ـــ اعه بجار — Principles of Physics (10th eddtion) (141)

Discussion ⇒ Chapter 7: kinetic Energy and work

Problems: 5, 9, 17, 35, 41, 46

Problem 5: A father racing his son has half the kinetic energy of the son, who has half the mass of father. The father speeds up by 1.0 m/s and then has the same kinetic energy as the son. what are the original speeds of (a) the father and b) the son.

sol: we denote the mass of the father as m and his intial speed v_i.

fat: father , S = son

معطى $\quad K_{f_{fat}} = \frac{1}{2} K_{son}$

K_{f_i}: kinetic energy of the father at speed v_i
سرعة الوالد الابتدائية

$$m_s = \frac{1}{2} m_f$$

$$\cancel{K_{ff} = K_{son}}$$

$$(K_{fat})_f = K_{son}$$

$(K_{fat})_f$: kinetic energy of the father at speed $v_f = v_i + 1\ m/s$
سرعة الوالد النهائية

a) $(K_{fath})_i = \frac{1}{2} K_{son}$ —— (1)

$(K_{fath})_f = K_{son}$ —— (2)

عوض K_{son} من (2) في (1)

$$(K_{fat})_i = \frac{1}{2} (K_{fath})_f$$

$$\cancel{\frac{1}{2}} \cancel{m}_{fat} v_i^2 = \cancel{\frac{1}{2}} \left(\frac{1}{2} \cancel{m}_{fath} v_f^2\right)$$

$$v_i^2 = \frac{1}{2} v_f^2$$

سامر بجار

$$v_i^2 = \frac{1}{2}(v_i+1)^2$$

$$v_i^2 = \frac{1}{2}(v_i^2+2v_i+1)$$

$$v_i^2 = \frac{1}{2}v_i^2 + v_i + \frac{1}{2}$$

$$\frac{1}{2}v_i^2 - v_i - \frac{1}{2} = 0$$ معادلة تربيعية

$a = \frac{1}{2}$ $b = -1$ $c = \frac{-1}{2}$

$$v_i = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{1 \pm \sqrt{(1)^2-(4 \times \frac{1}{2} * \frac{-1}{2})}}{2(\frac{1}{2})}$$

$$= \frac{1 \pm \sqrt{2}}{1}$$

$$v_i = \frac{1 + \sqrt{2}}{1} = 2.4\ m/s$$

or $$v_i = \frac{1-\sqrt{2}}{1} = -0.4\ m/s$$ غير مقبول

b) $(K_{fath})_i = \frac{1}{2}K_{son}$ من معادلة ①

$$\cancel{\tfrac{1}{2} m_{fath} v_{fath\,i}^2 = \tfrac{1}{2}(\tfrac{1}{2} m_{son} v_{son\,i}^2)}$$

$$\cancel{m_{fath}(v_{fat})_i^2 = \tfrac{1}{2} m_{fath}(v_{son})^2}$$

$$\cancel{2(v_{fath})_i^2 = (v_{son})_i^2}$$

/مساواة

$$(k_{fath})_i = \frac{1}{2} k_{son}$$

$$\cancel{\frac{1}{2}} m_{fath} v_{fath\,i}^2 = \cancel{\frac{1}{2}} \left(\frac{1}{2} m_{son} v_{son\,i}^2 \right)$$

$$m_{fath} v_{fath\,i}^2 = \frac{1}{2} m_{son} v_{son\,i}^2 \quad , \quad m_{son} = \frac{1}{2} m_{fath}$$

$$\cancel{m_{fath}} v_{(fath)\,i}^2 = \frac{1}{2} \left(\frac{1}{2} \cancel{m_{fath}} \right) v_{(son)\,i}$$

$$4 v_{(fath)\,i}^2 = v_{(son)\,i}^2$$

$$2 v_{fath)\,i} = v_{(son)\,i}$$

$$2 \times 2.4 = v_{(son)\,i}$$

$$4.8 \text{ m/s} = v_{(son)\,i}$$

Problem 9: The only force acting on a 2.0 kg canister that is moving in an xy plane has magnitude of 5.0 N. The canister initially has a velocity of 4.0 m/s in the positive x direction and some time later has a velocity of 6.0 m/s in the positive y direction. How much work is done on the cainster by the 5.0 N force during this time

