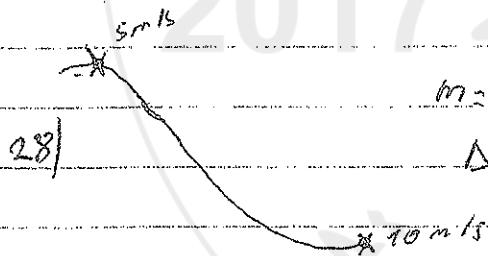
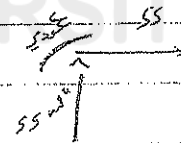
$$W = \int \vec{F} \cdot d\vec{r}$$



$$= \int F(x) dx + \int F(y) dy + \int F(z) dz$$

10) Kinetic energy is the same
 $K = \frac{1}{2} m v^2$

No. network bcs $\Delta K = 0$

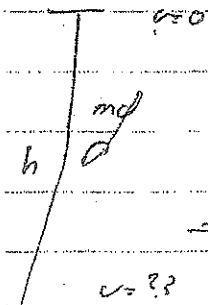


$$m = 60 \text{ kg}$$

$$\Delta K = \frac{1}{2} m (v_f^2 - v_i^2)$$

$$\Delta K = W_{\text{net}}$$

$$= \frac{1}{2} (60) (10^2 - 5^2) = 2250 \text{ J}$$

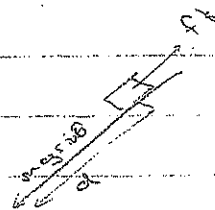


$$W = mgh = \frac{1}{2} m (v_f^2 - v_i^2)$$

$$v_f^2 = v_i^2 + 2gh$$

$$\Rightarrow \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = mgh$$

$$\frac{(mg \sin \theta - \mu_k) d}{\frac{1}{2} m (v_f^2 - v_i^2)} \Leftarrow$$



$$27) \frac{1}{2} m v_c^2 = \frac{1}{2} m c v_c^2$$

$$32) W = \vec{F} \cdot \vec{d} = 750(N) \times 200 m = 150000 = 1.5 \times 10^5 J$$

$$\bar{P} = \frac{dW}{dt} = \frac{1.5 \times 10^5}{5 \times 60} = 500 \text{ Watt}$$

$$P = \frac{500}{746} = 0.67 \text{ hp}$$

$$38) 1 \text{ Kw/m}^2$$

$$A = 15 \text{ m}^2$$

$$R = 1 \text{ Kw/m}^2 \times 15 \text{ m}^2$$

$$= 15 \text{ Kw}$$

$$\text{time} = \frac{40 \text{ Kw} \cdot \text{h}}{15 \text{ Kw}}$$

$$\text{time} = 2.67 \text{ h}$$

$$\frac{20 \times 10^3}{900}$$

$$39) P = \frac{W}{t}$$

$$t = \frac{W}{P} = \frac{20 \times 10^3 J}{900 W} = 22.2 \text{ s}$$

P.P. 82/ c → rate of work

83/ ~~100~~ c)

at 85, 145, 185 ~~ms~~ ms

84/ ~~100~~ ~~ms~~

OK = 550J ~~84~~ 5.50J a)

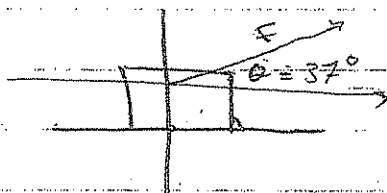
$$85/ OP = \vec{F} \cdot \vec{OO} = F \cos \theta$$

$$F = \frac{OP}{\cos \theta} \quad c)$$

$$W = \vec{F} \cdot \vec{\Delta r}$$

$$W = F(\Delta r) \cos \theta \quad \left(\begin{array}{l} \text{work done} \\ \text{by constant force} \end{array} \right)$$

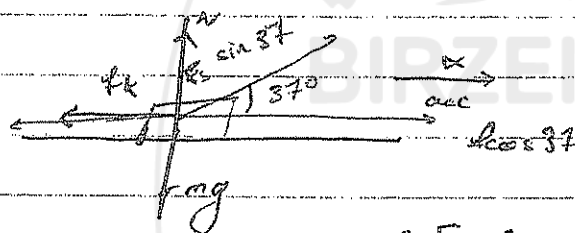
example:
 $m = 50 \text{ kg}$
 $\mu_k = 0.4$



$$a = 1.5 \text{ m/s}^2$$

Applied force act 37° above the horizontal
 moves the Mass at constant $a = 1.5 \text{ m/s}^2$
 for a displacement $= 100 \text{ m}$

Find the work done by each force



$$\sum F_y = 0$$

$$N + F \sin 37 - mg = 0$$

$$N = mg - F \sin 37 \quad \text{--- (1)}$$

$$\sum F_x = ma$$

$$ma = F \cos 37 - \mu_k N \quad \text{--- (2)}$$

① in ②

$$ma = F \cos 37 - \mu_k (-mg + F \sin 37)$$

$$ma = F \cos 37 - \mu_k mg + \mu_k F \sin 37$$

$$ma + \mu_k mg = F (\cos 37 + \mu_k \sin 37)$$

$$F = \frac{ma + \mu_k mg}{\cos 37 + \mu_k \sin 37} = 264.4 \text{ N}$$

from ①

$$N = mg - F \sin 37$$

$$= 50 \times 10 - 264.4 \times 0.6$$
$$= 331.4$$

$$f_k = \mu_k N$$

$$= 0.4 \times 331.4$$

$$= 132.4 \text{ N}$$

$$d = \Delta r$$

$$W_g = mgd \cos 90 = 0$$

$$W_N = Nd \cos 90 = 0$$

$$W_{f_k} = f_k \Delta r \cos 180 = (-)(132.4)(100)$$

$$= -13240 \text{ J}$$

$$W_F = F(\Delta r) \cos 37$$

$$F = (264.4) / (100) / (0.8)$$

$$W_F = 2115.2 \text{ J}$$

$$W_{\text{net}} = W_g + W_{f_k} + W_F + W_N$$
$$= +7912 \text{ J}$$

Work done By variable force

$$W = \int \vec{F} \cdot d\vec{r}$$

in one dimension

$$W = \int_{x_i} F_x dx$$

stretching a spring

$$F_s = -kx$$

$$F = kx \text{ (external force)}$$

Hooke's law

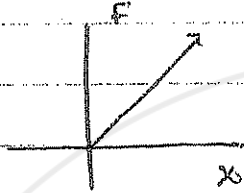
Work done in stretching the spring = $\int_{x_i}^{x_f} kx \, dx$

$$W = k \frac{x^2}{2} \Big|_{x_i}^{x_f}$$

$$W = \frac{1}{2} k x_f^2 - \frac{1}{2} k x_i^2$$

$x_i = 0$

$$W = \frac{1}{2} k x_f^2$$



work done by spring force
↓

$$W_s = \int_{x_i}^{x_f} -kx \, dx = -\frac{1}{2} k x_f^2 + \frac{1}{2} k x_i^2$$

Work done by variable force in 3 Dimensions -

$$W = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{r} \quad \vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

$$d\vec{r} = dx \hat{i} + dy \hat{j} + dz \hat{k}$$

$$W = \int_{x_i}^{x_f} F_x \, dx + \int_{y_i}^{y_f} F_y \, dy + \int_{z_i}^{z_f} F_z \, dz$$

Example -

$$\vec{F} = 2x \hat{i} + 3y \hat{j} \, \text{N}$$

acts on mass $m = 5 \, \text{kg}$

It moves it from $r_i = 2\hat{i} + 3\hat{j} \, \text{m}$

to $r_f = -4\hat{i} - 3\hat{j}$ find the work done by this force.

$$W = \int_{x_i}^{x_f} F_x \, dx + \int_{y_i}^{y_f} F_y \, dy = \int_2^{-4} (2x) \, dx + \int_3^{-3} 3 \, dy$$

$$= \left[\frac{2x^2}{2} \right]_2^{-4} + \left[3y \right]_3^{-3} = (32 - 8) + (9 - 9)$$

$$= 24 + 0 = 24 \, \text{J}$$

$$49) \vec{F} = 67\hat{i} + 23\hat{j} + 55\hat{k}$$

$$\vec{r}_1 = 16\hat{i} + 37\hat{j}$$

$$\vec{r}_2 = 21\hat{i} + 10\hat{j} + 14\hat{k}$$

$$W = \vec{F} \cdot \Delta \vec{r} = (67\hat{i} + 23\hat{j} + 55\hat{k}) \cdot (5\hat{i} - 21\hat{j} + 14\hat{k})$$

$$W = (67 \times 5)\hat{i} + (23 \times -21)\hat{j} + (55 \times 14)\hat{k}$$

$$622\hat{j}$$

$$58) l(x) = ax^{\frac{3}{2}} = 0.75 x^{\frac{3}{2}}$$

$$x=0 \rightarrow x=24\text{ m}$$

$$W = \int_0^{24} l(x) dx$$

$$= \int_0^{24} 0.75 x^{\frac{3}{2}} dx$$

$$= \frac{3}{2} \times 0.75 x^{\frac{5}{2}} \Big|_0^{24} = \left(\frac{0.75 \times (24)^{\frac{5}{2}}}{2.5} \right) + (0) = 22.0 \text{ J}$$

$$59) \vec{F}_1 \cdot \vec{F}_2 = \frac{1}{3} |\vec{F}|^2$$

$$c = \cos^{-1}\left(\frac{1}{3}\right)$$

$$\cos \theta = \frac{\vec{F}_1 \cdot \vec{F}_2}{|\vec{F}_1| |\vec{F}_2|} = \frac{\frac{1}{3} |\vec{F}|^2}{|\vec{F}|^2}$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} = \frac{\frac{1}{3} A^2}{A^2} = \frac{1}{3}$$

$$60) F = F_0 \left(\frac{L_0 - x}{L_0} - \frac{L_0^2}{(L_0 + x)^2} \right)$$

$$W = \int_0^{L_0} F(x) dx$$

$$W = F_0 \left(\frac{1}{L_0} (L_0 - x) dx - L_0^2 \int \frac{1}{(L_0 + x)^2} dx \right)$$

$$W = F_0 \left(\frac{1}{l_0} \left(L_0 x - \frac{x^2}{2} \right) - l_0^2 \int_0^x v^{-2} dv \right)$$

$$W = F_0 \left(\frac{1}{l_0} \left(L_0 x - \frac{x^2}{2} \right) + \left(\frac{l_0^2}{l_0 + x} \right) \right) \Big|_0^x$$

$$W = F_0 \left(x - \frac{x^2}{2l_0} \right) + \frac{l_0^2}{l_0 + x} - l_0$$

$$73) P = \frac{P_0 t_0^2}{(t + t_0)^2}$$

$$t = 0 \rightarrow \infty$$

$$W = P_0 t_0 \quad \text{show}$$

$$W = \int_0^{\infty} P dt$$

$$\int_0^{\infty} \frac{P_0 t_0^2}{(t + t_0)^2} dt$$

$$= P_0 t_0^2 \int_0^{\infty} \frac{dt}{(t + t_0)^2} \quad \text{let } v = t + t_0$$

$$dv = dt$$

$$W = P_0 t_0^2 \int_0^{\infty} v^{-2} dv$$

$$= P_0 t_0^2 \left(\frac{v^{-1}}{-1} \right) \Big|_0^{\infty}$$

$$= P_0 t_0^2 \left(\frac{1}{t + t_0} \right) \Big|_0^{\infty}$$

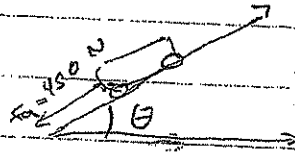
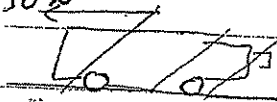
$$= P_0 t_0^2 \left(\frac{1}{\infty} - \frac{1}{t_0} \right)$$

$$= P_0 t_0^2$$

69) 1400 kg car

$$v = 60 \text{ km/h}$$

$$F_b = 450 \text{ N}$$



v is constant

$$W_{\text{net}} = 0$$

$$W_{\text{net}} = 0$$

$$P_{\text{net}} = 0$$

$$P_c - P_f - P_g = 0$$

$$P_c - \vec{F}_a \cdot \vec{v} - \vec{F}_g \cdot \vec{v} = 0$$

$$38 \times 10^3 - \frac{450 \times 60}{3.6} - \frac{1400 \times 10 \sin \theta \times 60}{3.6}$$

find $\sin \theta$??

2017

2016

$$y = -\sec u$$

$$u = x^2 + 7x$$

$$y' = \left(-\sec x^2 + 7x \tan x^2 + 7x \right) / (2x + 7)$$

$$y = \sec(\tan x)$$

$$y = \sec u$$

$$\tan x = u$$

$$= \left(\sec \tan x \tan(\tan x) \right) \times \sec^2 x$$

$$y = \cos^{-4} x$$

$$y = \frac{5x}{5u^{-4}}$$

$$y = \cos^{-4} u$$

$$+ 4 \cos^{-3} u \times \sin u$$

$$u = 5x$$

$$+ 4 \cos^{-3} 5x \times \sin 5x (5)$$

$$= 20 (\cos^{-3} 5x) (\sin 5x)$$

$$r = (\csc \theta + \cot \theta)^{-1}$$

$$\frac{1}{\tan} \quad \frac{\cos}{\sin}$$

$$-1 (\csc \theta + \cot \theta)^{-2} \times (\csc \theta \tan \theta + \tan \theta)$$

$$= \csc \theta \tan \theta + \tan \theta$$

$$\frac{1}{(\csc \theta + \cot \theta)^2}$$



$$y = -\sec u$$

$$u = x^2 + 7x$$

$$y' = \left(-\sec x^2 + 7x \tan x^2 + 7x \right) / (2x + 7)$$

$$y = \sec(\tan x)$$

$$y = \sec u$$

$$\tan x = u$$

$$= \left(\sec \tan x \tan(\tan x) \right) \times \sec^2 x$$

$$y = \sec^{-4} x$$

$$y = \frac{5x}{5u^4}$$

$$y = \cos^{-4} u$$

$$+4 \cos^{-3} u \times \sin u$$

$$u = 5x$$

$$+4 \cos^{-3} 5x \times \sin 5x (5)$$

$$= 20 (\cos^{-3} 5x) (\sin 5x)$$

$$r = (\csc \theta + \cot \theta)^{-1}$$

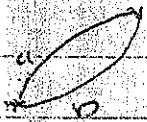
$$\frac{1}{\tan} \cdot \frac{\cos}{\sin}$$

$$= 1 (\csc \theta + \cot \theta)^{-2} \times (\csc \theta \tan \theta + \tan \theta)$$

$$= \frac{\csc \theta \tan \theta + \tan \theta}{(\csc \theta + \cot \theta)^2}$$

