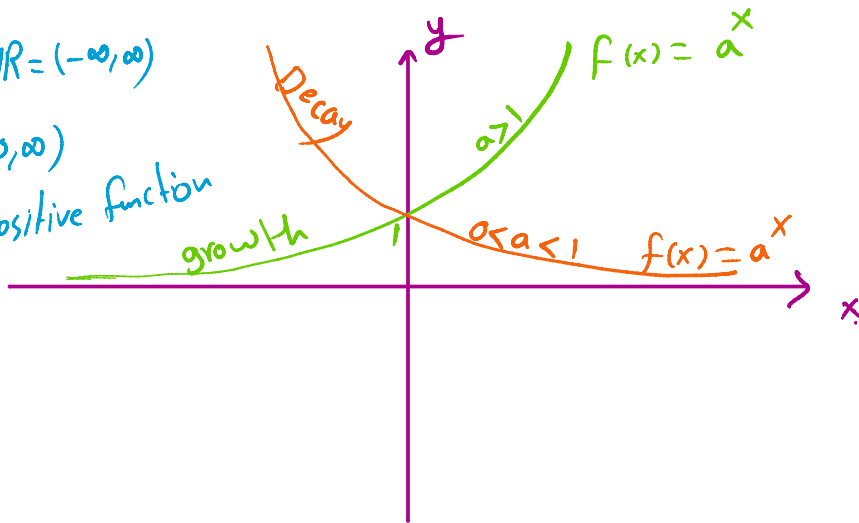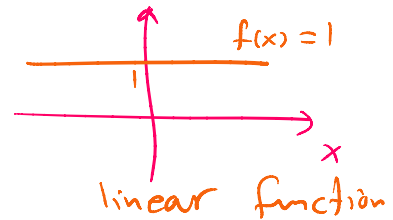The general form of the exponential function is

$$f(x) = a^x \quad, \quad a > 0 \quad, \quad a \neq 1$$

Domain $= \mathbb{R} = (-\infty, \infty)$

Range $= (0, \infty)$

↳ a^x is positive function

if $a = 1 \Rightarrow$

$f(x) = 1^x = 1$

$f(x) = 1$

linear function

Decay

growth

$a > 1$

$f(x) = a^x$

$0 < a < 1$ $f(x) = a^x$

__Exp__ Graph the exponential growth function $y = 2^x$ $a > 1$

$a = 2 > 1$

X	−3	−2	−1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

$y(-3) = 2^{-3} = \frac{1}{2^3} = \frac{1}{2 \cdot 2 \cdot 2} = \frac{1}{8}$

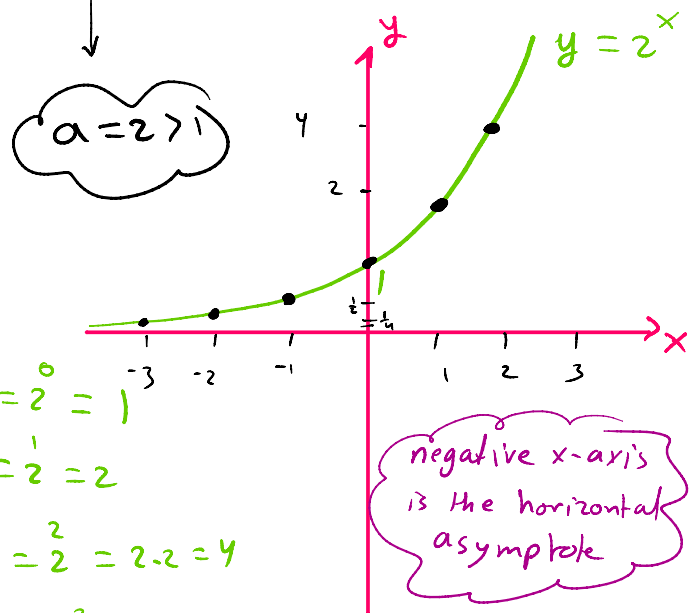$y(-2) = 2^{-2} = \frac{1}{2^2} = \frac{1}{2 \cdot 2} = \frac{1}{4}$

$y(-1) = 2^{-1} = \frac{1}{2}$

$y(0) = 2^0 = 1$

$y(1) = 2^1 = 2$

$y(2) = 2^2 = 2 \cdot 2 = 4$

$y(3) = 2^3 = 2 \cdot 2 \cdot 2 = 8$

$y = 2^x$

negative x-axis is the horizontal asymptote

__Exp__ Graph the exponential decay function $y = \left(\frac{1}{2}\right)^x$

$\underline{\underline{Exp}}$ Graph the exponential decay function 0 < ...

