

ditional exercises for the Lecture.

8.1) (14) $\int 4x \sec^2 2x dx$

Let $u = 2x \Rightarrow du = 2dx$ } No need for this step.

$\Rightarrow \int u \sec^2 u du$. Now by parts:

Let $u = 4x$, $dv = \sec^2 2x$
 $du = 4dx \rightarrow v = \tan 2x$

Let $w = u$ $dw = du$
 $dv = \sec^2 u du$
 $v = \tan u$

$\int \frac{\sin u}{\cos u} = \ln |\cos u|$

$\Rightarrow \int u \sec^2 u du = u \tan u - \int \tan u du$
 $= u \tan u - \ln |\sec u| + C$
 $= 2x \tan 2x - \ln |\sec(2x)| + C$

1) (50) $\int_0^{\frac{1}{\sqrt{2}}} 2x \sin^{-1}(x^2) dx$

Let $u = \sin^{-1}(x^2)$, $dv = 2x dx$

$du = \frac{2x dx}{\sqrt{1-x^4}}$

$v = x^2$

$\int_0^{\frac{1}{\sqrt{2}}} 2x \sin^{-1}(x^2) dx = x^2 \sin^{-1}(x^2) \Big|_0^{\frac{1}{\sqrt{2}}} - \int_0^{\frac{1}{\sqrt{2}}} \frac{2x^3}{\sqrt{1-x^4}} dx$

$x^2 \sin^{-1}(x^2) \Big|_0^{\frac{1}{\sqrt{2}}} - \int_0^{\frac{1}{\sqrt{2}}} \frac{du}{-2\sqrt{u}} = \frac{\pi}{12} + \left(\sqrt{1-x^4}\right) \Big|_0^{\frac{1}{\sqrt{2}}}$
 $= \frac{\pi}{12} + \sqrt{\frac{3}{4}} - 1$

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(1)

$$(32) \int_{-\pi}^{\pi} (1 - \cos^2 t)^{\frac{3}{2}} dt = \int_{-\pi}^{\pi} (\sin^2 t)^{\frac{3}{2}} dt$$

$$= \int_{-\pi}^{\pi} |\sin^3 t| dt = - \int_{-\pi}^0 \sin^3 t dt + \int_0^{\pi} \sin^3 t dt$$

$$= - \int_{-\pi}^0 (1 - \cos^2 t) \sin t dt + \int_0^{\pi} (1 - \cos^2 t) \sin t dt$$

$$= \int_{-1}^1 (1 - u^2) du - \int_1^{-1} (1 - u^2) du = 2 \int_{-1}^1 (1 - u^2) du$$

Let $u = \cos t$
 $du = -\sin t dt$

$$= 2 \left[u - \frac{u^3}{3} \right]_{-1}^1 = 2 \left[1 - \frac{1}{3} - \left[(-1) - \frac{(-1)^3}{3} \right] \right] =$$

$$2 \left(\frac{2}{3} - \frac{2}{3} \right) = 2 \left(\frac{4}{3} \right) = \boxed{\frac{8}{3}}$$

$$(17) \int \tan^5 x dx = \int \tan^4 x \tan x dx = \int (\sec^2 x - 1)^2 \tan x dx$$

$$= \int (\sec^4 x - 2\sec^2 x + 1) \tan x dx$$

$$= \int \sec^4 x \tan x dx - 2 \int \sec^2 x \tan x dx + \int \tan x dx$$

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$$= \int \sec^3 x \sec x \tan x dx - 2 \left[\frac{\tan^2 x}{2} \right] + \ln |\sec x| + C$$

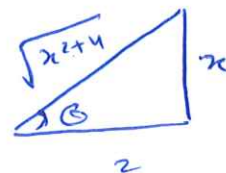
$u = \sec x$
 $du = \sec x \tan x dx$

$$= \frac{\sec^4 x}{4} - \tan^2 x + \ln |\sec x| + C.$$

8.3) (17) $\int \frac{x^3 dx}{\sqrt{x^2+4}}$

Trigonometric substitution.

Let $x = 2 \tan \theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$



$dx = 2 \sec^2 \theta d\theta$

$\sqrt{x^2+4} = \sqrt{4 \tan^2 \theta + 4} = 2 |\sec \theta| = 2 \sec \theta$

$\Rightarrow \int \frac{x^3 dx}{\sqrt{x^2+4}} = \int \frac{8 \tan^3 \theta \cdot (2 \sec^2 \theta) d\theta}{2 \sec \theta} = \int 8 \tan^3 \theta \sec \theta d\theta$

$= 8 \int \frac{\sin^3 \theta}{\cos^3 \theta} \cdot \frac{1}{\cos \theta} d\theta = 8 \int \frac{\sin^2 \theta \sin \theta}{\cos^4 \theta} d\theta$

$= 8 \int \frac{(1 - \cos^2 \theta) \sin \theta}{\cos^4 \theta} d\theta$

Let $u = \cos \theta$
 $du = -\sin \theta d\theta$

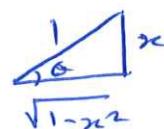
$= 8 \int \frac{-(1-u^2)}{u^4} du = 8 \int \frac{u^2-1}{u^4} du = 8 \left[\int \frac{1}{u^2} - \frac{1}{u^4} du \right]$

