$\boxed{6.3}$ Normal Approximation of Binomial probabilities "continuous"

- Recall the discrete binomial distribution $f(x) = \binom{n}{x} p^{x} (1-p)^{n-x}$ where the discrete random variable x is the number of successes in n trials within a binomial experiment.

- When the number of trials become large, evaluating the binomial probability function by hand or with calculator is difficult.

  $\Rightarrow$ In cases where $np \geq 5$ and $n(1-p) \geq 5$, the normal distribution provides an easy to use approximation of binomial probabilities, by using $\mu = np$ and $\sigma = \sqrt{np(1-p)}$.

Example: A company has a history of making errors in 10% of its invoices. (فواتير) A sample of 100 invoices has been taken.

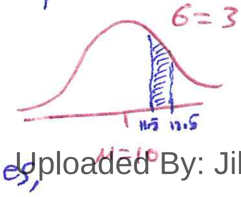
$\boxed{a}$ Find the prob. that 12 invoices contain error.

- We need the binomial prob. of 12 successes $= f(12) = \binom{100}{12}(0.1)^{12}(0.9)^{88}$ which is hard to calculate by hand or calculator.

- So we use the normal approximation: $\mu = np = 100(0.1) = 10$

  $\sigma = \sqrt{10(0.9)} = \sqrt{9} = 3$

Since here we are using continuous prob. distribution, the probabilities are computed as areas under the prob. density function.



$\Rightarrow$ So, the prob. of any single value is zero

$\Rightarrow$ Thus, to approximate the binomial prob. of 12 successes, we compute the area under the normal curve between 11.5 and 12.5.

$\Rightarrow$ The 0.5 we add and substract from 12 is called a continuity correction factor.

$$f(12) \approx P(11.5 \leq X \leq 12.5)$$
$$= P(0.5 \leq Z \leq 0.83)$$
$$= P(Z \leq 0.83) - P(Z \leq 0.5)$$
$$= 0.7967 - 0.6915 = 0.1052$$

$$Z = \frac{x-\mu}{\sigma} = \frac{11.5-10}{3} = 0.5$$

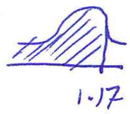
$$Z = \frac{x-\mu}{\sigma} = \frac{12.5-10}{3} = 0.83$$

$\boxed{b}$ Find the prob. of 13 or fewer errors.
(invoices, contain)

$P(X \le 13.5)$

$= P(z \le 1.17) = 0.8790$

$z = \frac{X-\mu}{\sigma} = \frac{13.5-10}{3} = 1.17$



Example (Q26 page 244) A binomial prob. distribution has $p = 0.2$ and $n = 100$

$\boxed{a}$ What are the mean and the standard deviation?

$\mu = np = 100(0.2) = 20$

$\sigma = \sqrt{np(1-p)} = \sqrt{20(0.8)} = \sqrt{16} = 4$

$\boxed{b}$ Is this situation one in which binomial probabilities can be approximated by the normal distribution? Explain.

Yes because $\mu = np = 20 \ge 5$ and $n(1-p) = 100(0.8) = 80 \ge 5$

$\boxed{c}$ What is the prob. of exactly 24 successes?

$P(23.5 \le X \le 24.5)$

$z = \frac{X-\mu}{\sigma} = \frac{23.5-20}{4} = 0.88$

$= P(0.88 \le z \le 1.13)$

$z = \frac{X-\mu}{\sigma} = \frac{24.5-20}{4} = 1.13$

$= P(z \le 1.13) - P(z \le 0.88)$

$= 0.8708 - 0.8106 = 0.0602$

$\boxed{d}$ What is the prob. of 18 to 22 successes?

$P(17.5 \le X \le 22.5)$

$z = \frac{X-\mu}{\sigma} = \frac{17.5-20}{4} = -0.63$

$z = \frac{X-\mu}{\sigma} = \frac{22.5-20}{4} = 0.63$

$= P(-0.63 \le z \le 0.63)$

$= P(z \le 0.63) - P(z \le -0.63) = 0.7357 - 0.2643 = 0.4714$

$\boxed{e}$ What is the prob. of 15 or fewer successes? "at most 15"

$P(X \le 15.5)$

$z = \frac{X-\mu}{\sigma} = \frac{15.5-20}{4} = -1.13$

$= P(z \le -1.13) = 0.1292$

$\boxed{f}$ What is the prob. of 15 or greater successes "at least 15"?

$z = \frac{X-\mu}{\sigma} = \frac{14.5-20}{4} = -1.38$

$P(X \ge 14.5) = P(z \ge -1.38) = 1 - P(z \le -1.38) = 1 - 0.0838 = 0.9162$