$$y = \left(\frac{1}{2}\right)^x = 2^{-x}$$

$0 < a = \frac{1}{2} < 1$

X	-3	-2	-1	0	1	2	3
y	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

$y(-3) = 2^{-(-3)} = 2^3 = 8$ $y(0) = 2^{-(0)} = 2^0 = 1$
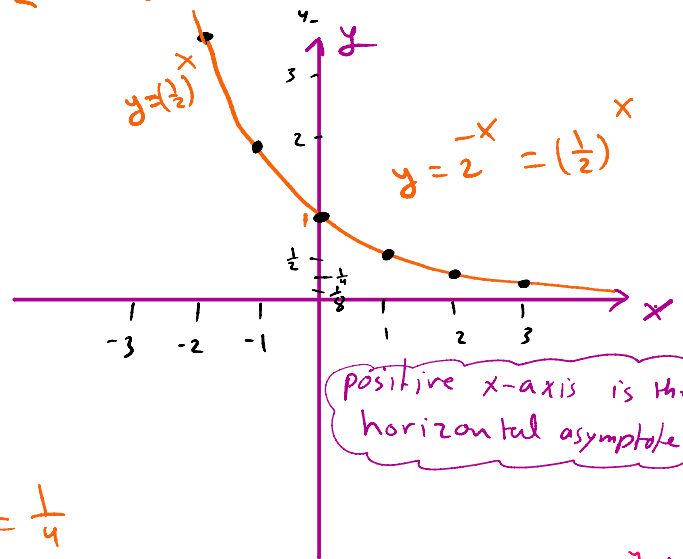
$y(-2) = 2^{-(-2)} = 2^2 = 4$ $y(1) = 2^{-1} = \frac{1}{2}$

$y(-1) = 2^{-(-1)} = 2^1 = 2$ $y(2) = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

$y(3) = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

$y = \left(\frac{1}{2}\right)^x$

$y = 2^{-x} = \left(\frac{1}{2}\right)^x$

positive x-axis is the horizontal asymptote

$\underline{\underline{Exp}}$ Find a in ① $\quad y = \left(\frac{3}{2}\right)^x \implies a = \frac{3}{2} > 1 \rightarrow$ growth

② $\quad y = \left(\frac{3}{2}\right)^{-x} \implies y = \left(\left(\frac{3}{2}\right)^{-1}\right)^x = \left(\frac{2}{3}\right)^x$

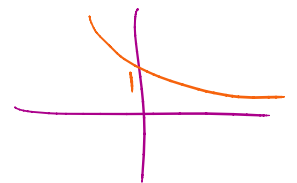$0 < a = \frac{2}{3} < 1 \rightarrow$ decay

③ $\quad y = 2^{-3x}$

$y = \left(2^{-3}\right)^x = \left(\frac{1}{2^3}\right)^x = \left(\frac{1}{2 \cdot 2 \cdot 2}\right)^x = \left(\frac{1}{8}\right)^x$

