

# Chapter 9 Asynchronous Sequential Logic

- **9-1 Introduction**
- **9-2 Analysis Procedure**
- **9-3 Circuits With Latches**
- **9-4 Design Procedure**



# Chapter 9 Asynchronous Sequential Logic

- **9-5 Reduction of State and Flow Tables**
- **9-6 Race-Free State Assignment**
- **9-7 Hazards**
- **9-8 Design Example**



# 9-1 Introduction

- **Synchronous Sequential Circuits**

The change of internal state occurs in response to the **synchronized clock pulse**.

Memory elements are **clocked flip-flops**.

- **Asynchronous Sequential Circuits**

The change of internal state occurs when there is a **change in the input variables**.

Memory elements are **unclocked flip-flops or time-delay elements**.



# 9-1 Introduction

- **Synchronous Sequential Circuits**

Timing problems are eliminated by triggering all flip-flops with pulse edge.

- **Asynchronous Sequential Circuits**

Care must be taken to ensure that each new state is stable even when a change in input exists.

Higher speed  
More economical



# 9-1 Introduction

## • Asynchronous Sequential Circuits

When an input variable changes in value, the  $y$  secondary variables do not change instantaneously.

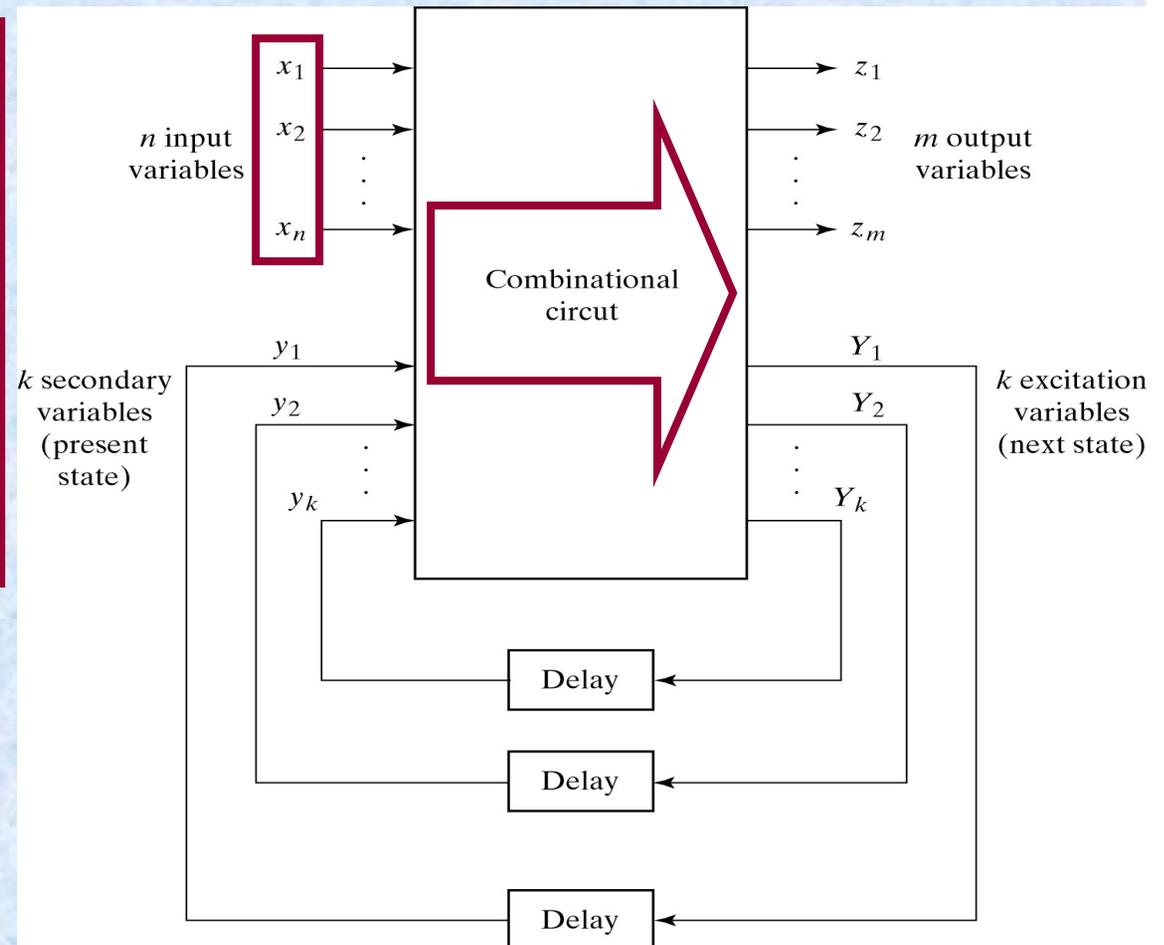


Fig. 9-1 Block Diagram of an Asynchronous Sequential Circuit

# 9-1 Introduction

## • Asynchronous Sequential Circuits

In steady-state condition, the  $y$ 's and the  $Y$ 's are the same, but during transition they are not.