Sol : By the work kinetic energy theorem

$$W = \Delta K$$

$$= k_f - k_i$$

$$= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$= \frac{1}{2} m (v_f^2 - v_i^2)$$

$$= \frac{1}{\cancel{2}} (\cancel{2}) (6^2 - 4^2)$$

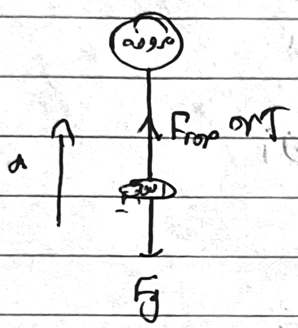
$$W = (36 - 16) \Rightarrow W = 20 \text{ J}$$

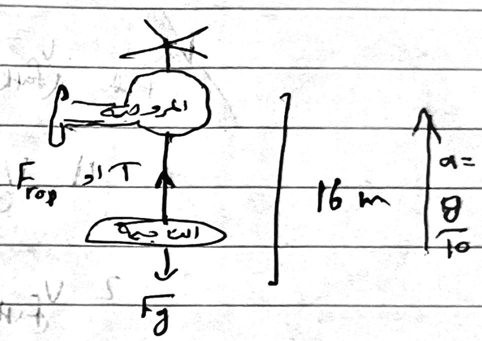
ملاحظة أن اتجاه السرعة لا يؤثر على W

مروحية عسكرية ناجياً من فيضانات

Problem 17: A military helicopter lifts a 75 kg flood survivor 16 m vertically from the river by a rope. If the acceleration of the survivors is g/10, how much work is done on the survivor by (a) the force from the helicopter and (b) the gravitational force on her? Just befor she reaches the helicopter, what are her (c) kinetic energy and (d) speed?

Sol:





a) بما أن التسارع لأعلى فيعتبر اتجاه موجب وبناءً على قانون نيوتن الثاني

According to Newton's second law

$$F_{rop} - F_g = ma$$

$$F_{rop} - mg = ma \Rightarrow F_{rop} = m(g+a)$$

$$\Rightarrow F_{rop} = m\left(g + \frac{g}{10}\right)$$

$$F_{rop} = \frac{11}{10} mg$$

Since the force of helicopter (force of the rope) and displacement $\vec{d}$ are in the same direction, the work done by force is

$$W_{F_{rop}} = \vec{F}_{rop} \cdot \vec{d} = F_{rop} d \cos zero = F_{rop} d \Rightarrow \frac{11 mg}{10} d$$

$$W_{F_{rop}} = \frac{11 \times 75 \times 9.8 \times 16}{10} = 12936 \text{ J} = 1.3 \times 10^4 \text{ J}$$

$= 13 \text{ kJ}$

ساعدني

b) The work done by the gravity is

$W_g = F_g d \cos 180 = -F_g d = -mgd$

$W_g = -75 \times 9.8 \times 16 = -11760 J = -1.2 \times 10^4 J = -12 kJ$

c) The total work done

$W_{net} = W_{F_{rop}} + W_{F_g} = \cancel{13 - 12 = 1 kJ}$

$= 12936 - 11760$

$= 1176 J$

$\simeq 1.2 kJ \rightarrow$ تقريب

since the ~~he~~ helicopter started from rest, the work-kinetic energy theorem tells us that this is her final kinetic energy.

$\cancel{\Rightarrow} k_f - k_i = W_{net} \Rightarrow k_f - \text{zero} = W_{net}$

$\Rightarrow k_f = W_{net} = 1.2 kJ$

d) $k_f = \frac{1}{2} m v_f^2$

$\cancel{1.2 \times 10^3}$

$1176 = \frac{1}{2} \times 75 \, v_f^2$

$\frac{2 \times 1176}{75} = v_f^2$

$31.36 = v_f^2$

$\boxed{5.6 m/s = v_f}$

Problem 35: The force on a particle is directed along an x axis and given by $F = F_0(x/x_0 - 1)$. Find the work done by the force in moving the particle from $x=0$ to $x=2x_0$ by (a) plotting $F(x)$ and measuring the work from the graph and (b) integrating $F(x)$

sol : a) plot $F = F_0\left(\frac{x}{x_0} - 1\right)$

ط أصفار، الأصفار (x - intercepts)

$$0 = F_0\left(\frac{x}{x_0} - 1\right)$$

$$0 = \frac{x}{x_0} - 1$$

$$1 = \frac{x}{x_0}$$

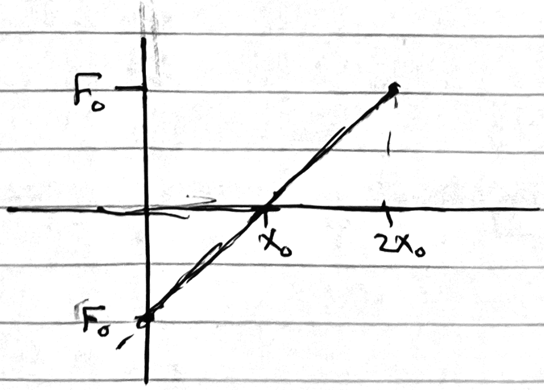
$$\Rightarrow \boxed{x_0 = x}$$

x صفر (y intercept)