$0 < a = \frac{1}{8} < 1 \rightarrow$ decay

$$\boxed{4} \quad y = (\sqrt{3})^{4x}$$

$$y = ((\sqrt{3})^4)^x = (\sqrt{3}\sqrt{3}\ \sqrt{3}\sqrt{3})^x$$

$$= (3 \cdot 3)^x$$

$$= 9^x \implies \boxed{a = 9 > 1} \quad \text{growth}$$

<u>Exp</u> Graph the following exponensial fuction

① $y = e^x \implies a = e \approx 2.718 > 1$

② $y = 5^x \implies a = 5 > 1 \implies$ Growth

$y = 5$
Growth
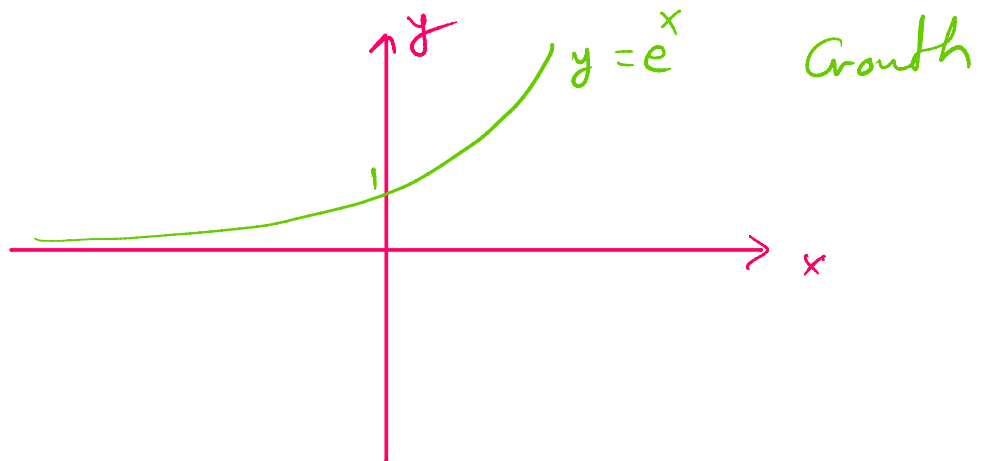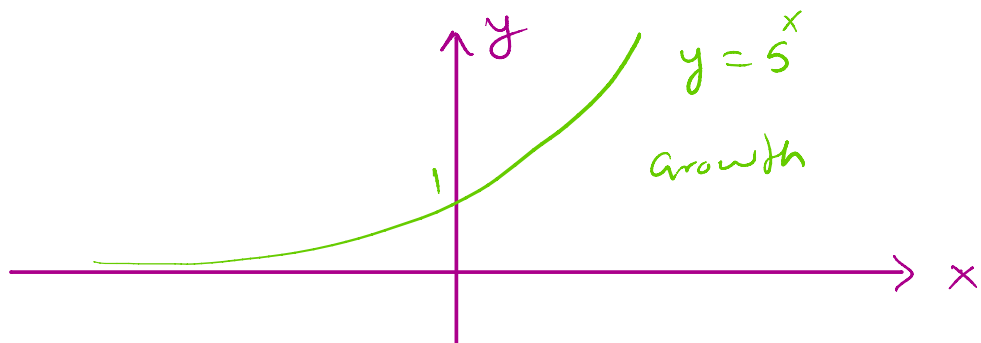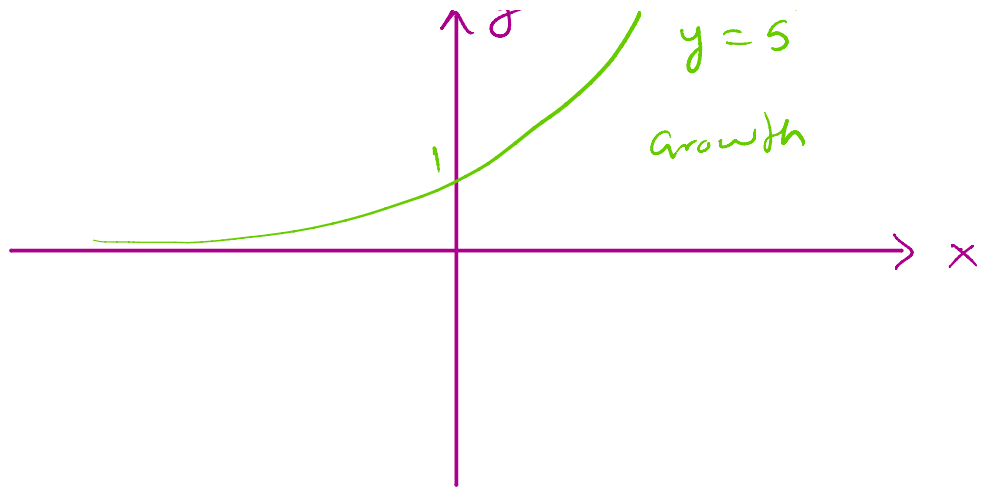
③ $y = 3e^{x}$

x	0	1	2	3	-1
e^{x}	1	e	e^{2}	e^{3}	$e^{-1} = \frac{1}{e}$
$3e^{x}$	3	$3e$	$3e^{2}$	$3e^{3}$	$3e^{-1} = \frac{3}{e}$

$a = e > 1$ growth
$3e^{2}$
$3e$
e^{2}
3
e
1
2
$3e^{x}$ e^{x}

④ $y = \left(\frac{3}{4}\right)^{x}$ $\Rightarrow$ $a = \frac{3}{4} < 1$ $\Rightarrow$ decay

