**Birzeit University**
**Faculty of Engineering and Technology**
**Department of Electrical and Computer Engineering**
**Network Analysis 2 – ENEE2315**
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## Question # 1:

Find the Y parameters for the network shown in Fig. 16.1 and then find the output voltage of the two-port when a 4mA current source is connected to the input port and a 4kΩ load is connected to the output port.

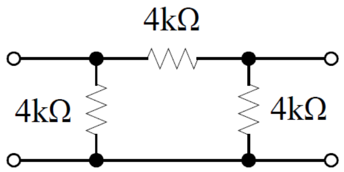


Fig. 16.1

## Question # 2:

Find the Z parameters for the circuit shown in Fig. 16.2, and then find the current in a –j4Ω capacitor connected to the output port when a $6\angle 0°$ V source is connected to the input port.

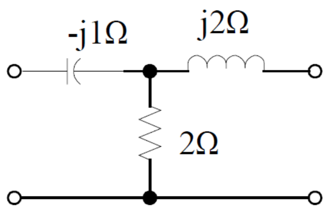


Fig. 16.2

## Question #3:

Find the transmission parameters of the network in Fig. 16.1 by treating the circuit as a cascade interconnection of elements.

## Question #4:

Check the validity of the answers obtained in problems Question #1 and Question #3 by using the parameter conversion formulas to convert the Y parameters in Question #1 to the transmission parameters in Question #3.

## Question # 1 Solution:

The equations for a two-port in terms of the Y parameters are

$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2$$

Since $y_{11} = \frac{I_1}{V_1}$ with $V_2 = 0$, the network in Fig. S16.1(a) is used to find $y_{11}$.

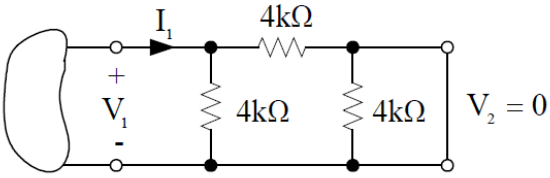


Fig. S16.1(a)

Since $V_2$ is made zero with the short-circuit, the 4kΩ resistor on the right is shorted and

$$V_1 = I_1 (4k \| 4k)$$

or

$$\left.\frac{I_1}{V_1}\right|_{V_2=0} = y_{11} = \frac{1}{2k} S$$

The parameter $y_{12}$ is found from the expression

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

The circuit in Fig. S16.1(b) is used to determine this parameter

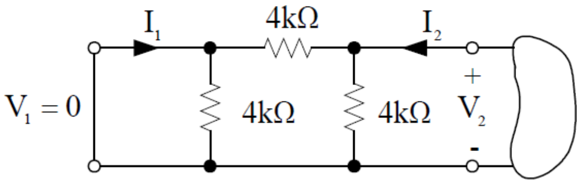


Fig. S16.1(b)

Note in this case, the 4kΩ resistor on the left is shorted and

$$-I_1 (4k) = V_2$$

or

$$\left.\frac{I_1}{V_2}\right|_{V_1 = 0} = y_{12} = \frac{-1}{4k} S$$

We could continue this procedure and determine $y_{21}$ and $y_{22}$ in the exact same manner, however, since the network looks the same from either port, we know that $y_{21} = \frac{-1}{4k} S$ and $y_{22} = \frac{1}{2k} S$. Therefore, the two-port equations for this network in terms of the Y parameters are

$$I_1 = \frac{1}{2k} V_1 - \frac{1}{4k} V_2$$
$$I_2 = \frac{-1}{4k} V_1 + \frac{1}{2k} V_2$$

If we now connect a 4mA current to the input and 4kΩ load to the output, the terminal conditions are

$$I_1 = \frac{4}{k} A$$
$$V_2 = -4k\, I_2$$

The two-port equations now become

$$\frac{4}{k} = \frac{1}{2k} V_1 - \frac{1}{4k} V_2$$
$$\frac{-V_2}{4k} = -\frac{1}{4k} V_1 + \frac{1}{2k} V_2$$

or

$$\frac{4}{k} = \frac{1}{2k} V_1 - \frac{1}{4k} V_2$$
$$0 = -\frac{1}{4k} V_1 + \frac{3}{4k} V_2$$

Simplifying

$$16 = 8V_1 - V_2$$
$$0 = -V_1 + 3V_2$$

or

$$16 = 8(3V_2) - V_2$$
$$V_2 = \frac{16}{23} V$$

The network with the terminal conditions attached is shown in Fig. S16.1(c).

