

Q4 $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n} = \sum_{n=1}^{\infty} \frac{(3(x-\frac{2}{3}))^n}{n} = \sum_{n=1}^{\infty} \frac{3^n (x-\frac{2}{3})^n}{n} \Rightarrow a = \frac{2}{3}$

نقطة داخل المجال التقارب المطلق

Apply RT $\Rightarrow \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(3x-2)^{n+1}}{(3x-2)^n} \cdot \frac{n}{n+1} \right| = |3x-2| \lim_{n \rightarrow \infty} \frac{n}{n+1} = |3x-2| \cdot 1 = |3x-2|$
 $|3x-2| < 1 \Rightarrow -1 < 3x-2 < 1 \Rightarrow 1 < 3x < 3 \Rightarrow \frac{1}{3} < x < 1$

IC = $(\frac{1}{3}, 1) \Rightarrow$ conv. Abs

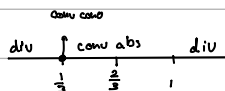
When $x = \frac{1}{3} \Rightarrow \sum_{n=1}^{\infty} \frac{(1-2)^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \Rightarrow$ Alternating p-series Test with $p=1 \Rightarrow$ conv. condi

When $x = 1 \Rightarrow \sum_{n=1}^{\infty} \frac{1^n}{n} = \sum_{n=1}^{\infty} \frac{1}{n} \Rightarrow$ div by Alternating p-test, $p=1$

$R = \frac{1}{3}$

The series conv. Abs on $(\frac{1}{3}, 1)$

" " conv. cond when $x = \frac{1}{3}$



Q12 $\sum_{n=0}^{\infty} \frac{3^n x^n}{n!} \Rightarrow a = 0$

Apply RT $\Rightarrow \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{3^{n+1} x^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n x^n} \right| = |3x| \lim_{n \rightarrow \infty} \frac{1}{n+1} = |3x| \cdot 0 = 0 \Rightarrow 0 < 1 \therefore$ conv. abs $\forall x$ so $R = \infty$, no conv. cond

Q23 $\sum_{n=1}^{\infty} (1 + \frac{1}{n})^n x^n \Rightarrow a = 0$

Apply RT $\Rightarrow \lim_{n \rightarrow \infty} \left| \frac{(1 + \frac{1}{n+1})^{n+1} x^{n+1}}{(1 + \frac{1}{n})^n x^n} \right| = |x| \lim_{n \rightarrow \infty} \frac{(1 + \frac{1}{n+1})^{n+1}}{(1 + \frac{1}{n})^n} = |x| \lim_{n \rightarrow \infty} \frac{(1 + \frac{1}{n+1})^{n+1}}{(1 + \frac{1}{n})^n} = |x| \frac{e}{e} = |x| \Rightarrow |x| < 1 \Rightarrow -1 < x < 1$

(1+1/n)^n is a constant

When $x = -1 \Rightarrow \sum_{n=1}^{\infty} (1 + \frac{1}{n})^n (-1)^n \Rightarrow$ div by n^{th} term test since $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n = e \neq 0$

When $x = 1 \Rightarrow \sum_{n=1}^{\infty} (1 + \frac{1}{n})^n \Rightarrow$ div by n^{th} term test since $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n = e \neq 0$

The series conv. Abs on $(-1, 1)$, $R=1$

There are no values for which the series conv. cond

Q24 $\sum_{n=2}^{\infty} \frac{x^n}{n(\ln n)^2} \Rightarrow a = 0$

Apply RT $\Rightarrow \lim_{n \rightarrow \infty} \left| \frac{x^{n+1}}{(n+1)(\ln(n+1))^2} \cdot \frac{n(\ln n)^2}{x^n} \right| = |x| \lim_{n \rightarrow \infty} \frac{n(\ln n)^2}{(n+1)(\ln(n+1))^2} = |x| \lim_{n \rightarrow \infty} \frac{n}{n+1} \left(\lim_{n \rightarrow \infty} \frac{\ln n}{\ln(n+1)} \right)^2 = |x| \cdot 1 \cdot 1 = |x| \Rightarrow |x| < 1 \Rightarrow -1 < x < 1$

When $x = -1 \Rightarrow \sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^2} \Rightarrow$ Alternating $\Rightarrow \lim_{n \rightarrow \infty} \frac{1}{n(\ln n)^2} = 0$ by IT $\Rightarrow$ conv. Abs. $\therefore$ for $x = -1$ the series conv. by IT