$y = \left(\frac{3}{4}\right)^{x} = \left(\frac{4}{3}\right)^{-x}$
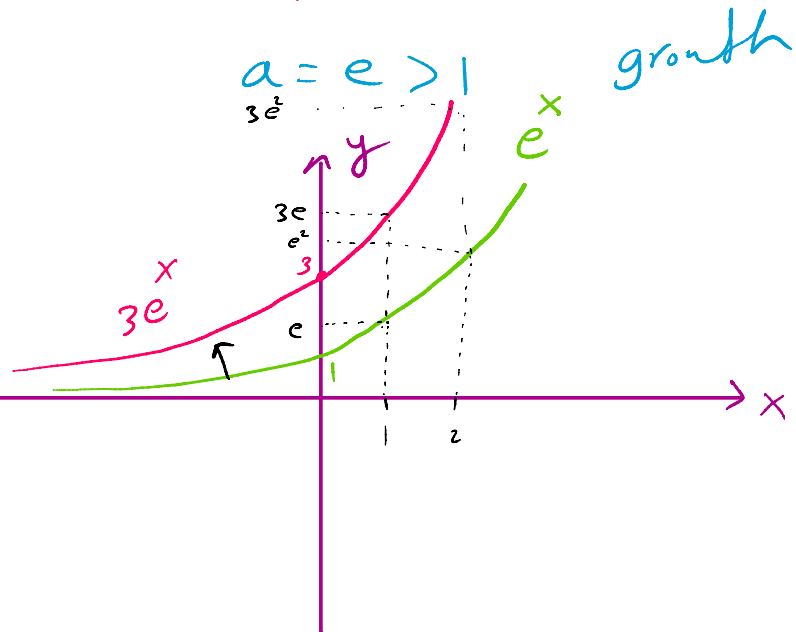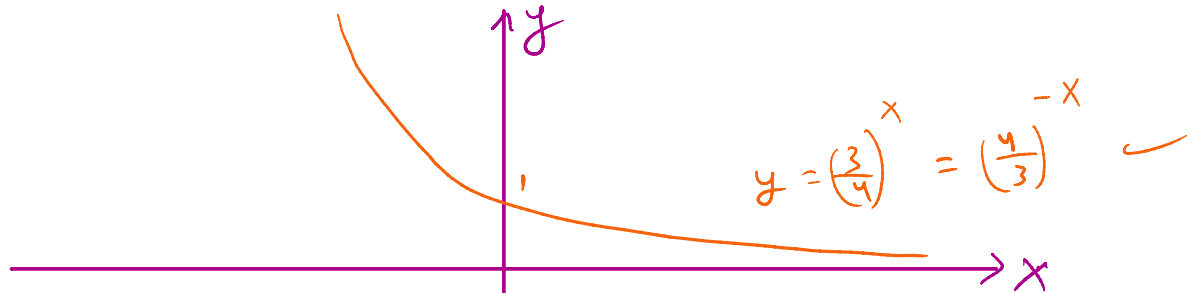
⑤ $y = e^{-x}$

$= \left(e^{-1}\right)^{x} = \left(\frac{1}{e}\right)^{x}$ $\Rightarrow$ $a = \frac{1}{e} < 1$ $\Rightarrow$ decay
$\downarrow_{2.718}$

$$= \left(e^{-1}\right) = \left(\frac{1}{e}\right) \implies a = \frac{1}{e}$$

$\underset{2.718}{}$

$y = e^{-x}$
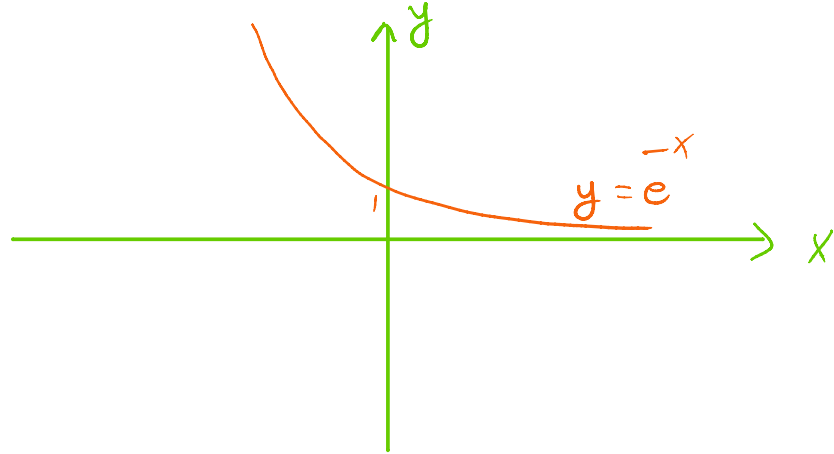
⑥ $y = 2^{x-1}$

$$= 2^{x} \, 2^{-1} = 2^{x} \, \frac{1}{2} = \frac{1}{2} \left(2^{x}\right) \longrightarrow \quad a = 2 > 1 \implies growth$$