$= 8 \left[-\frac{1}{u} + \frac{1}{3u^3} \right] + C = 8 \left[\frac{-1}{\cos \theta} + \frac{1}{3 \cos^3 \theta} \right] + C$

$= 8 \left[-\frac{\sqrt{x^2+4}}{2} + \frac{1}{3} \left(\frac{\sqrt{x^2+4}}{2} \right)^3 \right] + C$

8.3/ (23) $\int_0^{\sqrt{3}/2} \frac{4x^2 dx}{(1-x^2)^{3/2}}$

Let $x = \sin \theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

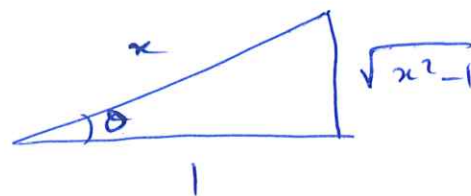


$$\sqrt{1-x^2} = \sqrt{1-\sin^2 \theta} = \cos \theta$$

$$\Rightarrow \int_0^{\pi/3} \frac{4 \sin^2 \theta \cos \theta d\theta}{\cos^3 \theta} = 4 \int_0^{\pi/3} \frac{(1-\cos^2 \theta) \cancel{\cos \theta} d\theta}{\cos^2 \theta}$$

$$= 4 \int_0^{\pi/3} \sec^2 \theta d\theta - 4 \int_0^{\pi/3} 1 d\theta = 4 [\tan \theta - \theta]_0^{\pi/3} = 4\sqrt{3} - \frac{4\pi}{3}$$

8.3/ (25) $\int \frac{dx}{(x^2-1)^{3/2}}$, $\frac{x}{a} \geq 1$



Let $x = \sec \theta$, $0 < \theta < \frac{\pi}{2}$

$$dx = \sec \theta \tan \theta d\theta$$

$$\sqrt{(x^2-1)} = \sqrt{\sec^2 \theta - 1} = \tan \theta$$

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$$\int \frac{\sec \theta \cancel{\tan \theta} d\theta}{\tan^{3/2} \theta} = \int \frac{\cos \theta}{\sin^2 \theta} d\theta = -\frac{1}{\sin \theta} + C$$

$$= -\frac{x}{\sqrt{x^2-1}} + C$$

$$8.4) (16) \int \frac{x+3}{2x^2-8x} dx = \int \frac{x+3}{2x(x+2)(x-2)}$$

$$= \int \left(\frac{A}{2x} + \frac{B}{x+2} + \frac{C}{x-2} \right) dx \quad \dots (*)$$

$$\Rightarrow x+3 = A(x+2)(x-2) + B(2x)(x-2) + C(2x)(x+2)$$

$$\Rightarrow x+3 = Ax^2 - 4A + 2x^2B - 4xB + 2x^2C + 4xC$$

$$\Rightarrow x+3 = (A+2B+2C)x^2 + (4C-4B)x - 4A$$

$$\Rightarrow \left. \begin{array}{l} A+2B+2C=0 \\ 4C-4B=1 \\ -4A=3 \end{array} \right\} \Rightarrow \begin{array}{l} A = -\frac{3}{4} \\ B = \frac{1}{16} \\ C = \frac{5}{16} \end{array}$$

$$(*) \Rightarrow \int \left(\frac{-\frac{3}{4}}{2x} + \frac{\frac{1}{16}}{x+2} + \frac{\frac{5}{16}}{x-2} \right) dx$$

$$= -\frac{3}{8} \ln|x| + \frac{1}{16} \ln|x+2| + \frac{5}{16} \ln|x-2| + C$$

$$8.4) (29) \int \frac{x^2}{x^4-1} dx = \int \frac{x^2}{(x^2+1)(x-1)(x+1)}$$

$$= \int \left(\frac{A}{x+1} + \frac{B}{x-1} + \frac{Cx+D}{x^2+1} \right) dx \quad \dots (*)$$

$$\begin{aligned} \Rightarrow x^2 &= A(x-1)(x^2+1) + B(x+1)(x^2+1) + (Cx+D)(x^2-1) \\ &= (A+B+C)x^3 + (-A+B+D)x^2 + \\ &\quad (A+B-C)x - A+B-D. \end{aligned}$$

$$\begin{aligned} \Rightarrow \left. \begin{aligned} A+B+C &= 0 \\ -A+B+D &= 1 \\ A+B-C &= 0 \\ -A+B-D &= 0 \end{aligned} \right\} \Rightarrow \begin{aligned} A &= -\frac{1}{4} \\ B &= \frac{1}{4} \\ C &= 0 \\ D &= \frac{1}{2} \end{aligned}$$

$$(*) \Rightarrow \int \frac{-\frac{1}{4}}{x+1} + \frac{\frac{1}{4}}{x-1} + \frac{\frac{1}{2}}{x^2+1} dx$$

$$= -\frac{1}{4} \ln|x+1| + \frac{1}{4} \ln|x-1| + \frac{1}{2} \tan^{-1} x + C.$$