Ch. 7

1) Cons force & nonconservative



$$W_{ab1} = W_{ab2}$$

$$W_{\text{round}} = 0$$

Potential Energy

$$DU = \int \vec{f}_c \cdot d\vec{r}$$

$$\Delta U(x) = \int \vec{f}_c(x) \cdot d\vec{x}$$

$$W_g = mgh \quad \Delta U(x) = mgh$$

$$W_s = \int_{x_i}^{x_f} (-kx) dx = -\frac{1}{2} k (x_f^2 - x_i^2)$$

$$\Delta K = W_{\text{tot}} = W_{\text{con}} + W_{\text{noncon}}$$

$$\Delta U = W_{\text{con}}$$

$$\Delta K = -\Delta U + W_{\text{noncons}}$$

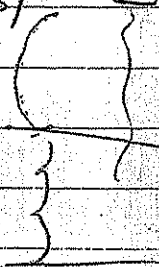
$$\Delta K + \Delta U = W_{\text{noncons}}$$

$$\Delta(K + U) = W_{\text{non}}$$

$$\Delta E = W_{\text{noncon}}$$

$E_i = E_f$ for cons forces

$$\Delta E = 0 \text{ (cons)}$$



$$\frac{1}{2} kx^2 - mgx - mgh = 0$$

$$E_i = E_f$$

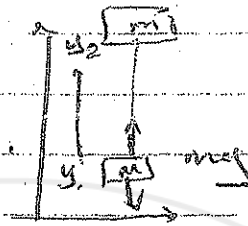
$$U_i + 0 = U_f + 0$$

$$mgh + U = \frac{1}{2} kx^2 - mgx$$

$$W_{\text{cons}} = -\Delta U$$

In this course there are 2 conservative forces:-

[1] $\vec{F} = m\vec{g}$



$$\Delta U = W_{\text{cons}}$$

$$\Delta U = \int_{y_1}^{y_2} mgy \, dy$$

$$U_2 - U_1 = - \int_{y_1}^{y_2} -mgy \, dy$$

$$U_2 - U_1 = mgy \Big|_{y_1}^{y_2}$$

$$U_2 - U_1 = mgy_2 - mgy_1$$

Take $U_1 = 0$ at $y_1 = 0$

$$U_2 = mgy_2$$

In general $U_g = mgy$

[2] $F_s = -kx$

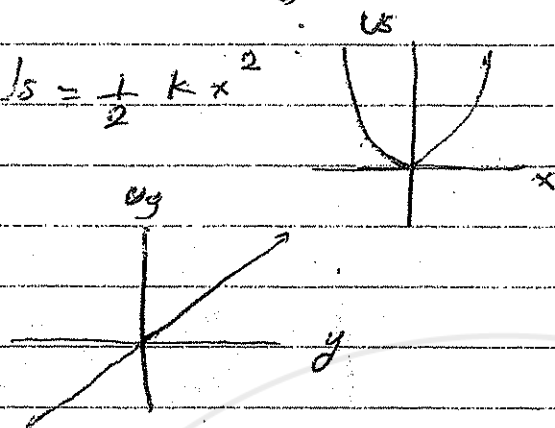
$$\Delta U = \int_{x_1}^{x_2} F_{\text{cons}} \, dx$$

$$U_2 - U_1 = - \int_{x_1}^{x_2} -kx \, dx$$

$$U_2 - U_1 = \frac{1}{2} kx_2^2 - \frac{1}{2} kx_1^2$$

Take $U_i = 0$ at $x_i = 0$

$$U_s = \frac{1}{2} k x^2$$



Conservation of mechanical energy :-
 Mechanical $\rightarrow E = K + U$

In any system there are a set of forces. some of them are conservative and the others are nonconservative.

$$W_{tot} = W_{con} + W_{non}$$

$$\Delta K = -\Delta U + W_{non}$$

$$\Delta K + \Delta U = W_{non}$$

$$W_{non} = \Delta E$$

F_{non} is friction force

$$\text{if } F_{non} = 0$$

$$\Delta E = 0$$

$$E_1 = E_2$$

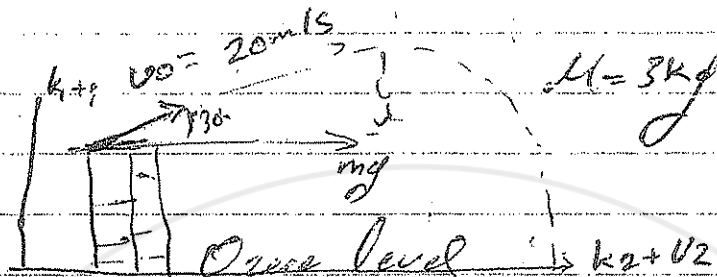
$$K_1 + U_1 = K_2 + U_2$$

$$K_1 + U_1 = K_2 + U_2$$

→ conservation of Mechanical energy

$$* F_{\text{fric}} = 0$$

Example:-



Find U_2 ?

$$K_1 + K_2$$

$$K_1 + U_1 = K_2 + U_2$$

$$\frac{1}{2} m_1 v_1^2 + m_1 g y = \frac{1}{2} m v_2^2 + 0$$

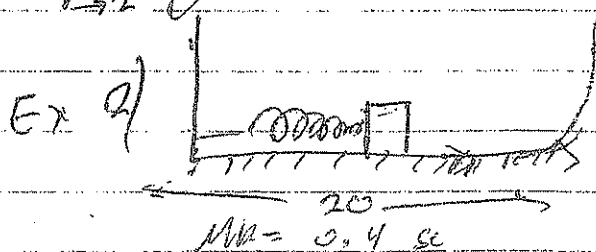
$$\frac{1}{2} (3) (20)^2 + (3) (10) (100) = \frac{1}{2} (3) (v_2)^2$$

$$600 + 300 = 1.5 v_2^2$$

$$v_2 = 48.9 \text{ m/s}$$

(2) Find Work done by gravity?

$$W_g = m g y = 3 (10) (100) = 3000 \text{ J}$$



$$R = 700 \text{ N/m}$$

$$m = 1000$$

$$k = 7 \text{ (m/s}^2 \text{)} + 1$$

horizontal surface

find h on the friction
on the bed plane.

the mass compressed the spring

30 cm

find h

$$W_b = PE$$

$$W_b = E(\omega_2 - \omega_1)$$

$$W_b = E_2 - E_1$$

$$= (K + 0)^2 - (K + 0)$$

$$= 0 + mgh + (0 + \frac{1}{2} K v^2)$$

$$= mgh - \frac{1}{2} K (0.3)^2$$

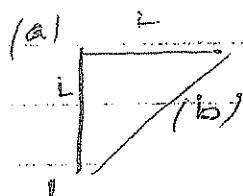
$$\frac{1}{2} K (20) \cos 180 = mgh - \frac{1}{2} K (0.3)^2$$

$$- \frac{1}{2} K m g (20) = mgh - \frac{1}{2} K (0.3)$$

$$- 8 = (0.1)(10) h - 31.5$$

$$23.5 = h$$

20) $W_a =$



$$W_a = -2L \int_0^L k = -2L (Mk \text{ mg})$$

Ragun-Te

$$W = -2L Fk$$

$$C_a = 2L Fk = 2L (Mk \text{ mg})$$

$$-2L (Mk \text{ mg})$$

17) $V = 210 \text{ J}$

$$= -2L Fk = -2L (Mk \text{ mg})$$

$$K = 1.4 \text{ KJ}$$

$$x = ?$$

$$V_s = \frac{1}{2} K x^2$$

$$0.55 \text{ m}$$

$$x = \sqrt{\frac{2V}{K}} = \sqrt{\frac{2(210)}{1400}} = 0.55 \text{ m}$$

$$\begin{aligned} W &= -2L Fk \\ &= -2L (Mk \text{ mg}) \\ &= -2L Fk \\ &= 2L (Mk \text{ mg}) \end{aligned}$$

18) $V = ?$

$$K = 0.046 \text{ KJ}/\mu\text{m}$$

$$x = 26 \mu\text{m}$$

$$0.046 \times 10^{-6}$$

$$\begin{aligned} V &= \frac{1}{2} K x^2 \\ &= \frac{1}{2} (0.046) \left(\frac{10^{-12}}{10^{-6}} \right) \times (26 \times 10^6)^2 \end{aligned}$$

$$= J$$

33) $F = -Kx + bx^2 - cx^3$

$$b = 4.1 \text{ N/m}^2$$

$$c = 3.1 \text{ N/m}^2$$

$$K = 223 \text{ N/m}$$

$$x_0 = 0 \quad x = 2.26$$

$$u(x) = \int_{x_0}^{x_0} F(x) dx$$

$$u(x) = - \left(-k \int_0^{2.62} x dx + b \int_0^{2.62} x^2 dx - c \int_0^{2.62} x^3 dx \right)$$

$$u(x) = - \left(\frac{-k x^2}{2} \Big|_0^{2.62} + \frac{b x^3}{3} \Big|_0^{2.62} - \frac{c x^4}{4} \Big|_0^{2.62} \right)$$

$$u(x) = +777.3$$

$$20/m = 10,000 \text{ kg}$$

$$K = 40 \text{ kN/m}$$

$$x = 25 \text{ m}$$

$$v = ??$$

$$E_i = E_f$$

$$0 + \frac{1}{2} m v^2 - \frac{1}{2} k x^2 = 0$$

$$v = \sqrt{\frac{k}{m} x^2}$$

$$= \sqrt{\frac{40,000}{10,000}} \times 25$$

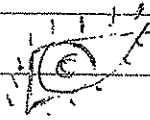
$$50 \text{ m/s} = 180 \text{ km/h}$$

$$21) E_i = E_f$$

$$\frac{1}{2} k x^2 = mgh$$

$$\frac{1}{2} (430) (0.71)^2 = (0.12) (90) h$$

$$h = 6.4 \text{ m}$$



$$23) \frac{1}{2} k x^2 = mgh$$

$$k = \frac{2mgh}{x^2} = \frac{(2)(0.065)(10)/(35)}{(0.14)^2} = 232 \text{ N/m}$$



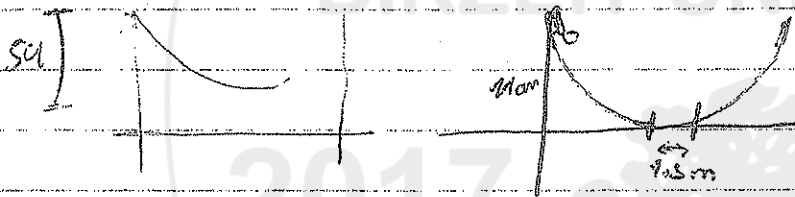
$$\frac{mv^2}{R} = mg + N$$

$$N = 0$$

$$v = \sqrt{Rg}$$

$$mgh = \frac{1}{2} mv^2 + mg(2R)$$

$$mgh = \frac{1}{2} mRg + 2mRg = \frac{5}{2} mRg$$



$$\begin{aligned} \Delta U_{\text{spring}} &= -\Delta U_{\text{grav}} \\ &= -\Delta mgh \end{aligned}$$

$$\Delta U = -mgh$$

$$= mg \times 0.11$$

$$= 0.61 \times 0.015 \text{ mg}$$

$$= \# \text{ of turns} = \frac{\Delta U}{W_{\text{fric}}}$$

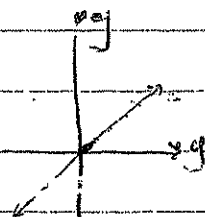
$$mg \times 0.11$$

$$mg \times 0.61 \times 0.015$$

$$= 12.022 \text{ turns}$$

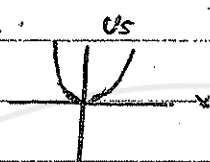
Ch.7 / Potential Energy curves

$$\vec{F}_g = -m\vec{g} \Rightarrow U_g = mgy$$



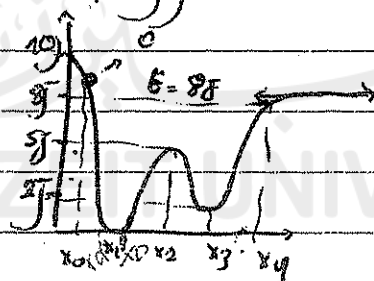
$$\vec{F}_s = -kx$$

$$U_s = \frac{1}{2} kx^2$$



for other conservative forces

Potential energy force could be



$$K + U = E$$

$$K = E - U$$

$$\text{let } E = 8J$$

$$\text{at } x_1, U = 0, K = 8J$$

$$x_2 = U_2 = 5J, K_2 = 3J$$

$$\text{at } x_0, U_0 = 8J, K_0 = 0$$

x_0 is called turning point

$$\text{at } x_4 \Rightarrow K_4 = 0, U_4 = 8J, K = 0$$

it stays at x_4

$$W_{\text{cons}} = -\Delta U$$

$$F_{\text{cons}} \cdot \Delta x = -\Delta U$$

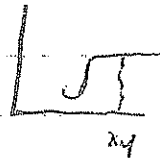
at x_0 F is positive

"to the right"

$$F_{\text{cons}} = -\frac{\Delta U}{\Delta x} \left(= -\frac{dU}{dx} \right)$$



at x_4
 $E=0$



x_4 is

called equilibrium point

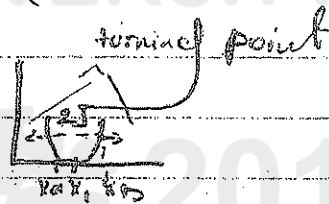
At the turning point $K=0$ condition
 $E=U$ potential energy

22) Let $E=2J$
 at x_{left} (x_4)

at x_a , $U=2J$
 K

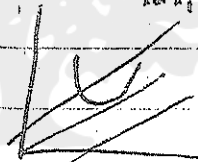
x_a is called a turning point

$F(x_a)$ is to the right



at x_{1b} , $U=2J$, $K=0$

x_{1b} is a turning point



at x_3 , $U=2$, $K=0$, $E=0$ equilibrium point

$F(x_4)=0$, (x_4 neutral equilibrium
 x_5)

x_2 unstable equilibrium

$F(x_2)=0$ unstable equilibrium point

$$Ex2) U(x) = \frac{6}{x^2} - \frac{1}{x} \quad x > 0$$

Let $E = 15 \text{ J}$

1) find the turning points

At the turning point

$$E = U \quad 15 \text{ J} = U$$

$$15 = \frac{6}{x^2} - \frac{1}{x}$$

$$15 = \frac{6-x}{x^2}$$

$$15x^2 = 6 - x$$

$$15x^2 - x + 6 = 0$$

$$\frac{(15x-5)(x+2)}{(x-3)(3x+2)} = 0$$

$$(15x-5)(x+2) = 0$$

$$x = -\frac{2}{3} \text{ m} \quad \text{--- / Rejected}$$

$$x > 0$$

$$x = \frac{3}{5} \text{ m} \quad \text{The turning point}$$

2) find the conservative force associated to U (corresponds)

$$F = -\frac{dU}{dx}$$

$$F(x) = -\left[\frac{6}{x^3} - \frac{1}{x^2} \right]$$

$$= -\left[\frac{-12}{x^3} + \frac{1}{x^2} \right]$$

$$= \frac{12}{x^3} - \frac{1}{x^2}$$

$$= \frac{12-x}{x^3}$$

3) find the equilibrium points

$$F=0$$

$$0 = \frac{12 - x}{x^3}$$

$$\underline{x = 12 \text{ m}}$$

$$x = 12 \text{ (boundary } x = 0)$$

Homework

$m = 0.2 \text{ kg}$ mouse
under the action of

F_{cons}

where U as a function of x

$$U(x) = 8x^2 + 2x^3 \text{ J}$$

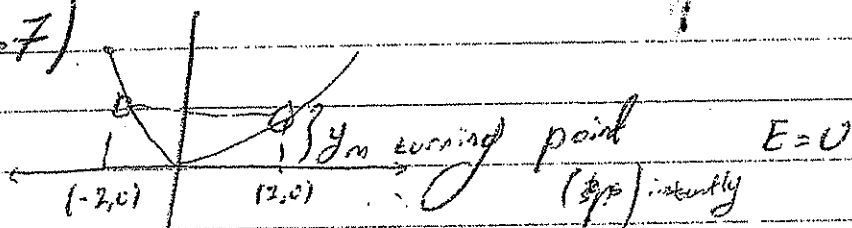
$$\text{at } x = 1 \text{ m}$$

$$v = 5 \text{ m/s}$$

① find the coordinates of the next turning point

$$\text{② find } U \text{ at } x = \frac{1}{2} \text{ m } 0.5 \text{ m}$$

Ch. 7)