$$F = F_0\left(\frac{0}{x_0} - 1\right)$$

$$F = F_0(0 - 1)$$

$$\boxed{F = -F_0}$$



$$F(2x_0) = F_0\left(\frac{2x_0}{x_0} - 1\right)$$

$$F = F_0(2-1)$$

$$F = F_0$$

$W =$ Area undere the plote

$= A_1 + A_2$

$= \frac{1}{2} x_0(-F_0) + \frac{1}{2}(2x_0 - x_0) F_0$

$= \frac{-x_0F_0}{2} + \frac{x_0F_0}{2}$

$=$ zero J

b) $W = \int_0^{2x_0} F_0\left(\frac{x}{x_0} - 1\right) dx$

$= F_0 \int_0^{2x_0} \left(\frac{x}{x_0} - 1\right) dx$

$= F_0 \left(\frac{x^2}{2x_0} - x\right)\Big|_0^{2x_0}$

$= F_0 \left[\left(\frac{(2x_0)^2}{2x_0} - 2x_0\right) - \left(\frac{2(0)}{x_0} - 0\right)\right]$

$= F_0 \left[\left(\frac{4x_0^2}{2x_0} - 2x_0\right) - \text{zero}\right]$

$= F_0 \left[2x_0 - 2x_0\right]$

$= F_0(\text{zero})$

$=$ zero J

Problem 41: only one force is acting on a 2.8 kg particle-like object whose position is given by $x = 4.0t - 5.0t^2 + 2.0t^3$ with x in meters and t in seconds. What is the work done by the force from $t = 0$ sec to $t = 6.0$ sec

Sol: $m = 2.8$ kg ,

ساره مجار

$W = k_f - k_i = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$

$x(t) = 4.0t - 5.0t^2 + 2.0\,t^3$

$v = \frac{dx}{dt} = 4 - 10t + 6t^2$

$v_i = v(t=0) = 4 - 10(0) + 6(0) = 4\,m/s$

$v_f = v(t=6) = 4 - 10(6) + 6(6)^2 = 160\,m/s$

$W = \frac{1}{2}(2.8)(160)^2 - \frac{1}{2}(2.8)(4)^2$

$= \frac{1}{2}(2.8)\left[160^2 - 4^2\right]$

$= 1.4 \times \cancel{25??}0\;25584$

$= 35817.6\ J$

$\simeq 36\ kJ$

كابينة المصعد

Problem 46: The loaded cab of an elevator has a mass of 5.0×10^3 kg and moves 210 m up the shaft (الشقة) in 23 sec at constant speed. At what average rate does the force from the cable do work on the cab?

sol: The mass of elevator $\Rightarrow m = 5.0 \times 10^3\ kg$

The distance is $d = 210\ m$

time, $\Delta t = 23\ sec$

* The rate at which force does the work on the object is called power due to ~~to~~ to the force

* The work done on a particle by a constant force during its displacement is given as

$W = \vec{F} \cdot \vec{d}$

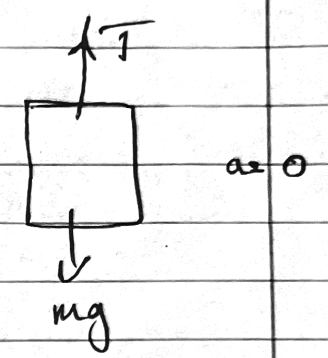
بالسؤال حكى يتحرك للأعلى بـ constant speed اي ان التسارع صفر

← بالتالي محصلة القوى عليه صفر (الجسم متزن)

نعتبر الأعلى موجب لانه باتجاه الأعلى .

$T - mg = 0$

$\boxed{T = mg}$



$\Rightarrow \quad W = F \cdot d$

$= T \cdot d$

$= mgd\cos 0$

لان اتجاه d مع اتجاه T الأعلى (θ=0) ولكن T كمقدار تساوي mg

$W = mgd$

$= 5\times10^{3} \times 9.8 \times 210$

$= 10290 \times 10^{3}$ J

The power due to force is the rate at which force does the work on the object.

$P_{avg} = \frac{W}{\Delta t} = \frac{10290\times10^{3}}{23} \frac{J}{sec}$

$= 447391.3$ W

$= 4.5\times10^{5}$ W