

$$1) f(x) = \frac{1}{1-2x}$$

$$f'(x) = \frac{2}{(1-2x)^2}$$

$$f''(x) = \frac{2 \cdot 4 (1-2x)}{(1-2x)^4} = \frac{8}{(1-2x)^3}$$

$$f'''(x) = \frac{8 \cdot 6 (1-2x)^2}{(1-2x)^6} = \frac{48}{(1-2x)^4}$$

$$\begin{aligned} \sum_{n=0}^{\infty} \frac{f^{(n)}(0) x^n}{n!} &= f(0) + f'(0)x + \frac{f''(0)x^2}{2} + \frac{f'''(0)x^3}{3!} + \dots \\ &= 1 + 2x + \frac{8x^2}{2} + \frac{48x^3}{3!} + \dots \\ &= 1 + 2x + 4x^2 + 8x^3 + \dots \end{aligned}$$

first 3:

$$\sum_{n=0}^{\infty} (2x)^n$$

$$\begin{aligned} r &< 1 \\ 2x &< 1 \\ x &< \frac{1}{2} \end{aligned}$$

$$2) \sum_{n=1}^{\infty} \frac{2^n + n}{n^3 + n^2}$$

$$\lim_{n \rightarrow \infty} \frac{2^n + n}{n^3 + n^2} \cdot \frac{n^2}{1} = 1$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} \text{ conv}$$

$$3) \sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$$

$$\lim_{n \rightarrow \infty} \frac{e^2}{n} = 0 < 1 \text{ conv by Root test}$$

$$4) \frac{1}{1+x} = 1 - x + x^2$$

$$E < 64 \times 10^{-6}$$

$$x^3 < 64 \times 10^{-6}$$

$$|x| < 0.04$$

Hauptkram
9. Stetigkeit

$$5) \sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

$$\lim_{b \rightarrow \infty} \int_2^b \frac{dx}{x \ln x}$$

$$u = \ln x \quad \frac{du}{dx} = \frac{1}{x} \quad \int \frac{du}{u} = \ln u$$

$$x \rightarrow b \rightarrow \ln b$$

$$x = 2 \rightarrow \ln 2$$

$$\lim_{b \rightarrow \infty} [\ln(\ln b) - \ln(\ln 2)]$$

$$= \infty$$

div by Integral test

$$e^{\ln 3} = 3$$

$$\ln 3 + \frac{(\ln 3)^2}{2!} + \frac{(\ln 3)^3}{3!} + \dots =$$

$$e^{\ln 3} = 3$$

$$\sum_{n=0}^{\infty} \frac{(\ln 3)^n}{n!}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

7) $\sqrt{1+x^3} = (1+x^3)^{\frac{1}{2}}$

$$= 1 + \sum_{k=1}^{\infty} \binom{m}{k} x^k \quad (1+x)^m$$

$$= 1 + \frac{1}{2} x^3 + \frac{-1}{8} x^6 + \frac{x^9}{16} \dots$$

8) $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$

$$x = -1 \rightarrow \left\{ (-1)^n \frac{1}{\sqrt{n}} \right\} \rightarrow \text{conv} \quad \text{IC conv} = [-1, 1)$$

$$x = 1 \rightarrow \left\{ \frac{1}{\sqrt{n}} \right\} \rightarrow \text{div}$$

9) $\sum_{n=0}^{\infty} \left(\frac{1}{2^n} + \frac{(-1)^n}{3^n} \right)$

$$\sum_{n=0}^{\infty} \left(\frac{1}{2} \right)^n + \sum_{n=0}^{\infty} \left(-\frac{1}{3} \right)^n = [1, 2] + \left[1, \frac{3}{4} \right] = \frac{4 \times 2}{4 \times 1} + \frac{3}{4} = \frac{11}{4}$$

10) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$

$$|x| < 0.1$$

$$E < \frac{(0.1)^3}{6} e^{0.1}$$

$$E < \frac{e^{0.1}}{6000}$$

$$11) \sum_{n=1}^{\infty} \frac{3}{n^2} - \frac{3}{(n+1)^2} = \left(\frac{3}{1} - \frac{3}{4}\right) + \left(\frac{3}{4} - \frac{3}{9}\right) + \left(\frac{3}{9} - \frac{3}{16}\right) + \dots$$

$$S_n = 3 - \frac{3}{(n+1)^2}$$

$$\lim_{n \rightarrow \infty} S_n = 3$$

$$12) \sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2} x^n$$

$$|x| \lim_{n \rightarrow \infty} \left(\frac{n}{n+1}\right)^n = |x| \lim_{u \rightarrow \infty} \left(1 - \frac{1}{u}\right)^u \left(1 - \frac{1}{u}\right)^{-1}$$

$$u = n+1$$

$$= \frac{|x|}{e} < 1$$

$$-1 < \frac{x}{e} < 1$$

$$-e < x < e$$

$$\begin{array}{c} + \\ -e \quad 0 \quad e \end{array}$$

$$R = e$$

$$13) a_n = \left(3 + \frac{3}{n}\right)^n$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$$