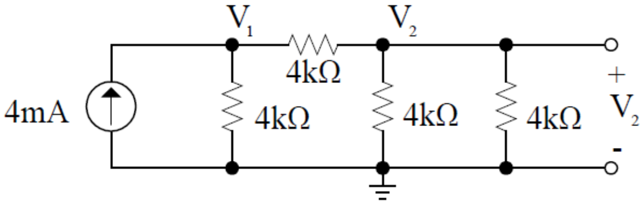


Fig. S16.1(c)

The nodal equations for this network are

$$V_1\left(\frac{1}{4k}+\frac{1}{4k}\right) - V_2\left(\frac{1}{4k}\right) = \frac{4}{k}$$
$$-V_1\left(\frac{1}{4k}\right) + V_2\left(\frac{1}{4k}+\frac{1}{4k}+\frac{1}{4k}\right) = 0$$

Note that these equations are identical to those obtained earlier.

## Question # 2 Solution:

The equations for a two-port in terms of the Z parameters are

$$V_1 = z_{11} I_1 + z_{12} I_2$$
$$V_2 = z_{21} I_1 + z_{22} I_2$$

Since $z_{11} = \frac{V_1}{I_1}$ with $I_2 = 0$, the network in Fig. S16.2(a) is used to derive $z_{11}$.

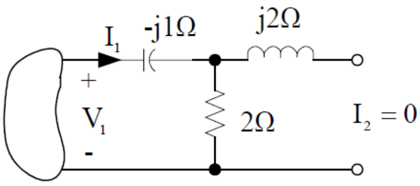


Fig. S16.2(a)

Note that with the output terminals open-circuited, $I_2 = 0$. Then

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0} = 2 - j1\Omega$$

Likewise, the parameter $z_{12}$ is found from the expression

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0}$$

The circuit used to derive this parameter is shown in Fig. S16.2(b).

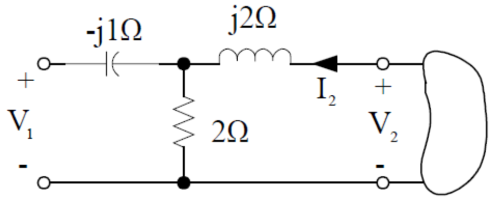


Fig. S16.2(b)

With the input terminals open-circuited, $I_1 = 0$. Since $I_1 = 0$, there is no current in the capacitor and therefore no voltage across it. Then $V_1$ is the voltage across the 2Ω resistor and

$$V_1 = 2I_2$$

and hence

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0} = 2\Omega$$

In a similar manner, we find that

$$z_{21} = 2\Omega$$
$$z_{22} = 2 + j2\Omega$$

Therefore, the two-port equations in terms of the Z parameters are

$$V_1 = (2 - j1)I_1 + 2I_2$$
$$V_2 = 2I_1 + (2 + j2)I_2$$

If we now apply the terminal conditions, the network is shown in Fig. S16.2(c).

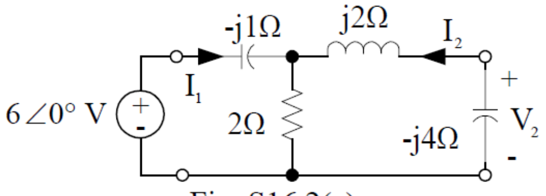


Fig. S16.2(c)

The terminal conditions are

$$V_1 = 6\angle 0° \text{ V}$$
$$V_2 = -(-j4)I_2$$

And the two-port equations are

$$6\angle 0° = (2 - j1)I_1 + 2I_2$$
$$0 = 2I_1 + (2 - j2)I_2$$

Solving the second equation for $I_1$ and substituting this value into the first equation yields

$$6 = (-1 + j)(2 - j) + 2I_2$$

or

$$I_2 = \frac{7 - 3j}{2}$$
$$= 3.81\angle -23.2° \text{ A}$$

**Question # 3 Solution:**

The network in Fig. 16.1 can be redrawn in the following manner as shown in Fig. S16.4(a).