When $x = 1 \Rightarrow \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} \Rightarrow$ conv. by IT $\Rightarrow$ conv. Abs $\therefore$ for $x = 1$ the series conv. by IT

$R = 1$

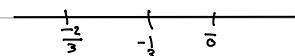
The series conv. Abs on $[-1, 1]$

There are no values for which the series conv. cond

Q32 $\sum_{n=1}^{\infty} \frac{(2x+1)^{n+1}}{2n+2} \Rightarrow a = -\frac{1}{2}$

Apply RT $\Rightarrow \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(2x+1)^{n+2}}{(2n+4)} \cdot \frac{(2n+2)}{(2x+1)^{n+1}} \right| = |2x+1| \lim_{n \rightarrow \infty} \frac{n+1}{n+2} = |2x+1| \Rightarrow |2x+1| < 1 \Rightarrow -1 < 2x+1 < 1 \Rightarrow -2 < 2x < 0 \Rightarrow -1 < x < 0$

When $x = -\frac{1}{2} \Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n+2} \Rightarrow$ conv. conv (lim $u_n = 0 \therefore$ conv)



When $x = 0 \Rightarrow \sum_{n=1}^{\infty} \frac{1}{2n+2} \Rightarrow \lim_{n \rightarrow \infty} \frac{1}{2n+2} = 0 \Rightarrow$ div by IT

$R = \frac{1}{2}$

The series conv. Abs on $(-\frac{1}{2}, 0)$

" " conv. cond when $x = -\frac{1}{2}$

Q40 $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2} x^n \Rightarrow a = 0$ R c- collig

Apply Root Test $\Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\left(\frac{n}{n+1}\right)^{n^2} x^n} = \lim_{n \rightarrow \infty} \left(\frac{n}{n+1}\right)^n |x| = |x| \lim_{n \rightarrow \infty} \left(1 - \frac{1}{n+1}\right)^n \Rightarrow \text{let } u = n+1 \Rightarrow |x| \lim_{u \rightarrow \infty} \left(1 - \frac{1}{u}\right)^{u-1} = |x| \lim_{u \rightarrow \infty} \frac{\left(1 - \frac{1}{u}\right)^u}{\left(1 - \frac{1}{u}\right)} = \frac{|x| \cdot e^{-1}}{1-0} = |x| e^{-1} \Rightarrow e^{-1} |x| < 1 \Rightarrow -1 < \frac{x}{e} < 1 \Rightarrow -e < x < e$
 $R = e$

Q46 $\sum_{n=0}^{\infty} (\ln x)^n \Rightarrow \lim_{n \rightarrow \infty} \left| \frac{(\ln x)^{n+1}}{(\ln x)^n} \right| = \lim_{n \rightarrow \infty} |\ln x| = |\ln x| < 1 \Rightarrow -1 < \ln x < 1 \Rightarrow e^{-1} < x < e$

when $x = e^{-1} \Rightarrow (\ln e^{-1})^n = (-1)^n \rightarrow \text{div by } n^{\text{th}} \text{ term test}$

when $x = e \Rightarrow (\ln e)^n = 1^n \rightarrow \text{div by } n^{\text{th}} \text{ term test}$

sum $\Rightarrow r = \ln x \Rightarrow \text{sum} = \frac{1}{1 - \ln x}$ where $e^{-1} < x < e$

Q49 $1 - \frac{1}{2}(x-3) + \frac{1}{4}(x-3)^2 + \dots + (-\frac{1}{2})^n (x-3)^n + \dots$

$r = -\frac{1}{2}(x-3) \Rightarrow |r| < 1 \Rightarrow \left| -\frac{1}{2}(x-3) \right| < 1 \Rightarrow -1 < -\frac{1}{2}(x-3) < 1 \Rightarrow -2 < x-3 < 2 \Rightarrow 1 < x < 5$, The sum when $1 < x < 5$ is $\frac{1}{1 + \frac{1}{2}(x-3)} = \frac{2}{x-1}$

$f'(x) = -\frac{1}{2} + \frac{1}{2}(x-3) + \dots + (-\frac{1}{2})^n n(x-3)^{n-1} + \dots$ is convergent when $1 < x < 5$ and diverges when $x=1$ or 5 . The sum for $f'(x)$ is $\frac{-2}{(x-1)^2}$ the derivative of $\frac{2}{x-1}$