$y = \frac{1}{2} \, 2^{x}$

x	0	1	2
2^{x-1}	$2^{-1}=\frac{1}{2}$	$2^{0}=1$	$2^{1}=2$
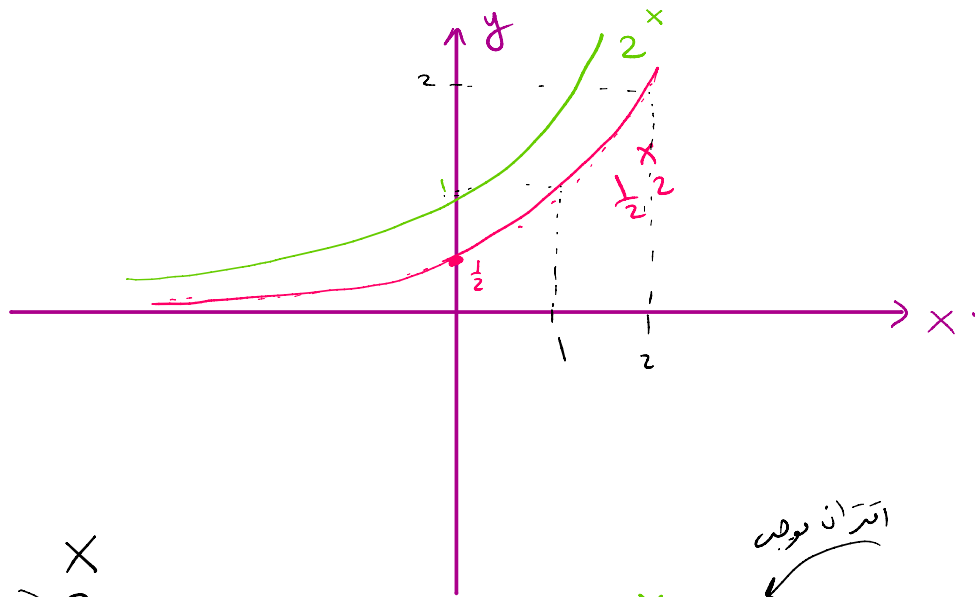
⑦ $y = -e^{x}$

inflection about x-axis

$y = e^{x}$

اقتران موجب

e^{x}

$-e^{x}$

اقتران سالب

$-e^x$ اقتران سالب

⑧ $y = -5^{-x} = -\left(5^{-1}\right)^x = -\left(\frac{1}{5}\right)^x$ $\Rightarrow a = \frac{1}{5} < 1$

$\Downarrow$
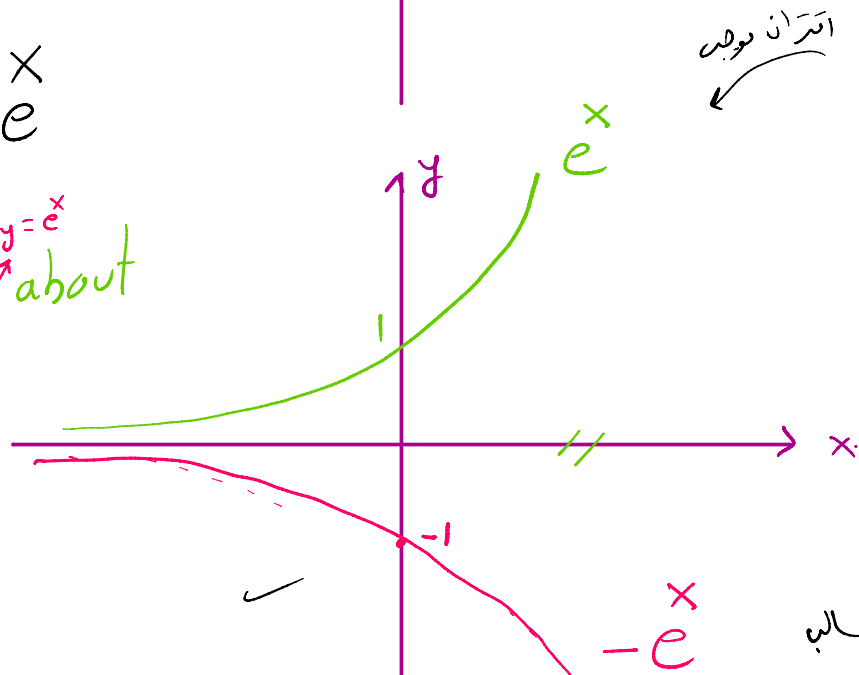
$y = \left(\frac{1}{5}\right)^x$

decay

inflection of $y = \left(\frac{1}{5}\right)^x$
about x-axis

$y = \left(\frac{1}{5}\right)^x$

$y = -\left(\frac{1}{5}\right)^x$

-1