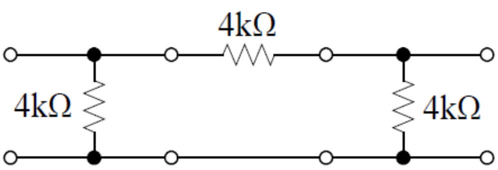

Fig. S16.4(a)

In this form we see that the original network can be drawn as a cascade connection of three networks. The general form of the transmission parameters is

$$V_1 = A\,V_2 - B\,I_2$$
$$I_1 = C\,V_2 - D\,I_2$$

Consider the network in Fig. S16.4(b).

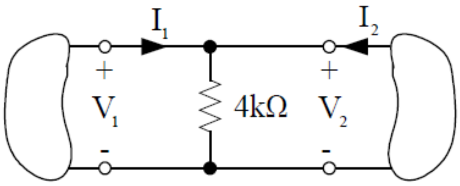

Fig. S16.4(b)

For this network

$$A = \left.\frac{V_1}{V_2}\right|_{I_2 = 0} = 1$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2 = 0} = 0$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2 = 0} = \frac{1}{4k}$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2 = 0} = 1$$

Next consider the network in Fig. S16.4(c).

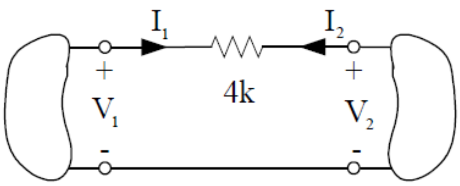

Fig. S16.4(c)

In this case

$$A = \left.\frac{V_1}{V_2}\right|_{I_2 = 0} = 1$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2 = 0} = 4k$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2 = 0} = 0$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2 = 0} = 1$$

Since the transmission parameters for the resistor on the right are the same as those for the resistor on the left, we have all the parameters for the individual networks. Now the transmission parameters for the entire network are

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{4k} & 1 \end{bmatrix} \begin{bmatrix} 1 & 4k \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{4k} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ \frac{1}{4k} & 1 \end{bmatrix} \begin{bmatrix} (1)(1) + (4k)\left(\frac{1}{4k}\right) & (1)(0) + (4k)(1) \\ (0)(1) + (1)\left(\frac{1}{4k}\right) & (0)(0) + (1)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ \frac{1}{4k} & 1 \end{bmatrix} \begin{bmatrix} 2 & 4k \\ \frac{1}{4k} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(2) + (0)\left(\frac{1}{4k}\right) & (1)(4k) + (0)(1) \\ \left(\frac{1}{4k}\right)(2) + (1)\left(\frac{1}{4k}\right) & \left(\frac{1}{4k}\right)(4k) + (1)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4k \\ \frac{3}{4k} & 2 \end{bmatrix}$$

## Question # 4 Solution:

The conversion formulas necessary to convert the Y parameters to the transmission parameters are as follows.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$$

where $\Delta y = y_{11}\, y_{22} - y_{12}\, y_{21}$. From the results of problem 16.1

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{2k} & -\frac{1}{4k} \\ -\frac{1}{4k} & \frac{1}{2k} \end{bmatrix}$$

and $\Delta y = \frac{1}{4k^2} - \frac{1}{16k^2} = \frac{3}{16k^2}$. Hence,

$$A = \frac{-y_{22}}{y_{21}} = \frac{-\frac{1}{2k}}{\frac{-1}{4k}} = 2$$

$$B = \frac{-1}{y_{21}} = \frac{-1}{\frac{-1}{4k}} = 4k$$

$$C = \frac{-\Delta y}{y_{21}} = \frac{\frac{-3}{16k^2}}{\frac{-1}{4k}} = \frac{3}{4k}$$

$$D = \frac{-y_{11}}{y_{21}} = \frac{\frac{-1}{2k}}{\frac{-1}{4k}} = 2$$

These results check with those obtained in the previous question.

*... With Best Wishes ...*