$$\lim_{n \rightarrow \infty} \left(3 + \frac{3}{n}\right)^n = e^3 \quad \text{Conv}$$

$$14) \sum_{n=1}^{\infty} \frac{n \ln n}{2^n}$$

$$\lim_{n \rightarrow \infty} \frac{(n+1) \ln(n+1)}{2^{n+1}} \cdot \frac{2^n}{n \ln n} = \lim_{n \rightarrow \infty} \frac{n+1}{2n} \cdot \frac{\ln(n+1)}{\ln(n)} = \frac{1}{2} \cdot 1 = \frac{1}{2} < 1 \quad \text{Conv}$$

$$15) \sum_{n=2}^{\infty} \frac{-2}{n(n+1)}$$

$$\frac{-2}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$$

$$= -\frac{2}{n} + \frac{2}{n+1}$$

$$\sum_{n=2}^{\infty} \left(-\frac{2}{n} + \frac{2}{n+1}\right) = \left(-1 + \frac{2}{3}\right) + \left(-\frac{2}{3} + \frac{2}{4}\right) + \dots$$

$$\lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \left[-1 + \frac{2}{n+1}\right] = -1$$

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{3^n+1}$$

$$b_n = \frac{(-2)^n}{(3)^n} = \left(-\frac{2}{3}\right)^n$$

$$\lim_{n \rightarrow \infty} \frac{(-2)^n}{3^n+1} \cdot \frac{3^n}{-2^n} = \lim_{n \rightarrow \infty} \frac{3^n}{3^n+1}$$

$$= \lim_{n \rightarrow \infty} \frac{3^n \ln 3}{3^n \ln 3} = 1$$

$$\therefore \sum_{n=1}^{\infty} \frac{(-2)^n}{3^n+1} \text{ CONV ABS}$$

$$\sum_{n=1}^{\infty} \left(-\frac{2}{3}\right)^n \text{ CONV } r = -\frac{2}{3} \in (-1, 1)$$

$$\sum_{n=1}^{\infty} \frac{1}{3^n+1} \quad \frac{1}{3^n+1} < \frac{1}{3^n}$$

$$(17) \sum_{n=1}^{\infty} \frac{e^n}{e^n+1}$$

$$\lim_{n \rightarrow \infty} \frac{e^n}{e^n} = 1 \neq 0 \text{ div by } n^{\text{th}} \text{ term test}$$

$$(18) f(x) = \ln(1+x^2)$$

$$f(x) = \ln x = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n}}{n}$$

$$(19) \sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$$

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{n^2+1} \cdot x^{\frac{3}{2}} = 1$$

CONV LCT

$$\sum_{n=1}^{\infty} \frac{1}{x^{\frac{3}{2}}} \text{ CONV}$$

$$(20) \sum_{n=1}^{\infty} (-1)^n \frac{n^2+1}{n^2+5}$$

$$\lim_{n \rightarrow \infty} b_n = \lim_{n \rightarrow \infty} \frac{n^2+1}{n^2+5} = 1 \neq 0 \text{ div by } n^{\text{th}} \text{ term test}$$

Question 2 :

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1+n}{n^2}$$

$$\lim_{n \rightarrow \infty} V_n = \lim_{n \rightarrow \infty} \frac{1+n}{n^2} = 0 \text{ CCHV cond}$$

$$\sum_{n=1}^{\infty} \frac{1+n}{n^2}$$

$$\lim_{n \rightarrow \infty} \frac{1+n}{n^2} \cdot \frac{n}{1} = 1$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \text{ div}$$

Question 3 :

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}$$

$$f(x) = \frac{1}{(1+x)^2}$$

Question 4: $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x+2)^n}{n 2^n}$

$$\lim_{n \rightarrow \infty} \frac{(x+2)^{n+1}}{(n+1) 2^{n+1}} \cdot \frac{n 2^n}{(x+2)^n} = \frac{|x+2|}{2} \lim_{n \rightarrow \infty} \frac{n}{n+1} = \frac{|x+2|}{2} < 1$$

$$x = -4 \rightarrow \sum_{n=1}^{\infty} (-1)^{n+1} \frac{-2^n}{n 2^n}$$

$$-1 < \frac{x+2}{2} < 1$$

$$-2 < x+2 < 2$$

$$-4 < x < 0 \quad \text{Conv Abs}$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^n (-1) - 2^n}{n 2^n} = \sum_{n=1}^{\infty} (-1)^n \frac{1}{n} \rightarrow \sum_{n=1}^{\infty} -\frac{1}{n} \rightarrow \text{div}$$

$$x = 0 \rightarrow \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} \rightarrow \text{Conv}$$



$$\text{IC Conv} \rightarrow -4 < x \leq 0$$

$$\text{Conv Cond} \rightarrow x = 0$$

$$\text{Radius Conv} \rightarrow \frac{0+4}{2} = 2$$

$$\text{Center} = \frac{0-4}{2} = -2$$