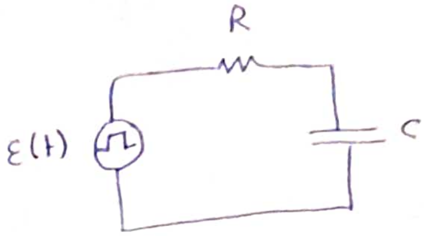
Exp 6 : Capacitors & Inductors
مواسع ملف

i. RC circuits



→ charging a capacitor :

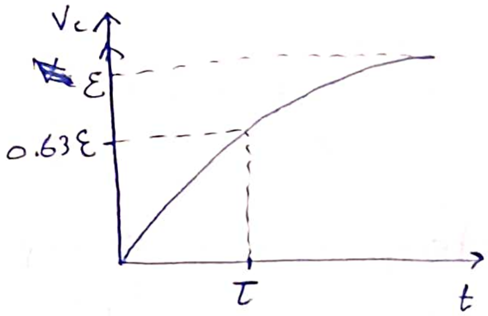
$$Q(t) = C\varepsilon(1 - e^{-t/RC})$$

(*) The Voltage across the capacitor :-

$$V_c = \frac{Q}{C} = \frac{C\varepsilon(1 - e^{-t/RC})}{C} = \varepsilon(1 - e^{-t/RC})$$

- at $t=0 \Rightarrow V_c = \varepsilon(1 - \underbrace{e^{0}}_{1}) = 0$
- at $t=\infty \Rightarrow V_c = \varepsilon(1 - \cancel{e^{-\infty}}^{0}) = \varepsilon$
- at $t = RC$

$$\Rightarrow V_c = \varepsilon(1 - e^{-1})$$
$$= \varepsilon(1 - 0.37)$$
$$= 0.63\,\varepsilon$$



$\tau = RC$ [time const.]

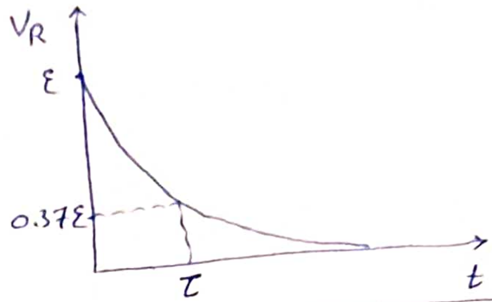
τ : indication of how fast the charging is

(*) The Voltage across the resistor :-

$$V_R = IR \quad , \text{ but } I = \frac{dQ}{dt} = \cancel{C}\varepsilon\left(\frac{1}{R\cancel{C}} e^{-t/RC}\right)$$
$$= \frac{\varepsilon}{R} e^{-t/RC}$$

$$V_R = \frac{\varepsilon}{\cancel{R}} e^{-t/RC} (\cancel{R}) = \varepsilon e^{-t/RC}$$

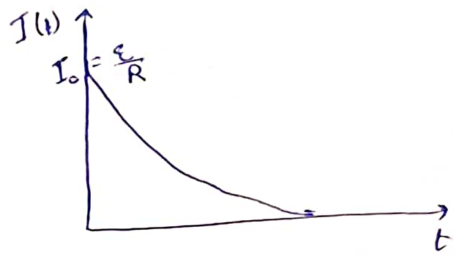
- at $t = 0 \Rightarrow V_R = \varepsilon e^{0} = \varepsilon$
- at $t = \infty \Rightarrow V_R = \varepsilon e^{-\infty} = 0$



- at $t = RC$

$\Rightarrow V_R = \varepsilon e^{-1} = 0.37\varepsilon$

$\boxed{\tau = RC \ : \text{time const.}}$

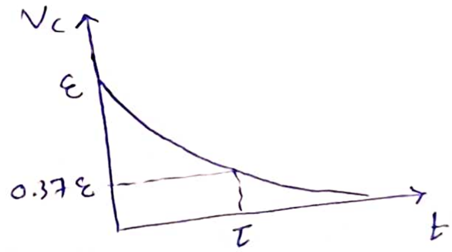


iii) Discharging a capacitor:

$Q(t) = C\varepsilon e^{-t/RC}$

- The voltage across the capacitor:-

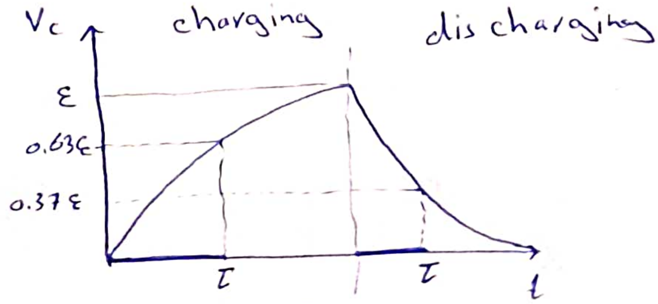
$V_c = \frac{Q}{C} = \varepsilon e^{-t/RC}$



$t = 0 \rightarrow V_c = \varepsilon$

$t = \infty \rightarrow V_c = 0$

at $t = RC \rightarrow V_c = 0.37\varepsilon$



$\boxed{\tau = RC}$

- The Voltage across Resistor :-

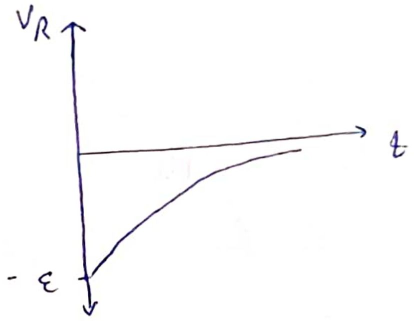
$V_R = IR$, $\quad I = \frac{dQ}{dt} = \frac{d}{dt}(C\mathcal{E} e^{-t/RC})$

