$\sum_{n=0}^{\infty} a_n(x-a)^n$: الصيغة العامة

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Q4 $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n} = \sum_{n=1}^{\infty} \frac{(3(x-\frac{2}{3}))^n}{n} = \sum_{n=1}^{\infty} \frac{3^n (x-\frac{2}{3})^n}{n} \Rightarrow a = \frac{2}{3}$

مافي داعي إرجع القيمة المطلقة لأنها $\lim$ لكل n

Apply RT $\Rightarrow \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{\cancel{(3x-2)^{n+1}}}{n+1} \cdot \frac{n}{\cancel{(3x-2)^n}} \right| = |3x-2| \lim_{n\to\infty} \boxed{\frac{n}{n+1}} = |3x-2| \cdot 1 = |3x-2|$ (1) دينادينها مع $|3x-2| < 1 \Rightarrow -1 < 3x-2 < 1 \Rightarrow 1 < 3x < 3 \Rightarrow \frac{1}{3} < x < 1$

IC $= (\frac{1}{3}, 1) \to$ conv Abs

when $x = \frac{1}{3} \Rightarrow \sum \frac{(1-2)^n}{n} = \sum \frac{(-1)^n}{n} \Rightarrow$ Alternating p-series Test with $P=1 \Rightarrow$ conv. condi

when $x = 1 \Rightarrow \sum \frac{1^n}{n} = \sum \frac{1}{n} \Rightarrow$ div by Alternating P-test, $P=1$

div | conv abs | div ; conv cond at $\frac{1}{3}$; $\frac{1}{3}$, $\frac{2}{3}$, 1

$R = \frac{1}{3}$

The series conv Abs on $(\frac{1}{3}, 1)$

" " conv. cond when $x = \frac{1}{3}$

Q12 $\sum_{n=0}^{\infty} \frac{3^n x^n}{n!} \Rightarrow a = 0$

Apply RT $\Rightarrow \lim_{n\to\infty} \left| \frac{3^{n+1} x^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n x^n} \right| = |3x| \lim_{n\to\infty} \frac{1}{n+1} = |3x| \cdot 0 = 0 \Rightarrow 0 < 1$ $\therefore$ conv abs $\forall x$ so $\to R = \infty$, no conv. cond

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Q23 $\sum_{n=1}^{\infty} (1+\frac{1}{n})^n x^n \Rightarrow a = 0$

Apply RT $\Rightarrow \lim_{n\to\infty} \left| \frac{(1+\frac{1}{n+1})^{n+1} \cancel{x^{n+1}}}{(1+\frac{1}{n})^n \cancel{x^n}} \right| = |x| \lim_{n\to\infty} \frac{(1+\frac{1}{n+1})^{n+1}}{(1+\frac{1}{n})^n} = |x| \frac{\lim_{t\to\infty} (1+\frac{1}{t})^t}{\lim_{n\to\infty}(1+\frac{1}{n})^n} = |x| \frac{e}{e} = |x| \to |x| < 1 \Rightarrow -1 < x < 1$

let $n+1 = t$

لما بدي أشوف إذا con أو div بطبق n^{th} على $(1+\frac{1}{n})^n$

when $x = -1 \Rightarrow \sum_{n=1}^{\infty} (1+\frac{1}{n})^n (-1)^n \Rightarrow$ div by n^{th} term test since $\lim_{n\to\infty} (1+\frac{1}{n})^n = e \neq 0$

when $x = 1 \Rightarrow \sum_{n=1}^{\infty} (1+\frac{1}{n})^n \Rightarrow$ div by n^{th} term test since $\lim_{n\to\infty} (1+\frac{1}{n})^n = e \neq 0$

. The series conv Abs on $(-1, 1)$, $R = 1$

. There are no values for which the series conv. cond

Q29 $\sum_{n=2}^{\infty} \frac{x^n}{n(\ln n)^2} \Rightarrow a = 0$

$\frac{\infty}{\infty} \to \frac{\frac{1}{n}}{\frac{1}{n+1}} = \frac{n+1}{n} = 1$

Apply RT $\Rightarrow \lim_{n\to\infty} \left| \frac{x^{\cancel{n+1}}}{(n+1)(\ln(n+1))^2} \cdot \frac{n(\ln n)^2}{\cancel{x^n}} \right| = |x| \lim_{n\to\infty} \frac{n(\ln n)^2}{(n+1)(\ln(n+1))^2} = |x| \left(\lim_{n\to\infty} \frac{n}{n+1}\right)\left(\lim_{n\to\infty} \frac{\ln n}{\ln (n+1)}\right)^2 = |x| \cdot 1 \cdot 1 = |x| \Rightarrow |x| < 1 \to -1 < x < 1$

when $x = -1 \Rightarrow \sum_{n=2} \frac{(-1)^n}{n(\ln n)^2}$ (Altrenating) $\Rightarrow \lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{1}{n(\ln n)^2}$ بجرب الـ n^{th} والـ I.T $\int \frac{1}{n(\ln n)^2}$ let $u = \ln n \to du = \frac{1}{n} dn \to dn = n\,du \Rightarrow \int \frac{1}{u^2} = \int u^{-2} = \frac{u^{-1}}{-1} \Big| = \frac{-1}{u} \Big| = \lim_{b\to\infty} \frac{-1}{\ln n}\Big|_2^b = \lim_{b\to\infty} \left[\frac{-1}{\ln b} - \frac{-1}{\ln 2}\right] = 0 - \frac{1}{\ln 2} = \frac{-1}{\ln 2}$ conv by IT

لكن احنا اوجدنا الـ lim لـ $|a_n|$ $\therefore$ Conv. Abs.

