

first 4:

$$1) \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{\sin x}} dx$$

$$u = \sin x$$

$$du = \cos x dx \quad \int \frac{du}{\sqrt{u}} = 2\sqrt{\sin x}$$

$$\lim_{c \rightarrow 0^+} [2\sqrt{\sin \frac{\pi}{2}} - 2\sqrt{\sin c}]$$

$$= 2 - 0 = 2 \text{ Conv}$$

d-

$$2) \int_1^{\infty} \frac{\ln x}{x^3} dx$$

Conv by DCT

$$\ln x < x$$

$$\frac{\ln x}{x^3} < \frac{1}{x^2}$$

a-

$$3) a_n = \frac{\ln(2n)}{\ln(3n)}$$

a-

$$\lim_{n \rightarrow \infty} \frac{\ln(2n)}{\ln(3n)} = \lim_{n \rightarrow \infty} \frac{2}{2n} \cdot \frac{3n}{3} = \frac{2}{3} \cdot \frac{3}{2} = 1 \text{ conv}$$

$$4) \int_2^{\infty} \frac{dx}{\sqrt{x^2-1}}$$

$$x^2-1 < x^2$$

$$\sqrt{x^2-1} < \sqrt{x^2}$$

$$\frac{1}{x} < \frac{1}{\sqrt{x^2-1}}$$

Haytham Stetade

c-

$$\lim_{n \rightarrow \infty}$$

div by DCT

$$5) \sum_{n=2}^{\infty} \frac{\sqrt{n}}{\ln n}$$

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\ln n} = \lim_{n \rightarrow \infty} \frac{1}{2\sqrt{n}} \cdot n = \infty \neq 0 \text{ div by } n^{\text{th}} \text{ term test}$$

a-

$$6) \sum_{n=1}^{\infty} \frac{n}{e^n}$$

$$\lim_{n \rightarrow \infty} \frac{n^{\frac{1}{n}}}{e} = \frac{1}{e} < 1 \text{ Conv by Root test}$$

c-

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n}-1}$$

$$\lim_{n \rightarrow \infty} V_n = 0 \quad \text{conv cond}$$

$$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}-1} \cdot \sqrt{n} = 1$$

$$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}} \quad \text{div}$$

$$8) \sum_{n=1}^{\infty} \frac{1}{n \ln n}$$

div by DCT

or

$$\lim_{n \rightarrow \infty} \frac{1}{n \ln n} \cdot n = \frac{1}{1-\frac{1}{n}} = 1$$

div by LCT

$$n - \ln n < n$$

$$\frac{1}{n} < \frac{1}{n - \ln n}$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \quad \text{div}$$

$$9) 1 - \frac{\pi^2}{2!} + \frac{\pi^4}{4!} - \frac{\pi^6}{6!} + \dots$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n (\pi)^{2n}}{n!}$$

$$10) \cos x \approx 1 - \frac{x^2}{2!}, \quad |x| < 0.1$$

$$E < \frac{(0.1)^4}{4!}$$

$$E < \frac{10^{-4}}{4!}$$

$$11) a_n = (2^n + 3^n)^{\frac{1}{n}}$$

$$\sqrt[n]{\lim_{n \rightarrow \infty} \left[\left(\frac{2}{3} \right)^n + 1 \right] 3^n} = 3 \quad \text{conv}$$

$$12) \sum_{n=1}^{\infty} \left(\frac{1}{2^n} + \frac{2}{3^n} \right)$$

$$= \left[\frac{1}{2} \cdot 2 \right] + \left[\frac{2}{3} \cdot \frac{3}{2} \right] = 1 + 1 = 2$$

$$(13) \sum_{n=1}^{\infty} \frac{4}{(2n-1)(2n+1)}$$

$$\frac{A}{2n-1} + \frac{B}{2n+1}$$

$$\sum_{n=1}^{\infty} \left[\frac{2}{2n-1} - \frac{2}{2n+1} \right]$$

$$\frac{2}{2n-1} - \frac{2}{2n+1}$$

$$S_n = 2 - \frac{2}{2n+1}$$

$$(14) \sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$$

$$\lim_{n \rightarrow \infty} \frac{((n+1)!)^2}{(2n+2)!} \cdot \frac{(2n)!}{(n!)^2} = \lim_{n \rightarrow \infty} \frac{(n+1)^2}{(2n+2)(2n+1)} = \lim_{n \rightarrow \infty} \frac{n^2+2n+1}{4n^2+2n+4n+2} = \frac{1}{4} < 1$$