at $x_1 \Rightarrow E=U$

$$y = 0.92 x^2$$

$$v = 8.5 \text{ m/s}$$

$$E = \frac{1}{2} m v_m^2$$

$$E = m g y_m$$

$$\frac{1}{2} m v_m^2 = m g y_m$$

$$\frac{1}{2} v_m^2 = g y_m$$

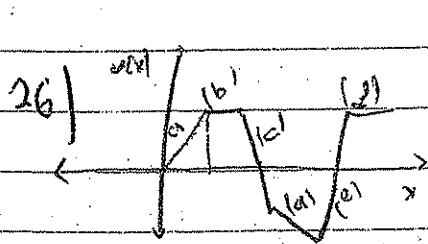
$$y_m = \frac{v_m^2}{2g}$$

$$y_m = \frac{(8.5 \text{ m/s})^2}{2(10)}$$

$$= \underline{\underline{3.5 \text{ m}}}$$

$$y_m = 0.92 x^2$$

$$x_m = \sqrt{\frac{y_m}{0.92}} = \sqrt{\frac{3.5 \text{ m}}{0.92}} \approx 2 \text{ m}$$



any function $U(x)$ is a cons force

$$F(x) = -\frac{dU}{dx} = -\frac{\Delta U}{\Delta x}$$

$$F_a = \frac{(3-0)}{(1.5-0)} = -2 \text{ N}$$

44.) $f_x = 5x - 2x^3$

$V=0$ $x=0$

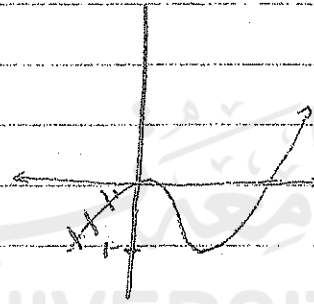
$$V(x) = - \int f(x) dx$$

$$= - \int (5x - 2x^3) dx$$

$$= - \left(\frac{5x^2}{2} - \frac{2x^4}{4} \right) + C$$

$V(0)=0$
 $C=0$

$$= - \frac{5x^2}{2} + \frac{x^4}{2}$$



Turning point

$V=E$
 $E=-1$

$$-\frac{5x^2}{2} + \frac{x^4}{2} = -1$$

$$5x^2 - x^4 = 2$$

$$x^2(5 - x^2) = 2$$

$x^2 = U$

$$5U - U^2 = 2$$

$$U^2 - 5U + 2 = 0$$

spring $h = 0.18x^2$

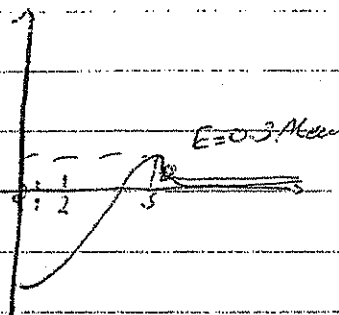
$E = K_{max} = \frac{1}{2} m v_{max}^2$

$v_{max} = 47 \text{ cm/s}$

$E = U = mgh$

$E_p = E_k$
 $mgh = \frac{1}{2} m v_{max}^2$
 $h = \frac{v_{max}^2}{2g} = \frac{(0.47)^2}{2(10)}$

$\frac{0.18x^2}{0.18} = \frac{0.1}{0.18}$
 $\sqrt{x^2} = \sqrt{0.5}$
 $= 0.7 \text{ m}$



68) $E=0$
at $\lambda=1, S_{H}$

69) α $k=0.3 MeV$

70) $E=3.3 MeV$

8) Gravity (chapter 8)

⇒ Universal Gravitation:-



between any two objects there is an attractive gravitational force

$$F = G \frac{m_1 m_2}{r^2}$$

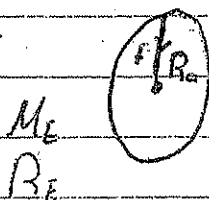
$$G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

G : Gravitational Universal Force constant

$F = G \frac{m_1 m_2}{r^2}$ { Force between 2 point Masses
 { Force between 2 spherical Masses
 { Force between 2 any shape Masses
 where r is very large

The acceleration of Gravity

1) Near the earth surface



$$F = \frac{G M_E m}{R_E^2} = m a$$

$$a = \frac{G M_E}{R_E^2}$$

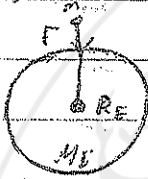
$$R_E = 6.37 \times 10^6 \text{ m}$$

$$M_E = 5.97 \times 10^{27} \text{ kg}$$

$$g = \frac{GM_E}{R_E^2} \quad \text{Near the surface}$$

$$9.81 \text{ m/s}^2$$

2) g at 380 km above the earth surface



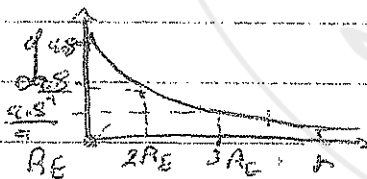
$$F_{\text{grav}} = \frac{GmM_E}{(R_E + R)^2}$$

$$= \frac{GM_E}{(R_E + R)^2}$$

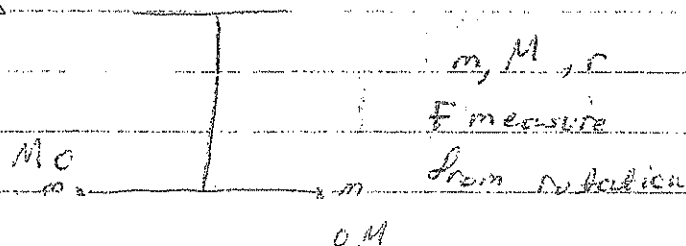
$$a = \frac{GM_E}{(R_E + R)^2}$$

$$= \frac{(6.67 \times 10^{-11}) (5.97 \times 10^{27})}{(6.37 \times 10^6 + 380 \times 10^3)^2}$$

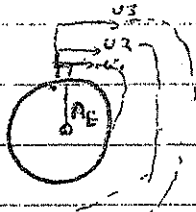
$$= 8.74 \text{ m/s}^2$$



The Cavendish experiment:



Orbital Motion



v_1, v_2, v_3 are not enough for m to rotate around the earth

Find $|v|$ for any Mass to Move in a ^{circular} Motion around the earth



$$F_{\text{grav}} = \frac{G m M_E}{r^2}$$

$r = R_E + \text{height above the surface of the earth}$

this force is a centripetal force

$$\frac{m v^2}{r} = \frac{G m M_E}{r^2}$$

$$v^2 = \frac{G M_E}{r}$$

$$v = \sqrt{\frac{G M_E}{r}} \quad (1)$$

$$v = \frac{2\pi r}{T}$$

T = Periodic time

$$\left(\frac{2\pi r}{T}\right)^2 = \frac{G M_E}{r}$$

$$T^2 = \frac{4\pi^2 r^3}{G M_E}$$

Chapter 8.1)

8) M_e, r, G, T

$$T^2 = \frac{4\pi^2 r^3}{GM_e} \quad \text{we can't find the mass from the information above!}$$

$$\frac{GM_e m_p}{r^2} = \frac{m_p v^2}{r}$$

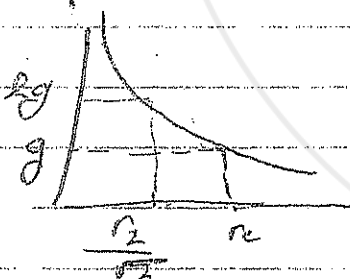
1) $M_u = M_e$

$v_u = 2v_e$

$$a = \frac{GM}{r^2}$$

$$\frac{g_u}{g_e} = \left(\frac{r_e}{r_u} \right)^2$$

$$r_u = \frac{r_e}{\sqrt{3}}$$

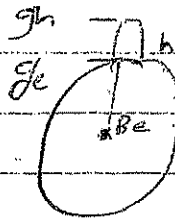


13) $g_u = 3g$

$$\frac{g_u}{g_e} = 3 = \left(\frac{r_e}{r_u} \right)^2$$

$$r_u = \frac{r_e}{\sqrt{3}}$$

17/



$$\frac{g_h}{g_e} = \frac{g}{g} \cdot \frac{1.36 \times 10^{-3}}{1}$$

$$= \left(\frac{R_e}{R_h} \right)^2 = \left(\frac{r_e}{r_e + h} \right)^2 \Rightarrow h ??$$

$$19) \quad T^2 = \frac{4\pi^2 r^3}{GM}$$

$$(24 \times 3600)^2 = \frac{4\pi^2}{GM_e} r^3 \Rightarrow r ??$$

$$\frac{GM_m}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM}{r}}$$

$$F = \frac{G m_1 m_2}{r^2}$$

$$U_2 = 0$$

$$U = -\frac{GmM_E}{r}$$

Geosynchronous orbit for a satellite

$$T = 24h$$

We calculate $r = 4.22 \times 10^7 m$

Example 1

Find the work done by in moving satellite of mass $m = 11 \times 10^3 kg$ from the earth surface to the geosynchronous orbit



$$W_g = -\Delta U$$

$$W_{done by} = \Delta U$$

$$W_{done} = 6.67 \times 10^{-11} \times 11 \times 10^3 \times 5.97 \times 10^{24} \times \left(\frac{1}{R_E} - \frac{1}{r} \right)$$

$$W_{done} = U_2 - U_1$$

$$W_{done} = -\frac{GmM_E}{r_2} - \left(-\frac{GmM_E}{R_E} \right)$$

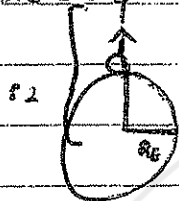
$$GmM_E \left(\frac{1}{R_E} - \frac{1}{r} \right)$$

$$W_{done} = 6.67 \times 10^{-11} \times 11 \times 10^3 \times 5.97 \times 10^{24} \left[\frac{1}{6.37 \times 10^6} - \frac{1}{9.22 \times 10^7} \right]$$

$$1 \rightarrow 2$$

$$= 5.8 \times 10^{11} \text{ J}$$

Example 5.3-9



$$v_2 = 3.1 \times 10^3 \text{ m/s}$$

$$(K+U)_1 = (K+U)_2$$

$$\frac{1}{2} m v_2^2 - \frac{G m M_E}{R_E} = 0 + - \frac{G m M_E}{r_2}$$

$$\frac{v_2^2}{2} - \frac{G M_E}{R_E} = - \frac{G M_E}{r_2}$$

6/9/14 :

$$r_2 = 6.9 \times 10^6 \text{ m}$$

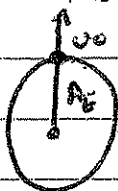
$$h = r_2 - R_E$$

$$= 6.9 \times 10^6$$

Q

→ Escape speed :-

Find the escape speed from earth surface?



outside of the gravity

$$K_2 = \text{zero}$$

$$U_2 = \text{zero}$$

$$K_0 + U_0 = (K + U)_2$$

$$\frac{1}{2} m v_0^2 - \frac{G m M_E}{R_E} = 0 + 0$$

$$v_0 = \sqrt{\frac{2 G M_E}{R_E}} = 11.2 \text{ km/s}$$

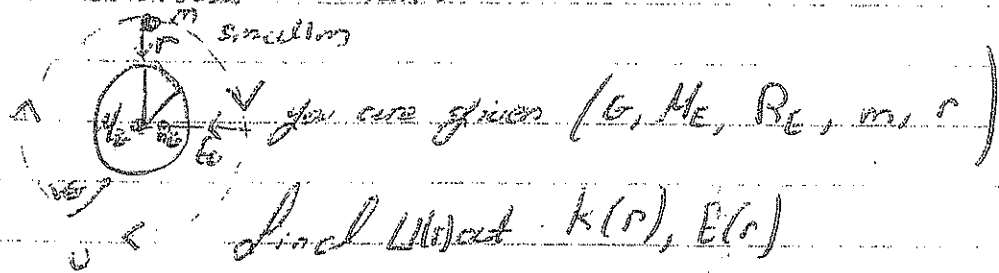
At the earth surface

$$K_0 = \frac{1}{2} m v_0^2$$

$$U_0 = - \frac{G m M_E}{R_E}$$

escape speed

Energy in circular orbits:-



$$U(r) = -\frac{GmM_E}{R_E r}$$

To find K

$$F = \frac{GmM_E}{r^2}$$

$$\frac{mv^2}{r} = \frac{GmM_E}{r^2} \Rightarrow \text{Multiply by half}$$

$$\frac{1}{2}mv^2 = \frac{1}{2} \frac{GmM_E}{r} = K$$

Note that $K = \frac{1}{2}U$, $U = -2K$

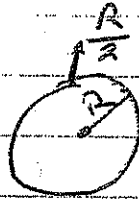
$$E = K + U$$

$$E = -\frac{GmM_E}{2R_E}, E < 0$$

When it is bound to go off
means it is bounded to Earth

$$33/ g = 22.5 \text{ m/s}^2$$

$$\text{Find } g' \text{ at } h = \frac{R}{2}$$



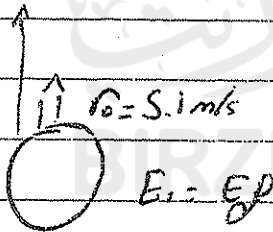
$$a_1 = \frac{GM}{r^2}$$

$$\frac{a_2}{a_1} = \frac{r_1^2}{r_2^2} = \left(\frac{\frac{3}{2}R}{\frac{R}{2}} \right)^2$$

$$\frac{a_2}{a_1} = \frac{9}{1} \cdot \frac{4}{9}$$

$$a_2 = \frac{9}{1} a_1 = \frac{9}{1} \times 22.5 = 10 \text{ m/s}^2$$

26



Find m

$$\frac{1}{2} m v_0^2 - \frac{GMm}{r_e} = - \frac{GMm}{r_m}$$

$$h = r_m - r_e$$

34

$$v_e = 7.7 \text{ km/s}$$

$$M = 2.9 \times 10^{24} \text{ kg}$$

$$r_p = ??$$

$$v_e = \sqrt{\frac{2GM}{r_p}}$$

$$7.1 \times 10^3 \text{ m/s} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 2.9 \times 10^{24}}{r_p}}$$

$$r_p = \frac{2 \times 6.67 \times 10^{-11} \times 2.9 \times 10^{24}}{(7.1 \times 10^3)^2}$$

$$= 7.6 \times 10^6 \text{ m} = 7660 \text{ km}$$

$$38) m = 1 \text{ kg}$$

$$W = \Delta U$$

$$= U_f - U_i$$

$$= \frac{GMm}{r_s} - \left(-\frac{GMm}{r_e} \right)$$

$$= GMm \left(\frac{1}{r_e} - \frac{1}{r_s} \right)$$

$$6.67 \times 10^{-11} \times 1 \times 97 \times 10^{24} \times \left(\frac{1}{6.37 \times 10^6} - \frac{1}{42.2 \times 10^6} \right)$$

$$49) \frac{1}{2} m v^2 = \frac{GMm}{r_e} - 0$$

$$v = \sqrt{\frac{2GM}{r_e}}$$

$$66) \frac{v_e}{v_a} = \frac{\sqrt{\frac{2GM_E}{R_E}}}{\sqrt{\frac{2GM_A}{R_A}}} = \sqrt{\frac{M_E R_A}{M_A R_E}}$$

$$= \frac{11.2 \times 10^3}{7.74}$$

$$= \sqrt{\frac{(15.47 \times 10^{24}) / 6}{\left(\frac{4}{3} \pi R_A^3 \right) / 6} / (6.57 \times 10^3)}$$

$$\left(\frac{11.2 \times 10^3}{7.74} \right)^2 = R_A = 650 \text{ km}$$

$$I_T = \frac{4\pi^2 r^3}{GM} \quad r = 20200m$$

$$v = \frac{2\pi r}{T}$$

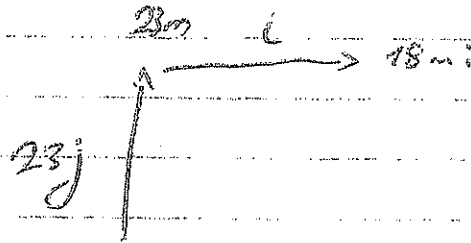
$$E = 0 = \frac{1}{2}mv^2 - \frac{GMm}{r_s}$$

$$v = \sqrt{\frac{2GM}{r_s}}$$

$$f = \frac{1}{T} \rightarrow 8.411 \times 10^{-4} \text{ Hz}$$

2017 - 2016

$$m = 650 \text{ kg}$$



$$CV = 8\text{Far}$$



Chapter 9:-

Systems of Particles

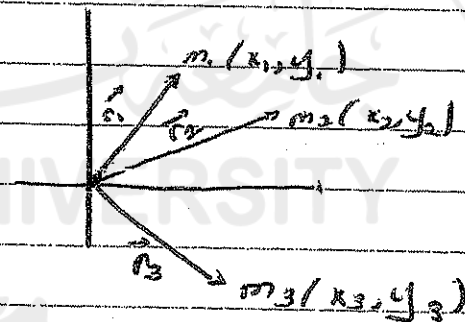
⇒ Centre of mass: "CM"

Centre of mass for a system containing many particles

is a point where all the masses are position (centre)

② $\Sigma \vec{F}$ are applied at that point

cm for many particles:



$$X_{cm} = \frac{m_1 x_1 + m_2 x_2 + \dots}{m_1 + m_2 + \dots}$$

$$X_{cm} = \frac{1}{M} \sum_{i=1}^n m_i x_i$$

$$y_{cm} = \frac{1}{M} \sum_{i=1}^n m_i y_i$$

total Mass

$$Z_{cm} = \frac{1}{M} \sum_{i=1}^n m_i z_i$$

$$\vec{r}_{cm} = \frac{1}{M} \sum_{i=1}^n m_i \vec{r}_i$$

ex: $m_1 = 0.5 \text{ kg}$ at $(1, 2) \text{ m}$ ⇒

$m_2 = 0.4 \text{ kg}$ at $(3, 2) \text{ m}$

$m_3 = 1.1 \text{ kg}$ at $(3, -1) \text{ m}$

Find $\vec{r}_{cm}$?

