Chapter 7.1, Problem 15E

Problem

Let *F* and *G* be functions from the set of all real numbers to itself. Define the product functions *F* • *G*: **R** → **R** and *G* • *F*: **R** → **R** as follows: For all *x* ∈ **R**,

(*F* • *G*)(*x*) = *F*(*x*) • *G*(*x*)

(*G* • *F*)(*x*) = *G*(*x*) • *F*(*x*)

Does *F* • *G* = *G* • *F*? Explain.

Step-by-step solution

Step 1 of 2

Consider the functions *F* and *G* are defined from real numbers to itself.

Define the product $F \cdot G : \mathbb{R} \to \mathbb{R}$ and $G \cdot F : \mathbb{R} \to \mathbb{R}$ for all $x \in \mathbb{R}$ as follows,

$$(F \cdot G)(x) = F(x) \cdot G(x)$$

$$(F \cdot G)(x) = (G \cdot F)(x)$$

Step 2 of 2

For all real number $x \in \mathbb{R}$

By the definition of $F \cdot G$,

$$(F \cdot G)(x) = F(x) \cdot G(x)$$

Now, apply the commutative law for multiplication of real numbers,

$$(F \cdot G)(x) = G(x) \cdot F(x)$$

Again by the definition of $G \cdot F$

$$(F \cdot G)(x) = (G \cdot F)(x)$$

Therefore, the functions F and G are satisfy the product $\boxed{F \cdot G = G \cdot F}$ for all real number $x \in \mathbb{R}$.