Conv ratio test

$$(15) \sum_{n=1}^{\infty} (\ln x)^n$$

Geometric Series
 $r \in (-1, 1)$

$$-1 < \ln x < 1$$

$$e^{-1} < x < e$$

$$(16) f(x) = 2^x \sum_{n=0}^{\infty} \frac{(\ln 2)^n x^n}{n!}$$

$$\begin{aligned} f(x) &= 2^x \\ f'(x) &= 2^x \ln 2 \\ f''(x) &= 2^x (\ln 2)^2 \\ f'''(x) &= 2^x (\ln 2)^3 \\ &\vdots \end{aligned}$$

$$(17) f(x) = (1+x)^{-\frac{1}{3}}$$

$$1 + \sum_{k=1}^{\infty} \left(-\frac{1}{3} \right)_k x^k$$

$$1 - \frac{1}{3}x + \frac{2}{9}x^2 - \dots$$

$$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{2}}}$$

$$\lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{n} \cdot \frac{n}{1} = 1$$

div by LCT

$$\sum_{n=1}^{\infty} \frac{1}{n} \text{ div}$$

19) $\frac{x^2}{(1-x)^2}$

$$\sum_{n=1}^{\infty} n x^{n+1}$$

$$f(x) = \frac{x^2}{(1-x)^2}$$

$$f'(x) = \frac{-2x^2 + 2x}{(1-x)^4}$$

$$f''(x) = \frac{(-4x+2)(1-x)^4 - 4(-2x^2+2x)(1-x)^3}{(1-x)^8}$$

$$f(0) = 0$$

$$f'(0) = 0$$

$$f''(0) = 2$$

Question 2:

$$\sum_{n=2}^{\infty} \frac{x^n}{n(\ln n)^2}$$

$$\lim_{n \rightarrow \infty} \frac{x^{n+1}}{(n+1)(\ln(n+1))^2} \cdot \frac{n(\ln n)^2}{x^n} = |x| \lim_{n \rightarrow \infty} \left[\frac{n}{n+1} \cdot \frac{(\ln n)^2}{(\ln(n+1))^2} \right] = |x| < 1$$

$$-1 < x < 1$$

$$x = -1 \rightarrow \sum_{n=2}^{\infty} (-1)^n \frac{1}{n(\ln n)^2} \rightarrow \text{CONV AST}$$

$$x = 1 \rightarrow \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} \rightarrow \text{CONV IT} \quad \begin{matrix} u = \ln x \\ du = \frac{dx}{x} \end{matrix}$$

$$f(x) = \frac{1}{x(\ln x)^2}$$

$$\int \frac{du}{u^2} = -\frac{1}{\ln x}$$

$$\lim_{b \rightarrow \infty} \left[-\frac{1}{\ln b} + \frac{1}{\ln 2} \right] = \frac{1}{\ln 2}$$

$$\lim_{n \rightarrow \infty} V_n = \lim_{n \rightarrow \infty} \frac{1}{n(\ln n)^2} = 0$$

$$\text{IC CONV} \rightarrow -1 \leq x \leq 1$$

$$\text{Radius of conv} \rightarrow R = 1$$

Question 3:

$$\frac{1}{1+t} = 1 - t + t^2 - t^3 + \dots \quad -1 < t < 1$$

$$a) \frac{1}{1+t^2} = 1 - t^2 + t^4 - t^6 + t^8 - t^{10} + \dots$$

$$b) \int_0^x \frac{dt}{1+t^2} = \int_0^x (1 - t^2 + t^4 - t^6 + t^8 - t^{10} + \dots) dt$$

$$\tan^{-1} t \Big|_0^x = t - \frac{t^3}{3} + \frac{t^5}{5} - \frac{t^7}{7} + \frac{t^9}{9} - \frac{t^{11}}{11} + \dots \Big|_0^x$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \frac{x^{11}}{11} + \dots$$

$$c) \int_0^1 \frac{\tan^{-1} x}{x} dx \quad E < 0.01$$

$$\frac{\tan^{-1} x}{x} = 1 - \frac{x^2}{3} + \frac{x^4}{5} - \frac{x^6}{7} + \frac{x^8}{9} - \frac{x^{10}}{11} + \dots$$

$$\int_0^1 \frac{\tan^{-1} x}{x} dx = \int_0^1 \left(1 - \frac{x^2}{3} + \frac{x^4}{5} - \frac{x^6}{7} + \frac{x^8}{9} - \frac{x^{10}}{11} + \dots \right) dx$$

$$= x - \frac{x^3}{9} + \frac{x^5}{25} - \frac{x^7}{49} + \frac{x^9}{81} - \frac{x^{11}}{121} + \dots \Big|_0^1$$

$$= 1 - \frac{1}{9} + \frac{1}{25} - \frac{1}{49} + \frac{1}{81} - \frac{1}{121} + \dots$$

$$\int_0^1 \frac{\tan^{-1} x}{x} dx \approx 1 - \frac{1}{9} + \frac{1}{25} - \frac{1}{49} + \frac{1}{81}$$

$$E < \frac{1}{121}$$