$\vec{r}_{cm} = 2.5\hat{i} + 0.35\hat{j} = 2.585 \text{ m}$

$$X_{cm} = \frac{1}{M} \sum m_i x_i$$

$$= \frac{1}{2} [0.5(1) + (0.4)(3) + (1.1)(3)]$$

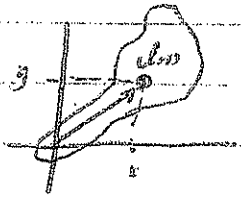
$$= \frac{5}{2} = 2.5 \text{ m}$$

$$y_{cm} = \frac{1}{2} [0.5 \times 2 + 0.4 \times 2 + 1.1 \times (-1)]$$

$$= 0.35 \text{ m}$$

cm for a rigid body?

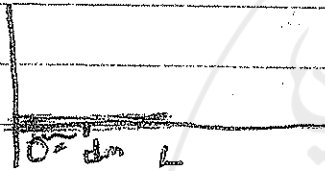
$$x_{cm} = \frac{1}{M} \int x dm$$



$$y_{cm} = \frac{1}{M} \int y dm$$

Example ones:-

Find x for cm for a uniform rod of length L & mass M



$$x_{cm} = \frac{1}{M} \int x dm, \quad dm = \lambda dx, \quad \lambda = \text{linear mass density}$$

$$\lambda = \frac{M}{L} \text{ kg/m}$$

$$= \frac{1}{M} \int_0^L x (\lambda dx)$$

$$= \frac{1}{M} \int_0^L x \left(\frac{M}{L} \right) dx$$

$$= \frac{1}{M} \int_0^L x \left(\frac{M}{L} dx \right)$$

$$x_{cm} = \frac{1}{L} \int_0^L x dx = \frac{1}{L} \left(\frac{L^2}{2} \right)$$

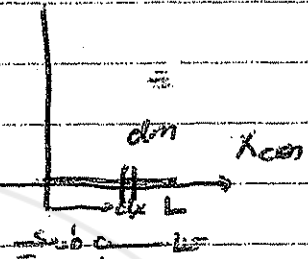
$$x_{cm} = \frac{L}{2}$$

example 2:

Find x_{cm} for a nonuniform rod of length

(1) $\lambda = \alpha x$
 $\uparrow$
any number

Find the mass of the rod?



$$M = \int_0^L dm = \int_0^L \lambda dx$$

$$= \int_0^L \alpha x dx$$

$$\left(\alpha \left(\frac{x^2}{2} \right) \right)_0^L = \frac{\alpha L^2}{2} \text{ kg}$$

$$x_{cm} = \frac{1}{M} \int x dm$$

$$= \frac{1}{M} \int_0^L x (\lambda dx)$$

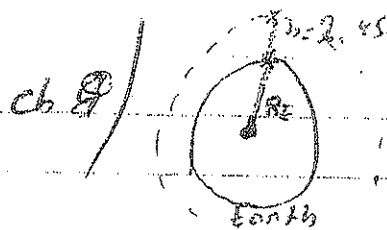
$$= \frac{1}{M} \int_0^L x (\alpha x) dx$$

$$= \frac{1}{M} \int_0^L \alpha x^2 dx$$

$$= \frac{\alpha}{M} \int_0^L x^2 dx$$

$$= \frac{\alpha}{M} \left[\frac{x^3}{3} \right]_0^L = \frac{\alpha}{M} \left[\frac{L^3}{3} \right] = \frac{\alpha L^3}{\alpha L^2/2 \cdot 3}$$

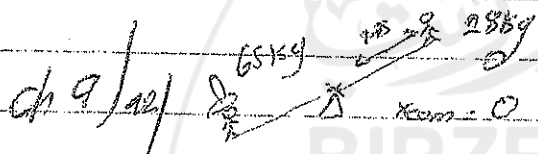
$$\boxed{x_{cm} = \frac{2L}{3}}$$



$$E_i = E_f$$

$$\frac{\frac{1}{2} m v^2 - G M m}{R_E} = - \frac{G M m}{R_E + h}$$

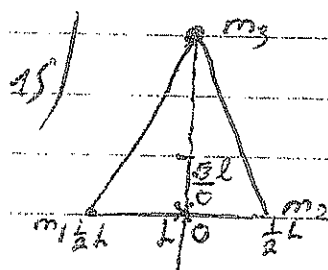
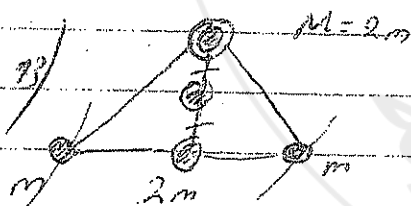
$$\frac{\frac{1}{2} m v^2}{R_E} - \frac{G M m}{R_E} = - \frac{G M m}{R_E + h}$$



$$x_{cm} = 0 = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = 0$$

$$\frac{28 \times 1.75 + 65(x_2)}{28 + 65} = 0$$

$$x_2 = - \frac{28 \times 1.75}{65} = -0.75 \text{ m}$$



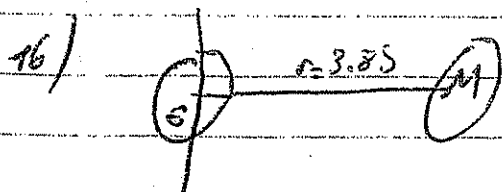
$$y_{cm} = \frac{m_1(0) + m_2(0) + m_3(h)}{3m} = \frac{1}{3} h$$

$$h_{cm} = h$$

$$\frac{\sqrt{3}}{2} L = h$$

$$y_{cm} = \frac{1}{3} \left(\frac{\sqrt{3}}{2} L \right)$$

$$E_{cm} = \left(0, \frac{\sqrt{3}}{2} L \right)$$



$$x_{cm} = \frac{M_E x_E + M_M x_M}{M_E + M_M}$$

$$= \frac{0.6735 \times 10^{24}}{10^{24} \times (5.97 + 0.0735)} = 960 \text{ km}$$

Chapter 9):

Newton's second law for a system of many particles

$$M \vec{r}_{cm} = m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots + m_n \vec{r}_n$$

$$\frac{d}{dt} [\quad]$$

$$M \vec{v}_{cm} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots + m_n \vec{v}_n$$

$$M \vec{a}_{cm} = m_1 \vec{a}_1 + m_2 \vec{a}_2 + \dots + m_n \vec{a}_n$$

$$M \vec{a}_{cm} = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n$$

$$M \vec{a}_{cm} = \vec{F}_{net}$$

⇒ Linear Momentum:-

$$\vec{p} = m \vec{v} \quad \text{kg m/s} \quad \vec{v}$$

* Linear Momentum for a system of many particles

$$M\vec{v}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2 + \dots + m_n\vec{v}_n$$

$$M\vec{v}_{cm} = \vec{p}_1 + \vec{p}_2 + \dots + \vec{p}_n$$

$$M\vec{v}_{cm} = \vec{P}$$

$$\vec{P} = M\vec{v}_{cm}$$

$$\vec{F}_{net} = M\vec{a}_{cm}$$

The relation between $\vec{F}_{net}$ & $\vec{P}$

$$\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + \dots \Rightarrow \frac{d}{dt} []$$

$$\frac{d\vec{P}}{dt} = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n$$

$$\frac{d\vec{P}}{dt} = \vec{F}_{net}$$

The net force is the time rate of change of linear momentum :-

If $\vec{F}_{net} = 0$ on the system

$$\frac{d\vec{P}}{dt} = 0 \Rightarrow \vec{P} = \text{constant}$$

$\vec{P}_i = \vec{P}_f$ (conservation of linear momentum)

Condition for Conservation of $\vec{P}$: "System is isolated" " $\vec{F}_{net} = 0$ "

Applications on $[P_i = P_j]$

- 1) All types of collisions
 - 2) All types of explosions
 - 3) All types of firing missiles
 - 4) Decay process
- of mass

$$\vec{P}_i = \vec{P}_j$$

$$\frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}$$

$$(\vec{v}_{cm})_i = (v_{cm})_i$$

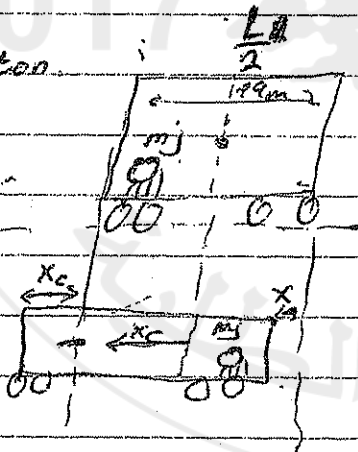
Ex 4. -

$m_j = 4.8 \text{ ton}$

$$m_c = 25 \text{ ton}$$

$$(V_{cm})_i = 0$$

$$L = 19 \text{ m}$$



$$f_{ext} = 0 \text{ (Jumbo + car)}$$

$$(x_{cm})_i = (x_{cm})_f$$

$$(\vec{v}_{cm})_i = (v_{cm})_f = 0$$

$$(X_{cm})_i = \frac{m_j x_{ji} + m_c v_{ci}}{m_j + m_c} = \frac{4.8(0) + M_c(E/2)}{m_j + m_c}$$

$$(X_{cm})_f = \frac{m_j x_{jf} + m_c x_{cf}}{m_j + m_c}$$

$$\underline{m_c = 15}$$

$$(X_{cm})_f = \frac{4.8(19 - x_c) + 15\left(\frac{1}{2} - x_c\right)}{m_j + m_c}$$

$$(X_{cm})_i = (X_{cm})_f$$

$$15\left(\frac{1}{2}\right) = 4.8(19) - 4.8x_c + 15\left(\frac{1}{2}\right) - 15x_c$$

$$0 = 4.8(19) - 19.8x_c$$

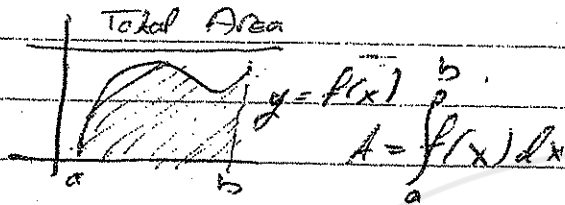
$$x_c = 4.6 \text{ m}$$

S.5, S.6)

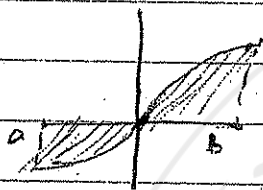
cek

Indefinite Integrals Substitution

Total Area:



2



$$= \left| \int_a^b f(x) dx \right| + \int_0^b f(x) dx$$

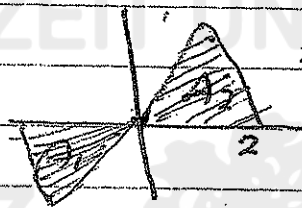
Ex: $f(x) = x^3 - x^2 - 2x$

x-axis $[-1, 2]$

$$f(x) = 0 \Rightarrow x(x^2 - x - 2) = 0$$

$$x(x-2)(x+1) = 0$$

$$x = 0, 2, -1$$



$$A = |A_1| + A_2$$

maen

$$\int_{-1}^0 (x^3 - x^2 - 2x) dx + \int_0^2 (x^3 - x^2 - 2x) dx$$

Symmetric functions:

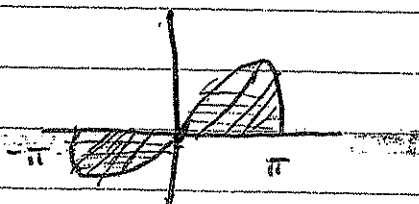
suppose that $f(x)$ is continuous on a symmetric interval $[-a, a]$

if $f(x)$ is even then $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$

||

if $f(x)$ is odd then $\int_{-a}^a f(x) dx = 0$

Ex: $\int_{-\pi}^{\pi} \sin x dx = 0$



$$\int_{-\pi/2}^{\pi/2} \cos x dx = 2 \int_0^{\pi/2} \cos x dx$$



$$= \int f(g(x)) g'(x) dx = \int f(u) du$$

$$u = g(x)$$

$$\int_{x=a}^{x=b} f(g(x)) g'(x) dx = \int_{g(a)}^{g(b)} f(u) du$$

Ex:

$$\int x \sqrt{2x+1} dx$$

$$u = 2x+1 \Rightarrow \frac{du}{2} = \frac{2}{2} dx$$

$$x = \frac{u-1}{2}$$

$$I = \int \frac{(u-1)}{2} \cdot u^{1/2} \cdot \frac{du}{2}$$

$$= \frac{1}{4} \left(u^{3/2} - u^{1/2} \right) du$$

$$\frac{1}{4} \times \frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} + C$$

$$= \frac{1}{4} \left(\frac{2}{5} (2x+1)^{5/2} - \frac{2}{3} (2x+1)^{3/2} \right) + C$$

$$\textcircled{2} \frac{1}{3} \int \frac{3x^2}{(x^3+5)^{5/2}} dx = \frac{1}{3} \int 3x^2 (x^3+5)^{-5/2} dx$$

$$= \left(\frac{1}{3} \right) \left(\frac{-2}{3} \right) (x^3+5)^{-3/2}$$

$$\textcircled{3} \int \frac{x^u}{x^3-1} dx$$

$$u = x^3 - 1$$

$$\frac{du}{3} = \frac{3x^2}{3} dx$$

$$\int \frac{x^2}{\sqrt{x^3-1}} dx$$

$$\frac{1}{3} \int u^{-1/2} du = \left(\frac{1}{3} \right) \left(\frac{1}{2} \right) (u^{1/2}) + C$$

$$= \frac{1}{6} (x^3-1)^{1/2} + C$$

$$(4) \int \sqrt{\frac{x-1}{x^5}} dx$$

$$= \int \sqrt{\frac{1}{x^4} - \frac{1}{x^5}} dx$$

$$\int \frac{1}{x^2} \sqrt{1 - \frac{1}{x}} dx$$

$$y = 1 - \frac{1}{x} \quad dy = \frac{1}{x^2} dx$$

$$\int \sqrt{y} dy = \frac{2}{3} y^{3/2} + C = \left(\frac{2}{3} \right) \left(1 - \frac{1}{x} \right)^{3/2} + C$$

$$(5) I = \int_0^{2\pi} \frac{\cos x}{\sqrt{4+3\sin x}} dx$$

$$y = 4 + 3\sin x \quad : \quad dx = \frac{1}{3} \int \frac{dy}{\frac{y}{3}}$$

$$\frac{dy}{3} = \frac{3}{3} \cos x dx$$

$$(6) \int_0^{3\pi/2} \sqrt{\theta} \cos^2 \theta^{3/2} d\theta$$

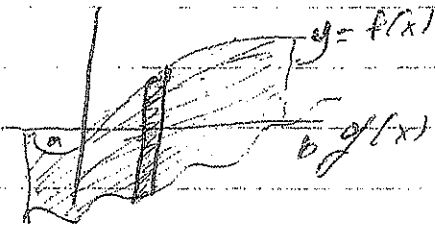
$$y = \theta^{3/2} \\ dy = \frac{3}{2} \theta^{1/2} d\theta$$

$$\frac{2}{3} dy = \sqrt{\theta} d\theta$$

$$I = \frac{2}{3} \int_0^{\pi} \cos^2 y dy$$

$$\begin{aligned} & \frac{2}{3} \int_0^{\pi} \left(\frac{1+\cos(2y)}{2} \right) dy \\ &= \frac{2}{3} \left[\frac{y}{2} + \frac{\sin 2y}{4} \right]_0^{\pi} \end{aligned}$$