When $x = 1 \Rightarrow \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} \Rightarrow$ conv by IT $\Rightarrow$ conv. Abs لو اخذنا الـ $\lim |a_n|$ نفس الجواب $\therefore$ بي في

. $R = 1$

. The series conv Abs on $[-1, 1]$

. There are no values for which the series conv. cond

Q32 $\sum_{n=1}^{\infty} \frac{(3x+1)^{n+1}}{2n+2} \Rightarrow a = -\frac{1}{3}$

Apply RT $\Rightarrow \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{(3x+1)^{n+2}}{2(n+1)+2} \cdot \frac{2n+2}{(x+1)^{n+1}} \right| = \lim_{n\to\infty} \left| \frac{\cancel{(3x+1)^{n}}(3x+1)^2}{\cancel{2}(n+2)} \cdot \frac{\cancel{2}(n+1)}{\cancel{(3x+1)^n}(3x+1)} \right| = |3x+1| \lim_{n\to\infty} \frac{n+1}{n+2} = |3x+1| \Rightarrow |3x+1| < 1 \Rightarrow -1 < 3x+1 < 1 \Rightarrow -2 < 3x < 0 \Rightarrow -\frac{2}{3} < x < 0$

when $x = -\frac{2}{3} \Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n+2}$ conv. conv ($\lim u_n = 0$ $\therefore$ conv)

$\frac{-2}{3}$, $-\frac{1}{3}$, 0

when $x = 0 \Rightarrow \sum_{n=1}^{\infty} \frac{1}{2n+1} \Rightarrow f(x) = \frac{1}{2x+1} \to \int_1^{\infty} \frac{1}{2x+1} dx \Rightarrow \lim_{a\to\infty} \int_1^a \frac{1}{2x+1} dx = \lim_{a\to\infty} \left(\frac{1}{2} \ln|2x+1|\right)\Big|_1^a = \lim_{a\to\infty} \left(\frac{1}{2} \ln|2a+1| - \frac{1}{2} \ln 3\right) = +\infty$ $\therefore$ div by IT

. $R = \frac{1}{3}$

. The series conv Abs on $(-\frac{2}{3}, 0)$

. " " conv. cond when $x = -\frac{2}{3}$

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Q40 $\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{n^2}x^n \Rightarrow a=0$ مطلوب R

Apply Root Test $\Rightarrow \lim_{n\to\infty}\sqrt[n]{a_n} = \lim_{n\to\infty}\sqrt[n]{\left(\frac{n}{n+1}\right)^{n^2}x^n} = \lim_{n\to\infty}\left(\frac{n+1-1}{n+1}\right)^n|x| = |x|\lim_{n\to\infty}\left(1-\frac{1}{n+1}\right)^n \Rightarrow$ let $u=n+1 \Rightarrow |x|\lim_{u\to\infty}\left(1-\frac{1}{u}\right)^{u-1} = |x|\lim_{u\to\infty}\frac{\left(1-\frac{1}{u}\right)^u}{\left(1-\frac{1}{u}\right)} = \frac{|x|\cdot e^{-1}}{1-0} = |x|e^{-1} \Rightarrow e^{-1}|x|<1 \Rightarrow -1<\frac{x}{e}<1 \Rightarrow -e<x<e$

$R=e$

Q46 $\sum_{n=0}^{\infty}(\ln x)^n \Rightarrow \lim_{n\to\infty}\left|\frac{(\ln x)^{n+1}}{(\ln x)^n}\right| = \lim_{n\to\infty}|\ln x| = |\ln x|<1 \Rightarrow -1<\ln x<1 \Rightarrow e^{-1}<x<e$

when $x=e^{-1} \Rightarrow \sum(\ln e^{-1})^n = (-1)^n \to$ div by n^{th} term test

when $x=e \Rightarrow \sum(\ln e)^n = 1^n \to$ div by n^{th} term test

sum $\Rightarrow r=\ln x \Rightarrow$ sum $=\frac{1}{1-\ln x}$ where $e^{-1}<x<e$

Q49 $1-\frac{1}{2}(x-3)+\frac{1}{4}(x-3)^2+\dots+\left(-\frac{1}{2}\right)^n(x-3)^n+\dots$

$r=-\frac{1}{2}(x-3) \Rightarrow |r|<1 = \left|\frac{1}{2}(x-3)\right|<1 \Rightarrow -1<\frac{1}{2}(x-3)<1 \Rightarrow -2<x-3<2 \Rightarrow 1<x<5$, The sum when $1<x<5 = \frac{1}{1+\frac{1}{2}(x-3)} = \frac{2}{x-1}$

$f'(x) = -\frac{1}{2}+\frac{1}{2}(x-3)+\dots+\left(-\frac{1}{2}\right)^n n(x-3)^{n-1}+\dots$ is convergent when $1<x<5$ and diverges when $x=1$ or 5. The sum for $f'(x)$ is $\frac{-2}{(x-1)^2}$, The derivative of $\frac{2}{x-1}$