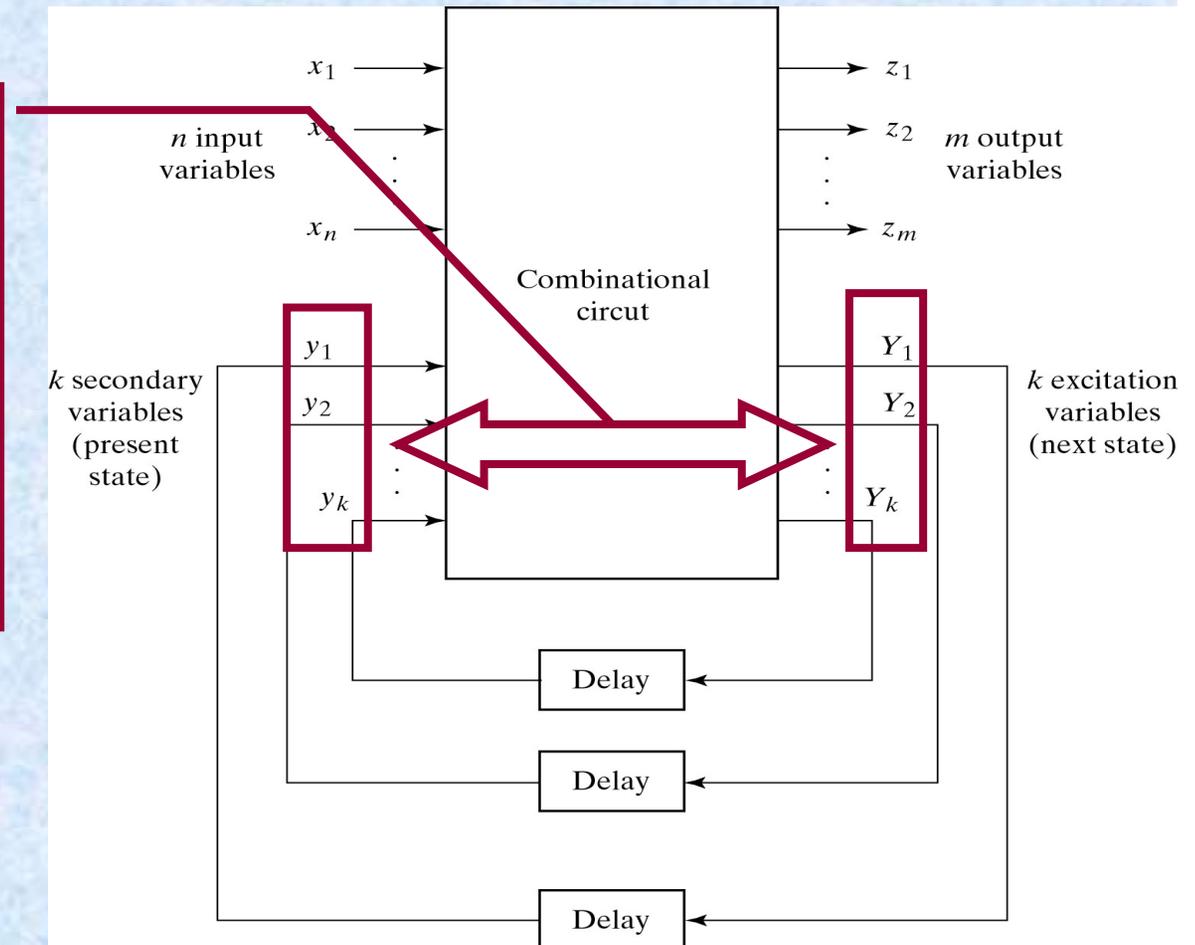


Fig. 9-1 Block Diagram of an Asynchronous Sequential Circuit

# 9-1 Introduction

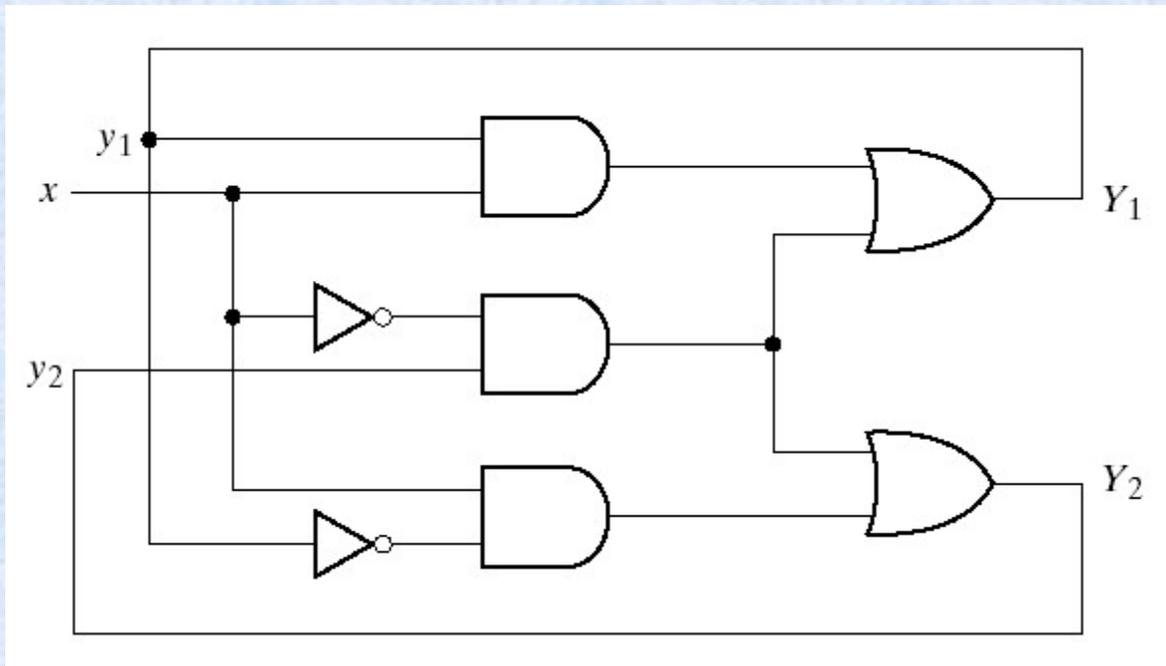
- **fundamental mode**

*fundamental mode* : Only one input variable can change at any one time and the time between two input changes must be longer than the time it takes the circuit to reach a stable state.