Area between 2 curves:-



$$\sum (f(x_k) - g(x_k)) \Delta x_k$$

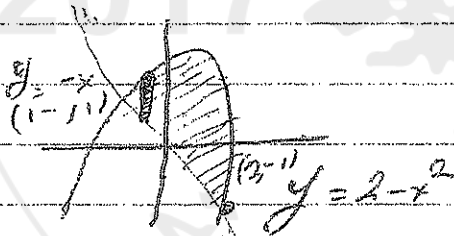
if $f(x) \geq g(x)$ is con $[a, b]$

$$f(x) \geq g(x)$$

$$A = \left| \int_a^b (f(x) - g(x)) dx \right|$$

Ex: find Area between

$$y = 2 - x^2 \text{ and } y = -x$$



$$x^2 = -x$$

$$x^2 + x = 0$$

$$x = 2, -1$$

$$\int_{-1}^2 (2 - x^2) - (-x) dx$$

$$\int_{-1}^2 2 - x^2 + x dx$$

Ex: find the area enclosed by $y = \sqrt{x}$

$$y = 0$$

$$y = x - 2$$



$$\sqrt{x} = x - 2$$

$$x^2 - 2x + 4 = x$$

$$x^2 - 3x + 4 = 0$$

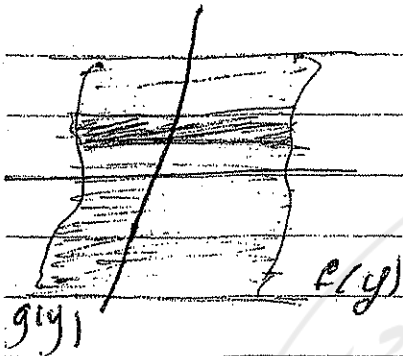
$$x = 4$$

$$x = 1$$

$$A = \int_0^2 \sqrt{x} dx + \int_2^4 (x - 2) dx$$

integration with respect to y :

$$x = f(y) \\ y = g(y)$$



$$A = \int_{y=c}^{y=d} (f(y) - g(y)) dy$$

$$A = \int_0^2 (y+2 - y^2) dy$$

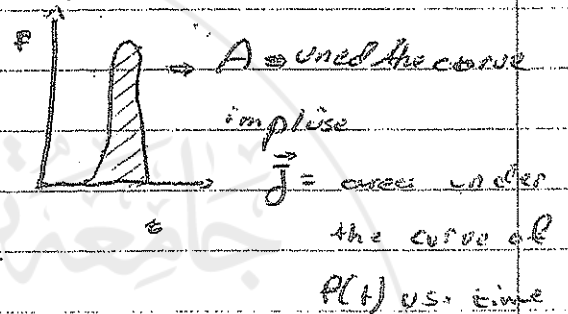
⇒ Impulse:- $\vec{F} \cdot \Delta t$

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

$$d\vec{p} = \vec{F}_{net} \cdot dt$$

$$\Delta \vec{p} = \int_{t_i}^{t_f} \vec{F}(t) dt = \vec{J} \quad \text{if } \vec{F} \text{ is constant}$$

$$\vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} \vec{F} dt = \vec{J} \quad \text{N.s}$$



Collision:-

* Linear momentum is conserved

external force = 0 " ideal case "

In all types of collisions

$$\vec{p}_i = \vec{p}_f$$

Collision's types:-

Elastic collision

elastic collision

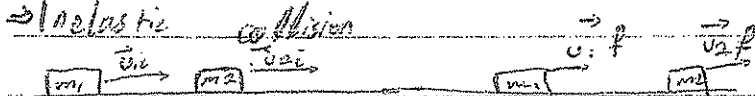
$$K_i = K_f$$

Inelastic collision

Inelastic collision

$$K_i \neq K_f$$

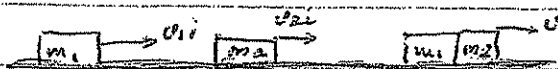
⇒ Inelastic collision



$$K_i \neq K_f$$

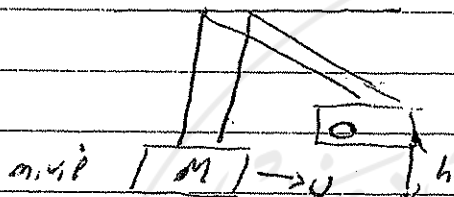
$$m_1 u_{1i} + m_2 u_{2i} = m_1 v + m_2 v$$

* Completely inelastic collision:



$$m_1 u_{1i} + m_2 u_{2i} = (m_1 + m_2) v_f \quad K_i > K_f$$

Example (9): Ballistic Pendulum



in the collision:

$$P_i = P_f \\ = m u_{1i} = (m + M) v \quad (1)$$

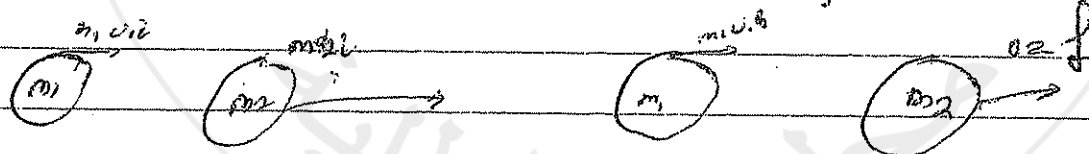
moves to a height h

$$(K + U)_i = (K + U)_f$$

$$\frac{1}{2}(m + M)v^2 = 0 + (m + M)gh$$

$$u = \sqrt{2gh} \quad \text{bullet @ find } u$$

Elastic collision in One Dimension:



$$m_1 u_{1i} + m_2 u_{2i} = m_1 u_{1f} + m_2 u_{2f} \quad (1)$$

$$K_i = K_f$$

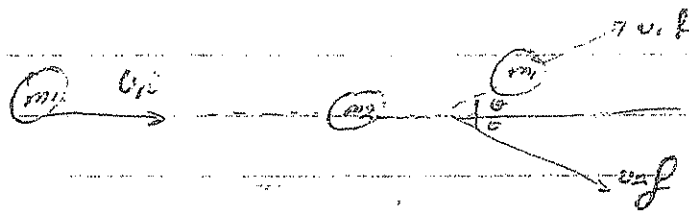
$$(1 \rightarrow 2)$$

$$\frac{1}{2} m_1 u_{1i}^2 + \frac{1}{2} m_2 u_{2i}^2 = \frac{1}{2} m_1 u_{1f}^2 + \frac{1}{2} m_2 u_{2f}^2 \quad (2)$$

from 1 & 2 you can get

$$u_i - u_{2i} = u_{1f} - u_{2f} \quad (3)$$

Collision 2 Dimensions:



$$(P_i)_x = (P_f)_x$$

$$P_i = P_f$$

$$m_1 u_1 = m_1 v_1 \cos \theta + m_2 v_2 \cos \phi$$

$$(P_i)_y = (P_f)_y$$

$$0 = m_1 v_1 \sin \theta + m_2 v_2 \sin \phi$$

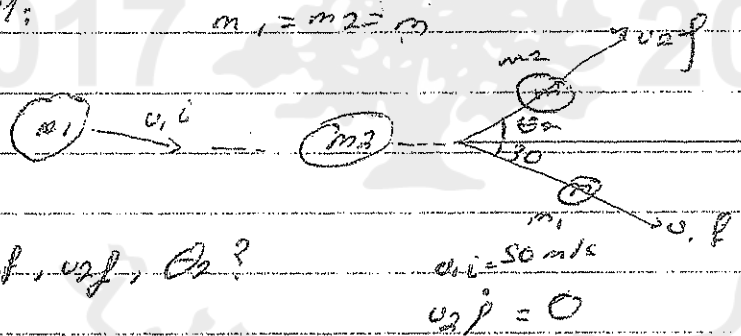
for elastic collision add

$$K_i = K_f$$

$$\frac{1}{2} m_1 u_1^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

example 11:

$$m_1 = m_2 = m$$



Find u_f, v_f, θ ?

$$(P_i)_x = (P_f)_x$$

$$m u_1 = m v_1 \cos \theta + m v_2 \cos 30$$

$$50 = v_1 \cos \theta + v_2 \cos 30 \quad (1)$$

$$(P_i)_y = (P_f)_y$$

$$0 = m v_1 \sin \theta + m v_2 \sin 30$$

$$v_2 \sin 30 = v_1 \sin \theta \quad (2)$$

(2) Elastic collision $K_i = K_f$

in One dimension

$$\frac{1}{2} m (u_1)^2 = \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2$$

$$v_{ii}^2 = v_{if}^2 + v_{2f}^2 \quad (3)$$

$$\vec{p}_i = \vec{p}_f$$

$$m \vec{v}_{ii} = m \vec{v}_{if} + m \vec{v}_{2f}$$

$$\vec{v}_{ii} = \vec{v}_{if} + \vec{v}_{2f}$$

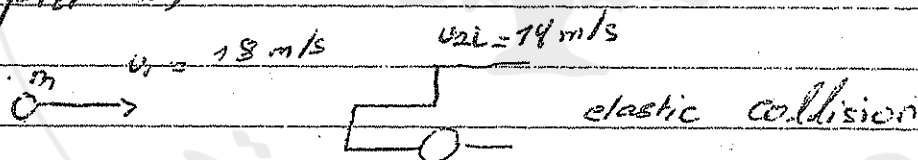
$$v_{ii} \cdot v_{ii} = (\vec{v}_{if} + \vec{v}_{2f}) \cdot (\vec{v}_{if} + \vec{v}_{2f})$$

$$v_{ii}^2 = v_{if}^2 + v_{2f}^2 + 2 \vec{v}_{if} \cdot \vec{v}_{2f}$$

$$v_{ii}^2 = v_{if}^2 + v_{2f}^2 + 2 v_{if} v_{2f} \cos(\theta_2 + 30^\circ)$$

$$\cos(\theta_2 + 30^\circ) = 0$$

(Chapter 9):



$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2 m_2}{m_1 + m_2} v_{2i}$$

$$m_1 < m_2 = -v_{1i} + 2v_{2i}$$

$$= -18 + 2(-14)$$

$$= -46 \text{ m/s}$$

* Chapter 9 *

Fain L

* 26-11-2014

(physics 141 - chapter 9)

* Systems of Particles $\rightarrow$ center of mass $\rightarrow$ (مركز الكتلة)
 $\uparrow$
 $(C.M.)$

* center of mass for a system containing many particles is a point where 1 All the masses are positioned (centered)

2 $\sum \vec{F}$ are applied at that point

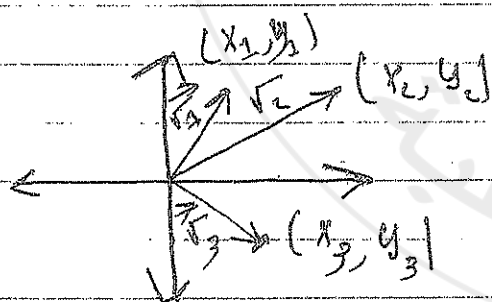
* First $\Rightarrow$ center of mass for many particles $\rightarrow$ 1

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \dots}{m_1 + m_2 + m_3} \quad * \quad y_{cm} = \frac{m_1 y_1 + m_2 y_2 + \dots}{\sum m} \quad \text{---} \frac{\text{مركز الكتلة}}{\sum m} = \rho \sigma$$

$$x_{cm} = \frac{1}{M} * \sum_{i=1}^n m_i x_i \quad * \quad z_{cm} = \frac{m_1 z_1 + m_2 z_2 + \dots}{\sum m} \quad \text{---} \frac{\text{مركز الكتلة}}{\sum m} = \rho \sigma$$

total mass $\downarrow$

(x_{cm}, y_{cm}) or (x_{cm}, y_{cm}, z_{cm})



$$* \vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots}{\sum m} \quad \text{---} \text{ (1)}$$

$$* \vec{r}_{cm} = x_{cm} \hat{i} + y_{cm} \hat{j} + z_{cm} \hat{k} \quad \text{---} \text{ (2)}$$

* Ex. $m_1 = .5 \text{ kg}$ at $(1, 2)$ $m_2 = .4 \text{ kg}$ $(3, 2)$ $m_3 = 1.1 \text{ kg}$ $(3, -1)$

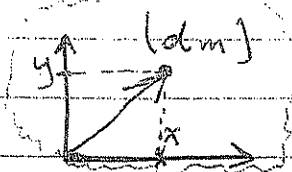
Find $\vec{r}_{cm}$? $\rightarrow x_{cm} = 2.5 \text{ m}$ $y_{cm} = .35 \text{ m}$

$$\vec{r}_{cm} = 2.5 \hat{i} + .35 \hat{j}$$



#26-11-2014 (physics 141 chapter 9)

Second $\Rightarrow$ ^{cm} $\Rightarrow$ x_{cm} For a rigid body $\rightarrow$ (2)



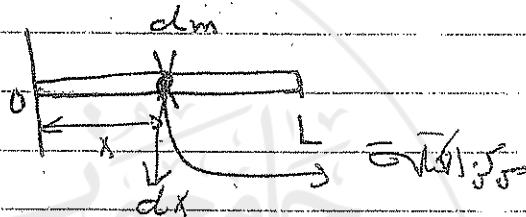
$$x_{cm} = \frac{1}{M} \int x dm \quad y_{cm} = \frac{1}{M} \int y dm \rightarrow M \rightarrow \text{total mass}$$

$\rightarrow dm \rightarrow a dx, a dy \rightarrow a$

constant given in question

* Ex. Find x_{cm} For a Uniform Rod of length = L , mass = M

$$x_{cm} = \frac{1}{M} \int x dm$$



$$dm = \lambda dx \rightarrow \lambda = \frac{M}{L} \rightarrow \frac{kg}{m} \rightarrow \text{Linear mass density}$$

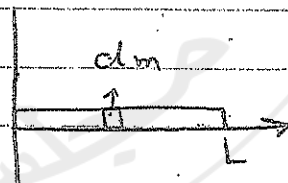
$$x_{cm} = \frac{1}{M} \int_0^L x \cdot \frac{M}{L} dx = \frac{1}{L} \int_0^L x dx = \frac{1}{L} \cdot \frac{L^2}{2} = \frac{L}{2}$$

$$x_{cm} = \frac{L}{2}$$

$dm \rightarrow$ constant
 dx, dy & dz are small
 x -axis or y -axis

* Ex. Find x_{cm} For a Non Uniform Rod of length L and $\lambda = \alpha x$

and Find the mass of the rod?