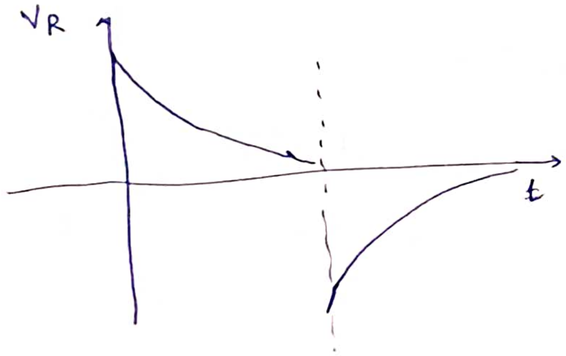
$= -\frac{\mathcal{E}}{R} e^{-t/RC}$

$V_R = -\mathcal{E} e^{-t/RC}$

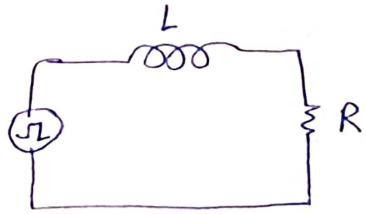
$t = 0 \rightarrow V_R = -\mathcal{E}$

$t = \infty \rightarrow V_R = 0$





ii) RL Circuits :



$I = \frac{\mathcal{E}}{R}(1 - e^{-\frac{Rt}{L}})$

- The Voltage across the inductor :

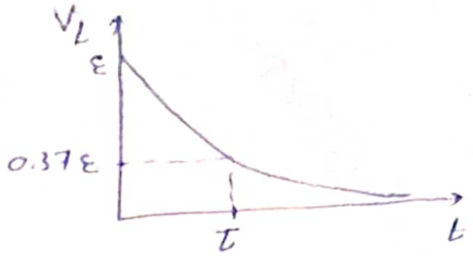
$V_L = L\frac{dI}{dt} = \cancel{L}\left(\frac{\mathcal{E}}{\cancel{R}}\left(\frac{\cancel{R}}{\cancel{L}} e^{-\frac{Rt}{L}}\right)\right)$

$= \mathcal{E} e^{-\frac{Rt}{L}}$

- at $t = 0 \rightarrow V_L = \mathcal{E}$
- at $t = \infty \rightarrow V_L = 0$

→ at $t = \frac{L}{R}$ → $V_L = \mathcal{E} e^{-1}$

$= 0.37\mathcal{E}$



$\tau = \frac{L}{R}$: time const.

for RL-circuit

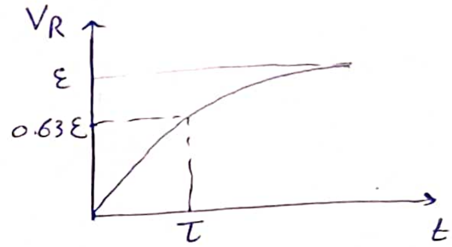
* Voltage across Resistor :

$V_R = IR = \mathcal{E}(1 - e^{-\frac{Rt}{L}})$

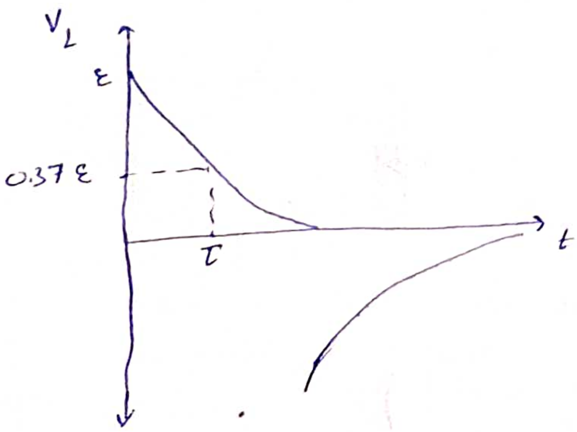
* at $t = 0$ → $V_R = 0$
* at $t = \infty$ → $V_R = \mathcal{E}$

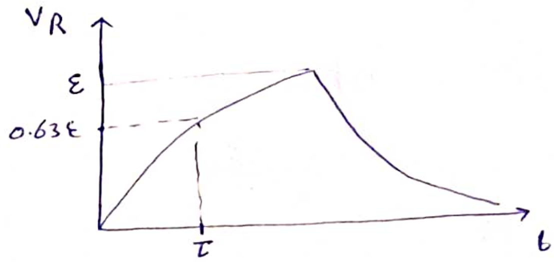
→ at $t = \frac{L}{R}$ ⇒ $V_R = \mathcal{E}(1 - e^{-1})$

$= 0.63\,\mathcal{E}$

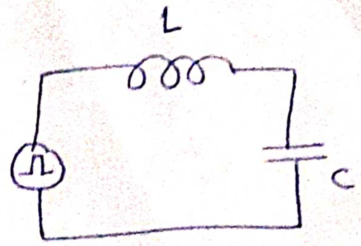


$\tau = \frac{L}{R}$





iii) LC circuit :-



LC-Oscillator

$Q(t) = Q_0 \cos(\omega t + \varphi)$

$V_c = \frac{Q}{c} = \frac{Q_0}{c} \cos(\omega t + \varphi)$

$= V_0 \cos(\omega t + \varphi)$ [Simple Harmonic Oscillator]

$\omega = \frac{1}{\sqrt{LC}}$: natural angular frequency

$\omega = 2\pi f$

$\Rightarrow f = \frac{1}{2\pi\sqrt{LC}}$