# 9-2 Analysis Procedure

## Transition Table



$$Y_1 = xy_1 + x'y_2$$

$$Y_2 = xy'_1 + x'y_2$$

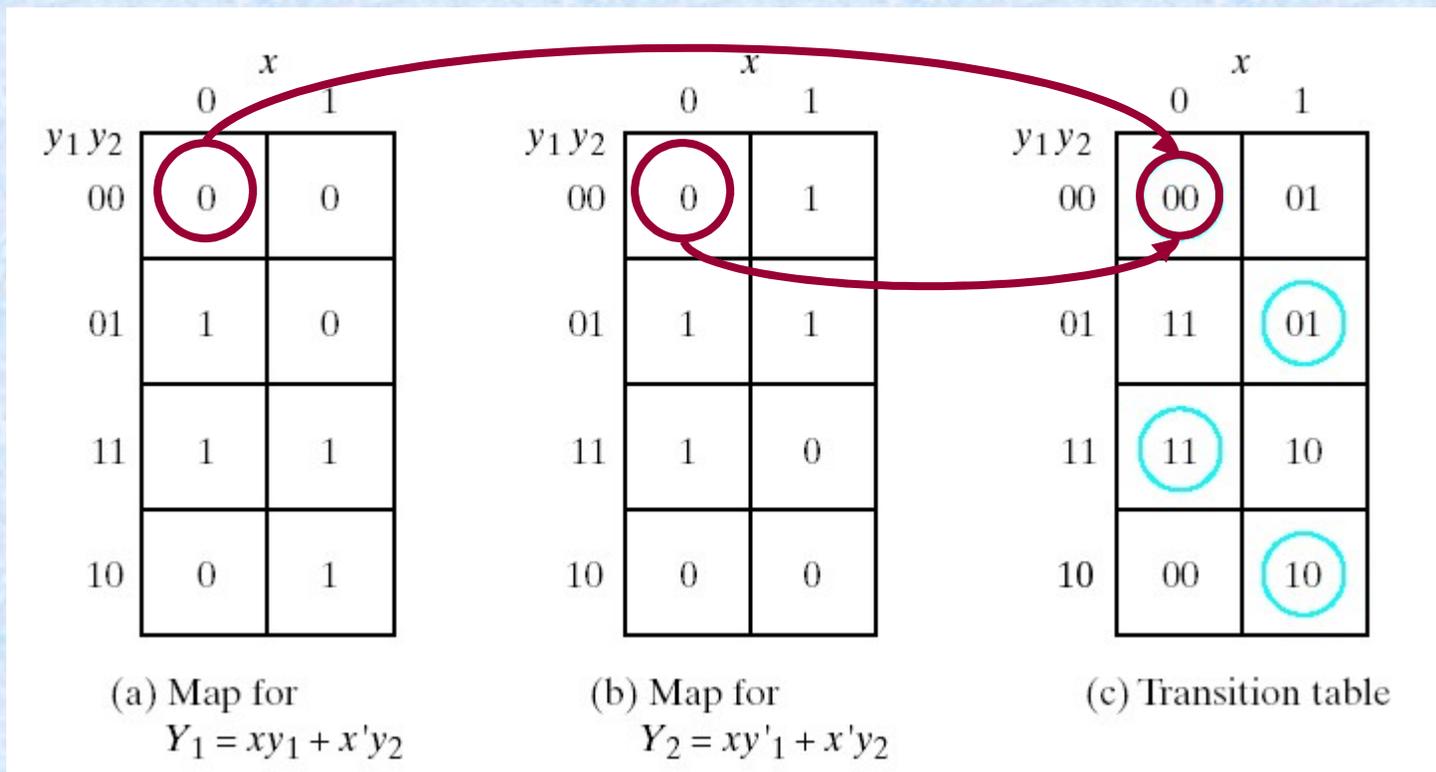


# 9-2 Analysis Procedure

## Transition Table

$$Y_1 = xy_1 + x'y_2$$

$$Y_2 = xy'_1 + x'y_2$$



# 9-2 Analysis Procedure

## Transition Table

For a state to be stable, the value of  $Y$  must be the same as that of  $y = y_1y_2$

	$x$	
	0	1
$y_1y_2$		
00	00	01
01	11	01
11	11	10
10	00	10