$$M = \int_0^L dm = \int_0^L \lambda dx = \int_0^L \alpha x dx$$

$$M = \frac{\alpha L^2}{2} \quad x_{cm} = \frac{1}{M} \int_0^L x \lambda dx \rightarrow \frac{\alpha}{M} \int_0^L x^2 dx$$

$$x_{cm} = \frac{\alpha}{M} \cdot \frac{L^3}{3} \quad \text{but } M = \frac{\alpha L^2}{2}$$

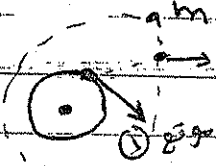
$$\frac{\alpha}{\frac{\alpha L^2}{2}} \times \frac{L^3}{3} = \frac{2L}{3}$$

الجواب

* 12-2014

(physics 141 → chapter 9)

* [28] $T = 24 \text{ h} \rightarrow T^2 = \frac{4\pi^2 r^3}{GM_E} \rightarrow r = 4.2 \times 10^7 \text{ m}$ $V = \frac{2\pi r}{T} = 3070 \text{ m/s}$

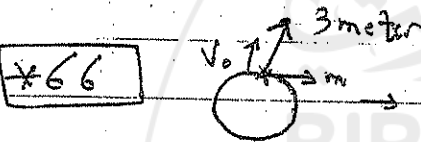


* Work done by this case $W = \Delta E$

$W = \Delta E$

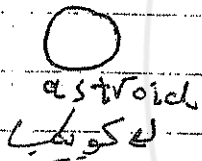
$W = \Delta U_g + \Delta K$ $U_2 - U_1 + \frac{1}{2} K V_2^2 \rightarrow V_2 = 0$

$W = -\frac{GM_E}{r} + \frac{GM_E}{R_E} + \frac{1}{2} K V_2^2 =$



* [66] $mgh = \frac{1}{2} m V_1^2$ $V_1 = 6 \text{ g}$ $\rightarrow \rho \rightarrow \text{Roof of asthenite}$
 $= 2500 \text{ kg/m}^3$

* Fuel V_2 on asteroid $\rightarrow$ we suppose $V_1 = V_2$



$V_2 = \frac{2GM_a}{R_a} \rightarrow 6 * g = \frac{2GM_a}{R_a}$ but $M_a = \frac{\rho * \frac{4}{3} \pi R_a^3}{}$

$R_a = \sqrt{\frac{18}{8\pi G * \rho}} = 6500 \text{ m}$

*

1-2-2014

(physics 141 → chapter 9)

① Newton's second law for a system of many particles

$$\vec{V}_{cm} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots}{\sum m} \rightarrow M * \vec{V}_{cm} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots$$

Then we make the derivative → $M * \vec{V}_{cm} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots$
 المشتقة الأولى

→ then the second derivative → $M * \vec{a}_{cm} = m_1 \vec{a}_1 + m_2 \vec{a}_2 + \dots$

$$* M * \vec{a}_{cm} = F_1 + F_2 + F_3 + \dots \quad * M * \vec{a}_{cm} = \vec{F}_{net} \rightarrow \text{القوة المحركة}$$

② Linear Momentum (كمية التحرك الخطي)

$$\vec{P} = m \vec{v} \rightarrow \text{kgm/s}$$

$$* M \vec{V}_{cm} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots \rightarrow M * \vec{V}_{cm} = \vec{P}_1 + \vec{P}_2 + \dots \quad \textcircled{1}$$

$$* M * \vec{V}_{cm} = \sum \vec{P} \quad * \vec{P} = M * \vec{V}_{cm} \quad * \vec{F}_{net} = M * \vec{a}_{cm} \quad \textcircled{2}$$

* [3] The relation between $\vec{F}_{net}$ and $\vec{P}$

$$\vec{P} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 \dots \quad \frac{d\vec{P}}{dt} = \vec{F}_{net} \quad \left[\vec{F}_{net} = \vec{F}_{ext} \right]$$

$$\frac{d\vec{P}}{dt} = m_1 \vec{a}_1 + m_2 \vec{a}_2 + \dots$$

* The net force is the time rate of changing linear momentum

* 2-12-2014 (physics 141 → chapter 9)

* if $\vec{F}_{\text{net}} = 0$ on the system $\rightarrow \frac{d\vec{P}}{dt} = 0 \rightarrow \vec{P} = \text{constant}$

* In the case $\vec{P}_i = \vec{P}_f$ (we called it → conservation of linear momentum)
Initial $\hookrightarrow$ Final

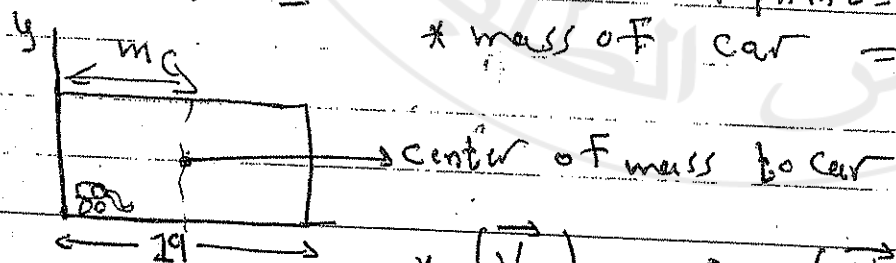
* Condition for conservation of $\vec{P}$ system is isolated ($\vec{F}_{\text{net}} = 0$)

* Application on conservation
① All types of collisions
② All types of explosions
③ All types firing missiles
④ Decaying process of mass

* $\vec{P}_i = \vec{P}_f \rightarrow m_1 \vec{V}_{1i} + m_2 \vec{V}_{2i} = m_1 \vec{V}_{1f} + m_2 \vec{V}_{2f}$ then divided by M

$$* \left(\vec{V}_{\text{cm}} \right)_i = \left(\vec{V}_{\text{cm}} \right)_f \rightarrow \boxed{3}$$

* Example 4 $\Rightarrow$ * mass of elephant = 4.8 ton * car's tail
* mass of car = 15 ton equal = 19



$$* \left(\vec{V}_{\text{cm}} \right)_i = 0 = \left(\vec{V}_{\text{cm}} \right)_f$$

$$\left(X_{\text{cm}} \right)_i = \frac{4.8 \times 0 + 15 \times 9.5}{29.8} \quad L/2$$

$$\left(X_{\text{cm}} \right)_f = \left(X_{\text{cm}} \right)_i$$

$$\left(X_{\text{cm}} \right)_f = 4.8 \times (19 - x_c) + 15 \times (L/2 - x_c)$$

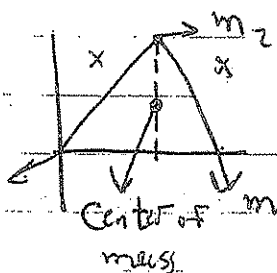
* discussion *

$$\frac{v_1 \hat{i} + v_2 \hat{j}}{v_1^2 + v_2^2} = \rho \hat{u}$$

3-12-2014

(physics 141 → chapter 9)

* 13

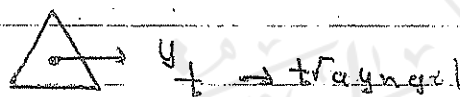
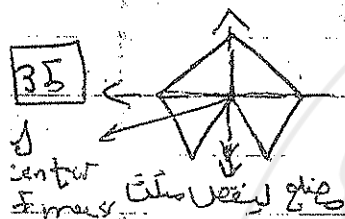


Find $m_2 \rightarrow y = \frac{0 + 0 + m_2 L}{2m + m_2}$

$$\frac{L}{2} = \frac{m_2 * L}{2m + m_2}$$

$$2m + m_2 = 2m_2 \rightarrow m_2 = 2m$$

* 35



* $y_c = \frac{y_t * m + 4m * y_c}{5m}$ but $y_c = 0$

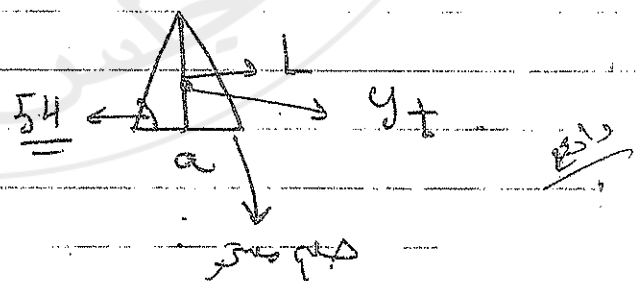
$$0 = y_t m + 4m y_c \rightarrow y_c = -\frac{1}{4} y_t$$

Note → ① $\Delta \rightarrow \theta = \frac{180}{3}$ ② $\frac{360}{4} \leftarrow \square$

مثال قانون الزاوية الداخلية → الزاوية الزاوية الداخلية = 540

then we return to Example 3

$$\tan 54 = \frac{L}{\frac{1}{2}a} \rightarrow L = \frac{1}{2}a \tan 54$$



$$y_t = \int x dm = \frac{2}{3} L$$

$$y_t = -\frac{2}{3} \tan 54 * \frac{a}{2}$$

??

$$y_c = \frac{1}{4} \left(\frac{2}{3} \tan 54 * \frac{a}{2} \right) = \frac{a}{12} \tan 54$$

* Lecture *

* 4-12-2014

(physics 141 - chapter 9)

* Impulse (قوة الدفع) * $\vec{F}_{net} = \frac{d\vec{p}}{dt} \rightarrow d\vec{p} = \vec{F}_{net} dt$ then take

the integration $\rightarrow \Delta \vec{p} = \int_{t_i}^{t_f} \vec{F}(t) dt = \vec{J} \rightarrow \boxed{\vec{p}_f - \vec{p}_i = \vec{J}} \text{ N.s}$

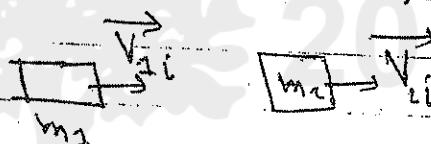
* Impulse = Area under the curve of $F(t)$ VS time

* Collision (تصادم)

In All types of collisions $\rightarrow \vec{p}_i = \vec{p}_f$

* Types of Collision $\rightarrow$ 1 Elastic ($K_i = K_f$) $\rightarrow$ (التصادم المرن)

2 Inelastic $\rightarrow (K_i \neq K_f) \rightarrow$ (التصادم غير المرن)

* Inelastic Collision $\rightarrow$ 

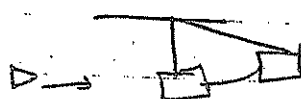
$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

* Completely Inelastic Collision (التصادم غير المرن التام)

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2) \vec{V} \rightarrow K_i > K_f$$

* Example $\rightarrow$ Ballistic Pendulum (البندول القذافي)

هو بندول مربوط بكلمة خشبية يمكن ان يسقط من سرعة الرصاص



$$p_i = p_f \rightarrow m_1 v_{1i} = (m_1 + m_2) V$$

$$\frac{1}{2} (m_1 + m_2) V^2 = \frac{1}{2} m_1 v_{1i}^2$$

1-12-2014

(physics 141 → chapter 9)

One dimension

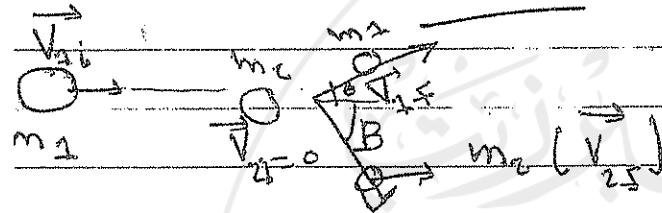
* Elastic collision in one Dimension

$$m_1 \vec{V}_{1i} + m_2 \vec{V}_{2i} = m_1 \vec{V}_{1f} + m_2 \vec{V}_{2f} \rightarrow (1)$$

$$\frac{1}{2} m_1 \vec{V}_{1i}^2 + \frac{1}{2} m_2 \vec{V}_{2i}^2 = \frac{1}{2} m_1 \vec{V}_{1f}^2 + \frac{1}{2} m_2 \vec{V}_{2f}^2 \rightarrow (2)$$

$$(3) \vec{V}_{1i} - \vec{V}_{2i} = -(\vec{V}_{1f} - \vec{V}_{2f})$$

* Collision in two Dimension



$$(\vec{V}_{cm})_i = (\vec{V}_{cm})_f$$

In any type of Collision

$$(P_i)_x = (P_f)_x \rightarrow m_1 V_{1i} \cos \theta + 0 = m_1 V_{1f} \cos \theta + m_2 V_{2f} \cos \theta$$

$$(P_i)_y = (P_f)_y \rightarrow 0 = m_1 V_{1i} \sin \theta + m_2 V_{2f} \sin \theta$$

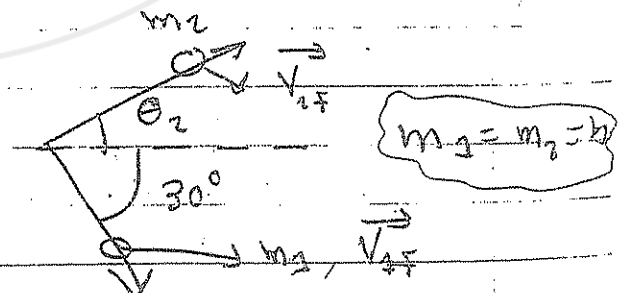
* ملاحظة إشارة السرعات في المعادلات

* For elastic collision add $\rightarrow K_i = K_f$

$$\frac{1}{2} m_1 V_{1i}^2 = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} V_{2f}^2 m_2$$

* Example 11 $\rightarrow V_{1i} = 50 \text{ m/s}$

Find V_{1f} , V_{2f} , θ_2 ?



$$50 = m V_{1f} \cos 30 + m V_{2f} \cos \theta$$

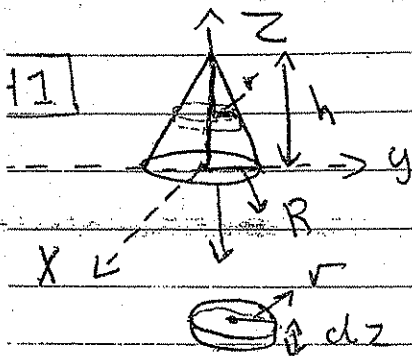
$$V_{2f} \sin \theta = V_{1f} \sin 30$$

$$V_{1i}^2 = V_{1f}^2 + V_{2f}^2$$

* discussion *

-12-2014

(physics 141 → chapter 9)



$$Z_{cm} = \frac{\int_0^h z \cdot \rho \cdot r^2 \pi dz}{\int_0^h R^2 \pi \cdot h \cdot \rho}$$

mass = $\rho \cdot V$

$$Z_{cm} = \frac{\int Z dm}{M}$$

$$dm = \rho \cdot dV$$

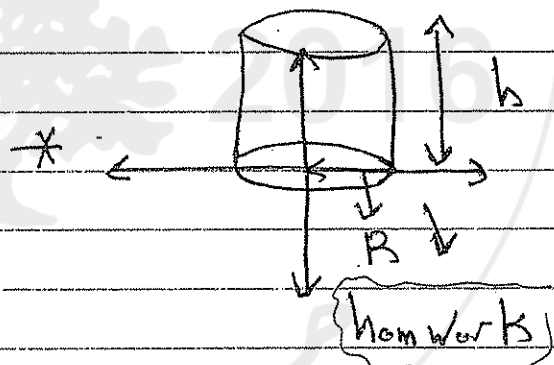
$$dm = \rho \cdot r^2 \pi dz$$

$$m = V \cdot \rho \quad \text{or} \quad m = \int dm$$

$$Z_{cm} = \frac{\int_0^h z \cdot \rho \cdot r^2 \pi dz}{\int_0^h \rho \cdot r^2 \pi dz} \quad \text{but } r \rightarrow 0 \text{ at } z=0$$

$$\frac{h-z}{h} = \frac{r}{R} \Rightarrow r = \frac{R}{h}(h-z)$$

$$\frac{\int_0^h z (h-z)^2 dz}{\int_0^h (h-z)^2 dz} = Z_{cm}$$



Find the center of mass

$$\rho = \text{constant}$$

constant

* Chapter 10 *

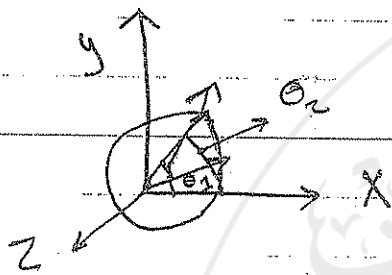
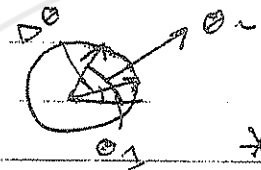
* 9-12-2014 (physics 141 → chapter 10)

* Rotational Motion (الحركة الدورانية)

الزاوية = الزاوية

* Angular quantities * Initial angular $\rightarrow$ Position $= \theta_1 \text{ rad}$

* Final Angular Position $= \theta_2 \text{ rad}$ * Angular displacement $= \Delta\theta = \theta_2 - \theta_1$



* Average Angular Velocity

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} \text{ rad/second}$$

* $\bar{\omega}$ is positive if $\bar{\omega}$ is counter clockwise الزاوية عكس عقارب الساعة
 * $\bar{\omega}$ is negative if $\bar{\omega}$ is clockwise الزاوية مع عقارب الساعة

2017 2016

$$-6 + 6t \quad \text{---} \quad a(2) = 6$$

$$a(4) = 18$$

-12-2014

(physics 141 → chapter 10)

• example → $\Theta(t) = 4t - 3t^2 + t^3$



* 9-12-2014 (physics 141 → chapter 10)

example 2, $\alpha(t) = 6t^4 - 4t^2$ at $t=0 \rightarrow \theta_0 = +1.5 \text{ rad}$

$$\alpha = \frac{d\omega}{dt} \rightarrow \int_{\omega_0}^{\omega} d\omega = \int_{t_0}^t \alpha dt \rightarrow \omega - \omega_0 = \int_0^t (6t^4 - 4t^2) dt$$

$\omega_0 = +2.5 \text{ rad/s}$

$$\omega = \frac{6}{5}t^5 - \frac{4}{3}t^3 + \omega_0 \rightarrow \boxed{1} \omega = +2.5 + \frac{6}{5}t^5 - \frac{4}{3}t^3$$

$$\boxed{2} \omega = \frac{d\theta}{dt} = \int_{\theta_0}^{\theta} d\theta = \int_{t_0}^t \omega dt = \theta - \theta_0 = \int_0^t (2.5 + \frac{6}{5}t^5 - \frac{4}{3}t^3) dt$$

$$\theta(t) = 1.5 + 2.5t + \frac{t^6}{5} - \frac{t^4}{3}$$

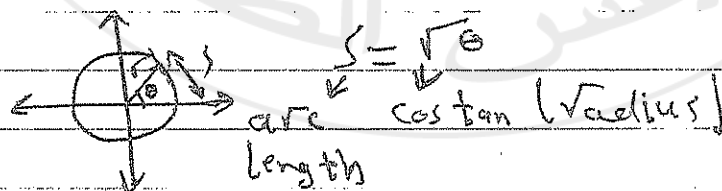
* Rotational Motion with constant α

$$\boxed{1} \omega = \omega_0 + \alpha t \quad * \boxed{2} \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

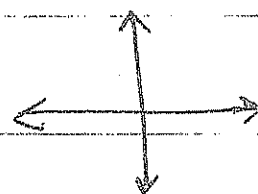
$$\boxed{3} \theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2 \quad * \boxed{4} \omega = \frac{\omega + \omega_0}{2}$$

* Relation between linear quantities and Rotational

quantities



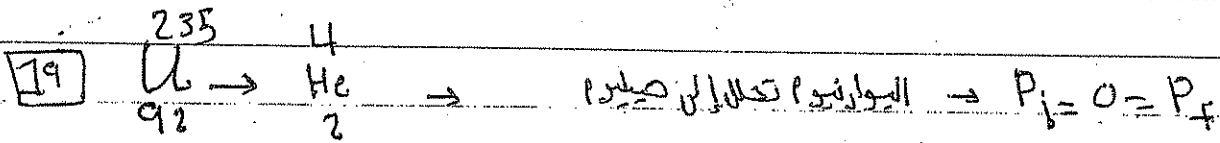
$$\frac{ds}{dt} = r \frac{d\theta}{dt} \rightarrow \boxed{V = r\omega}$$



discussion

10-12-2014

(physics 141 → chapter 9)



* Kinetic Energy for He = 5.15 MeV Find the Velocity of U

$$m_u V_u + m_{\text{He}} V_{\text{He}} = 0 \rightarrow 5.15 \times 10^6 \times 1.6 \times 10^{-19} = \frac{1}{2} \times 4 \times 1.6 \times 10^{-19} V_{\text{He}}^2$$

المعادلة الثانية

$$V = 1.575 \times 10^7 \text{ m/s}$$

$$V_u = -\frac{m_{\text{He}} V_{\text{He}}}{m_u} = -2.7 \times 10^5 \text{ m/s}$$

* [25] Impulse = $F \times \Delta t \rightarrow J = 5.64 \text{ N.s} \rightarrow F = 135 \text{ mN}$

$$\Delta t = \frac{J}{F} = \frac{5.64}{135 \times 10^{-3}} = 42 \text{ second}$$

* [27]

$m_1 V_{1i} + m_2 V_{2i} = m_1 V_{1f} + m_2 V_{2f} \rightarrow V_{2f} = \frac{V_{1i}}{2}$

$$\Delta K = \frac{1}{2} m V_{2f}^2 - \frac{1}{2} m V_{1i}^2 = \Delta K = \frac{1}{2} \times 2m \times \frac{V_{1i}^2}{4} - \frac{1}{2} m V_{1i}^2$$

$$\Delta K = \frac{1}{4} m V_{1i}^2 - \frac{1}{2} m V_{1i}^2 = -\frac{1}{4} m V_{1i}^2 = -\frac{1}{2} \times K_1 \rightarrow \Delta K = -\frac{1}{2} K_1 = -50\%$$

* [31]

18 m/s
 m_1

14 m/s
 m_2

$$1 = 14 - V_{1f} - -32 = 14 - V_{1f}$$

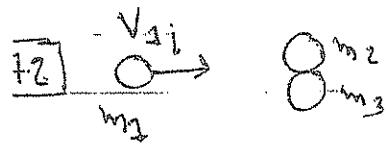
$$V_{1f} = 46 \text{ m/s}$$

We suppose that $V_{2i} = V_{2f}$

* [48] → Collision in 2 Dimension

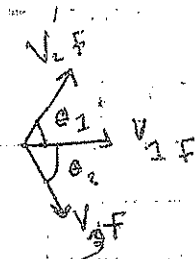
10-12-2014

(physics 141 → chapter 9)



$$V_{2f} = V_{3f} = 0$$

$$m_1 = m_2 = m$$



$$mV_{1i} = mV_{1f} + mV_{2f} \cos \theta_1 + mV_{3f} \cos \theta_2$$

$$V_{1i} = V_{1f} + V_{2f} \cos \theta_1 + V_{3f} \cos \theta_2$$

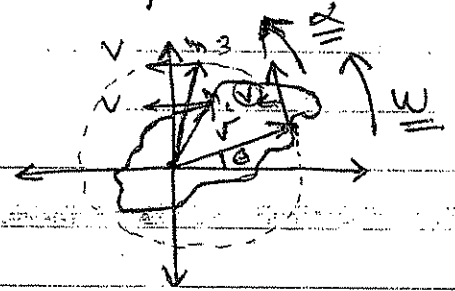
$$\text{In y direction} \rightarrow V_2 \sin \theta_1 = V_3 \sin \theta_2$$

* Lecture *

* 16-12-2014

(physics 141 → chapter 10)

$$* W = \frac{\Delta \theta}{\Delta t}, \quad W = \frac{d\theta}{dt}, \quad \alpha = \frac{dW}{dt}$$



* Rotational Kinetic Energy

$$K_{rot} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2 + \dots$$

$$= \frac{1}{2} m_1 (W r_1)^2 + \frac{1}{2} m_2 (W r_2)^2 + \frac{1}{2} m_3 (W r_3)^2 + \dots$$

$$= \frac{1}{2} W^2 (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots)$$

$$* v = W r$$

$$* \alpha = \frac{dW}{dt}$$

(I → moment of inertia)

$$I = \sum m_i r_i^2$$

the unite → Kg · m²

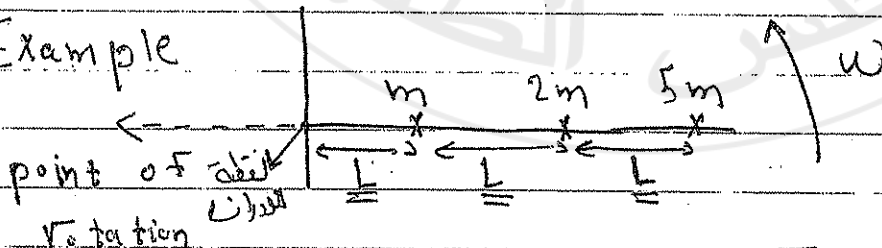
$$K_{rot} = \frac{1}{2} I W^2$$

Very Important

moment of inertia

$$* \text{Kinetic Energy of Rotation} = \frac{1}{2} * \underline{I} * W^2$$

* Example



$$* I = m L^2 + 2m (2L)^2 + 5m (5L)^2$$

$$* I = 54 m L^2$$

$$* K = \frac{1}{2} 54 m L^2 * W^2$$

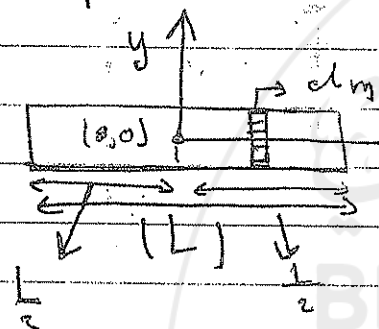
16-12-2014 (physics 141 → chapter 10)

Moment of Inertia for a rigid body (جسم صلب)

$$I = \int r^2 dm$$

$$I = \int r^2 dm$$

Example → Find I_{cm} for a rod of length = L and mass = m

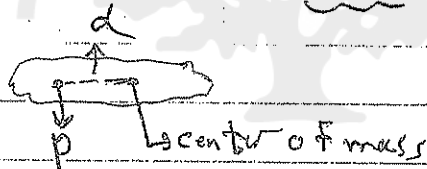


$$dm = \lambda dx \rightarrow \lambda = \frac{M}{L}$$

$$I = \int_{-L/2}^{L/2} x^2 \cdot \frac{M}{L} dx = \frac{M}{L} \int_{-L/2}^{L/2} x^2 dx = \frac{1}{12} M L^2$$

Parallel axis Theorem (عبارتي I جلد)

is given



??

Find I_p at a point p at a distance d from cm

$$I_p = I_{cm} + md^2$$

→ عبارت I جلد حول cm

d → distance between cm and the point

* 16-12-2014 (physics 141 → chapter 10)

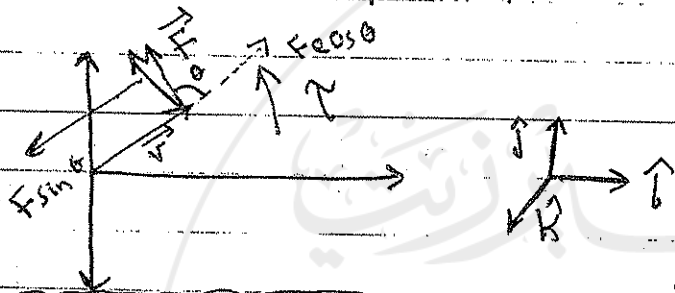
* Torque (العزم) ⇒ Torque is the Cause of Rotation

$$\vec{\tau} = \vec{r} \times \vec{F} \rightarrow \tau \rightarrow \text{Torque} \rightarrow \text{N.m}$$

distance Force

$$\vec{\tau} = A_x B_y \hat{i} + A_y B_x \hat{j} + A_z B_z \hat{k} \quad (1)$$

$$\tau = |\vec{r}| * |\vec{F}| * \sin \theta \quad * \quad \tau = |\vec{r}| * |\vec{F}| * \sin \theta \quad (2)$$



* $\vec{\tau} \neq h(\vec{r} \text{ and } \vec{F})$
 * العزم يكون عموديا على المساحة والقوة

$$\hat{i} \times \hat{j} = 1 * 1 * \sin 90 = 1 \rightarrow \hat{k}$$

$$\hat{j} \times \hat{k} = 1 * 1 * \sin 90 = 1 \rightarrow \hat{i}$$

$$\hat{i} \times \hat{k} = -\hat{j} = \hat{k} \times \hat{i} \rightarrow \text{يساوي بالقياس وبكافة الاتجاه}$$

* Example ⇒ $\vec{F} = 3\hat{i} - 4\hat{j} + 5\hat{k}$ N acts on a body

at point $\vec{r} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ m find $\tau = (3\hat{i} - 4\hat{j} + 5\hat{k}) \times (3\hat{i} - 2\hat{j} + 4\hat{k})$

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 5 \\ 3 & -2 & 4 \end{vmatrix} \quad \text{محددات}$$

$$= \hat{i}(-10 + 20) - \hat{j}(12 - 15) + \hat{k}(-12 + 10)$$

2/

* Lecture *

18-12-2014 (physics 141 → chapter 10)

Newton's second law for rotation

$$\tau_{\text{net}} = I \alpha \rightarrow \text{angular acceleration}$$

$\hookrightarrow$ moment of inertia

Work done by $\tau = \int_{\theta_i}^{\theta_f} \tau d\theta \rightarrow (\text{for variable } \tau)$

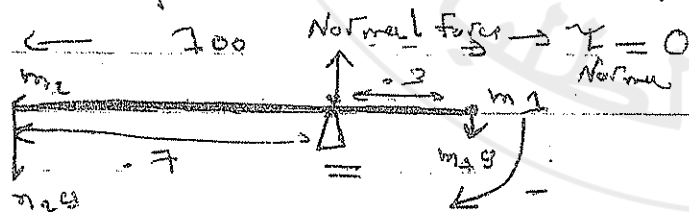
Work done by net $\tau = \Delta K = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2$

Work done by $\tau = \tau \Delta \theta$ (for constant τ)

Power = $\frac{d \text{work}}{dt} = \tau \frac{d\theta}{dt} = \tau \omega$

$P = \tau \omega$ Watt

* Example → masses rod of length



$$\begin{cases} L = 1.00 \text{ m}, m_1 = 0.3 \text{ kg} \\ L_1 = 0.3 \text{ m}, m_2 = 0.4 \text{ kg} \\ L_2 = 0.7 \text{ m} \end{cases}$$

* مع عقارب الساعة ← سالب (العزم)

* ضد عقارب الساعة ← موجب (العزم)

* Find τ_{net} around 0

$$\tau_{\text{net}} = \tau_1 + \tau_2 = -m_1 g L_1 + m_2 g L_2$$

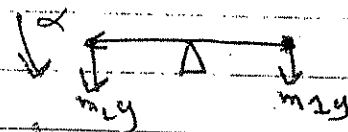
τ_1

* الذراع مع مركز الدوران وأصابع اليدين
اليدين مع الساعة وباصم اليدين القوة
فبع المتجه

$= 2.2 \text{ N.m}$

* 18-12-2014 (physics 141 → chapter 10)

* [2] find the initial angular acceleration?



$$\tau_{\text{net}} = I * \alpha \rightarrow I = m_1 r_1^2 + m_2 r_2^2 = .214 \quad \alpha = \frac{2.2}{.214} = 10.3 \text{ rad/s}^2$$

* [3] find tangential acceleration at the initial moment for m_1, m_2

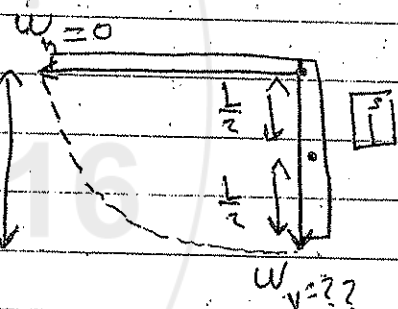
$$* a_{t1} = \alpha r_1 = 3.1 \text{ m/s}^2 \quad * a_{t2} = \alpha r_2 = 7.2 \text{ m/s}^2$$

* Example [2] Rod of length = 1m of mass = .8 Kg in horizontal

Position finally the rod is in vertical Position?

* We can make $E_1 = E_2 \rightarrow mgy \rightarrow \text{conservative}$

$$(K_1 + U_1) = (K_2 + U_2) = 0 + mgyL = \frac{1}{2} I \omega_v^2 + mgy \frac{L}{2}$$



$$= \text{center of mass} \rightarrow \text{conservative}$$

$$mgyL = \frac{1}{2} \left(\frac{1}{3} mL^2 \right) \omega_v^2 + mgy \frac{L}{2} \quad [2] \text{ When the rod is vertical find}$$

$$I = \frac{1}{3} mL^2$$

$$V_{\text{cm}} \text{ and } V_{\text{end}}$$

$$* V_{\text{cm}} = \omega \frac{L}{2} \quad * V_{\text{end}} = \omega * L$$

[3] Find τ at horizontal Position and vertical Position

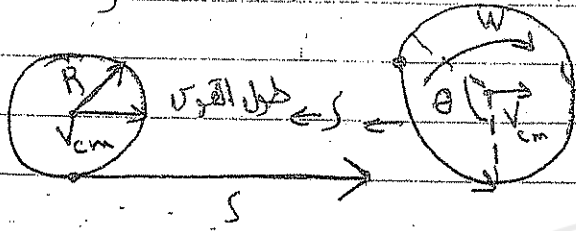
$$\tau_h = mgyL \sin 90 \quad \tau_v = mgy * L * \cos 90 = 0$$

18-12-2014

(physics 141 → chapter 10)

قانون لحركه القوس المائيه

Rolling Motion



$$* V_{cm} = \frac{s}{t} \rightarrow s = R \theta$$

$$\frac{ds}{dt} = R \frac{d\theta}{dt} \rightarrow V_{cm} = R \omega$$

Rolling Motion = Translational motion of the center

mass + Rotational motion around the (Cm)
 حركة انتقاليه
 دوران

$K = K_{for translation} + K_{for rotational}$
 Rolling

$$K_{Rolling} = \frac{1}{2} M V_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$

الحركه الانتقاليه (الدوران حول المركز)

* 22-12-2014

(physics 141 → chapter 10)

* 41 $\omega_0 = 0$ * 2π of revol * 18 rpm = $\frac{18 \times 2\pi}{60} = 1.88$
 $\ominus = -2 \times 2\pi = -4\pi$ 2π minute

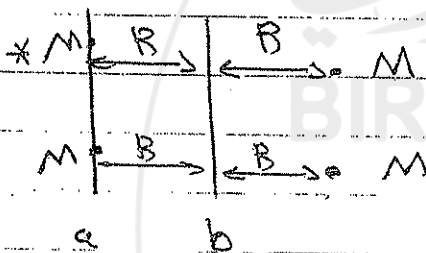
$\omega^2 = \omega_0^2 + 2\alpha\theta = 0^2 + 2 \times 1.88 \times 4\pi = 6.9$

* $\alpha = 1.88$

* 20 diameter = 1m Find the torque around the center



$\tau = \frac{1}{2} \times 320 \times \sin 90 = 160 \text{ N.m}$ المoment
القوة × المسافة



$I = \sum m_i r_i^2$ about $\underline{a}$
 $= M \times (2R)^2 + M \times (2R)^2 + 0 + 0 = 8R^2 M$

* about $b \rightarrow MR^2 + MR^2 + MR^2 + MR^2 = 4MR^2$

* 30 solid sphere $\rightarrow I = \frac{2}{5} MR^2 = \frac{2}{5} \times 5.97 \times 10^{24} \times (6.37 \times 10^6)^2 = 9.7 \times 10^{37}$

* 30 → b → 1 per century → dT المدة الزمنية Find the τ and F

$\frac{dT}{dt} = \frac{1}{1 \times 100 \times 365.25 \times 24 \times 3600} = 3.2 \times 10^{-10}$

$\omega = \frac{2\pi}{T} \rightarrow \frac{d\omega}{dt} = 2\pi \times \frac{1}{T^2} \times \frac{dT}{dt} = \alpha$

$(24 \times 60 \times 60)^2$

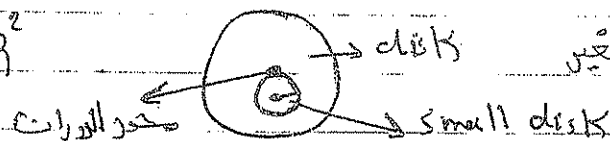
$\tau = \alpha \times I$

$\tau = F \times R_E \rightarrow F = \frac{\tau}{R_E} = 10 \text{ N}$

(physics 141 $\rightarrow$ chapter 10)

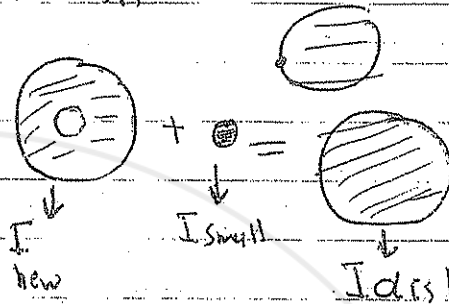
$$\boxed{65} \quad I = \frac{1}{2} M R^2$$

disk



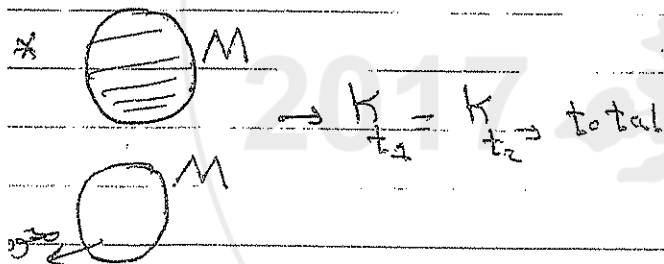
لین $\frac{I}{cm}$ بعد قطع الدسلة العنبر

$$I_{\text{small disk}} = \frac{1}{2} M R^2$$



$$I = I + M(2R)$$

Small disk em-small



* Lecture *

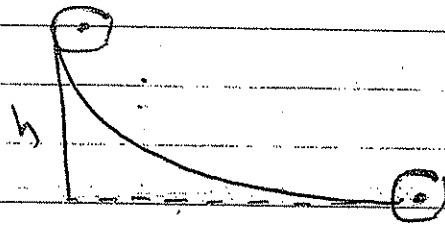
* 23-12-2014

(physics 141 → chapter 10)

* Example 12

* Find V_{cm} of the ball

$E_1 = E_2$



$$(K+U)_1 = (K+U)_2 = mgh = \left(\frac{1}{2} m V_{cm}^2 + \frac{1}{2} I \omega^2 \right) + 0$$

$$mgh = \frac{1}{2} m V_{cm}^2 + \frac{1}{2} \times \frac{2}{5} M R^2 \times \frac{V_{cm}^2}{R^2}$$

$V_{cm} = \omega R$

I spherical ball

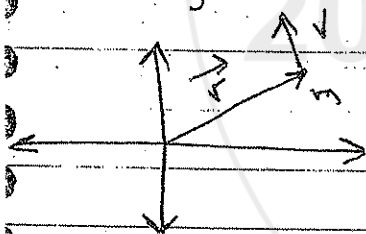
$$I = \frac{2}{5} M R^2$$

$$V_{cm} = \sqrt{\frac{10}{7} gh}$$

* 23-12-2014

(physics 141 → chapter 11)

* Angular Momentum (كمية الزخم الزاوي)



$$\vec{L} = \vec{r} \times \vec{p} \quad \text{but} \quad \vec{p} = m \vec{v}$$

angular momentum

$$L = |\vec{r}| * |\vec{p}| * \sin \theta$$

but if $\theta = 90^\circ \rightarrow L = r * p$

but $\vec{p} = m \vec{v}$ and $v = \omega r$

$$L = r * m v \quad \text{but} \quad v = \omega r$$

$$L = m r^2 \omega$$

$$L = m * r^2 * \omega$$

angular

velocity

I

angular momentum

mass of body

$$\text{but } I = m r^2$$

$$L = I \omega$$

The new law of

Angular Momentum

*23-12-2024

(physics 141 → chapter 11)

* $\vec{L} = I * \vec{\omega}$ → Newtons second Law For rotational

motion → $\vec{L}_{net} = I * \vec{\alpha}$ then take the derivative to the

equation → $\vec{L}_{net} = I * \frac{d\alpha}{dt} = \vec{L}_{net} = \frac{dL}{dt} * (I * \frac{d\omega}{dt}) = dL$

$\vec{L}_{net} = \frac{dL}{dt}$ → we take $\dot{f}(x)$ to first equation by
time → $\frac{dL}{dt} = I * \frac{d\omega}{dt} \rightarrow I * \alpha = \frac{dL}{dt}$

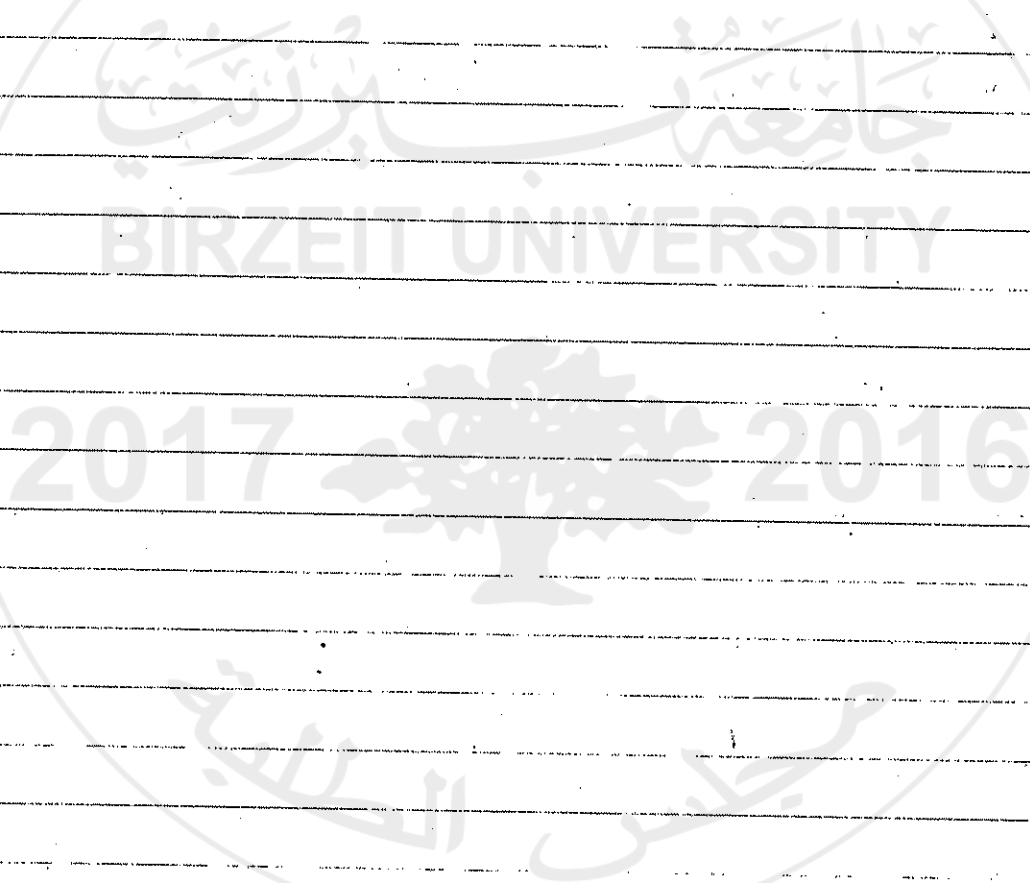
$\vec{L}_{net} = \cancel{I} * I = \vec{L}_{net} = \frac{dL}{dt}$ Very important

* Example 1

*23-12-2014 (physics 141 → chapter 11)

*Example 2 mass = 3 kg and its position given by $\vec{r} = 4t^2\hat{i}$

$\vec{r} = 4t^2\hat{i} - (2t + 6t^2)\hat{j}$ m find $\vec{L}(t)$ and $\vec{\tau}(t)$



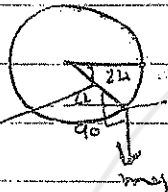
* discussion *

5-1-2014

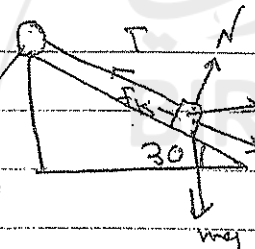
(physics 141 → chapter 10)

39 $\frac{K_r}{K_r + K_t} = \frac{\frac{1}{2} I \omega^2}{\frac{1}{2} I \omega^2 + \frac{1}{2} m v^2}$ but $\omega = \frac{v}{r}$

$$\frac{\frac{1}{2} I \omega^2}{\frac{1}{2} I \omega^2 + \frac{1}{2} m \omega^2 r^2} = \frac{I \frac{1}{2} m r^2}{\frac{1}{2} m r^2 + m r^2} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}$$

24  $\tau = F * r * \sin \theta$

$$= m g * r * \sin(90 + 24) =$$

57  $m_1 = 2.4$ Final F_k, M_k

$K = 0.85 K_B$ $a = 1.6 \text{ m/s}^2$

* $F_{\text{net}} = m a \rightarrow m g \sin 30 - T - m g \cos 30 = m a$

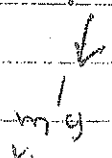
* $v * \alpha = a \rightarrow \alpha = \frac{a}{r} = \frac{1.6}{0.05} = 32 \text{ s}^{-2}$ (في البكرة ثابتة)

$\alpha = \frac{\tau}{I} \rightarrow \tau = \alpha * \frac{1}{2} M R^2 \quad \tau = .034$

* $\tau = F * r * \sin 90 \Rightarrow F = \frac{\tau}{r} = .68 \text{ N} \rightarrow \text{Tension Force}$

Passenger Problem page. chapter 4

$m a = N - m g$ $m u = \frac{(m' - m) g}{m} = \frac{.5 * 10}{5} = 1 \text{ m/s}^2$





* 5-11-8014

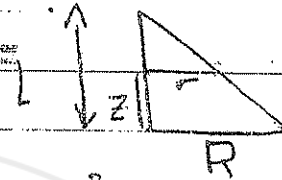
(physics 141 → chapter 10)

* Chapter 9



$$z_{cm} = \frac{1}{M} * \int_{z_b}^{z_t} z dm$$

$$dm = \rho * dv \rightarrow dv = (\pi r^2) dz$$



$$\frac{r}{R} = \frac{L-z}{L}$$

$$r = \left(\frac{L-z}{L} \right) * R \rightarrow dv = \pi * \left(\frac{R}{L} \right)^2 * (L-z)^2 dz$$

$$= \frac{\int z * \pi * \left(\frac{R}{L} \right)^2 * (L-z)^2 dz * \rho}{\int \rho * \pi * \left(\frac{R}{L} \right)^2 * (L-z)^2 dz} = \frac{\int z (L-z)^2 dz}{\int (L-z)^2 dz}$$

$$y = L - z \quad \cancel{z = L - y} \quad \cancel{dz = -dy}$$

$$= \int_0^L y^3 L$$