(c) Transition table



# 9-2 Analysis Procedure

## Transition Table

The Unstable states,  
 $Y \neq y$

	$x$	
$y_1 y_2$	0	1
00	00	01
01	11	01
11	11	10
10	00	10

(c) Transition table



# 9-2 Analysis Procedure

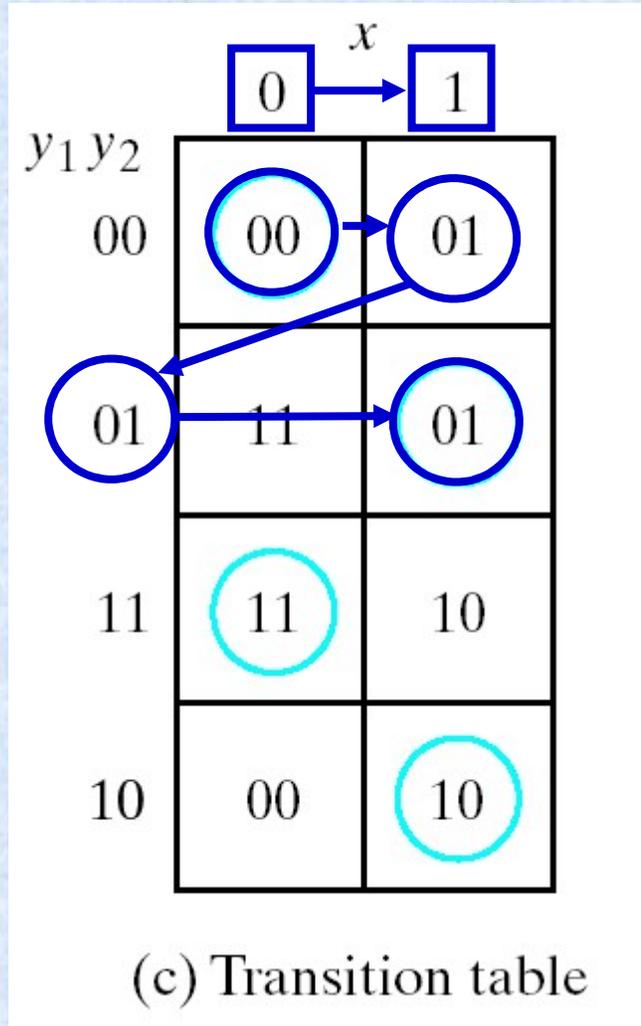
## Transition Table

Consider the square for  $x = 0$  and  $y = 00$ . It is stable.

$x$  changes from 0 to 1.

The circuit changes the value of  $Y$  to 01. The state is unstable.

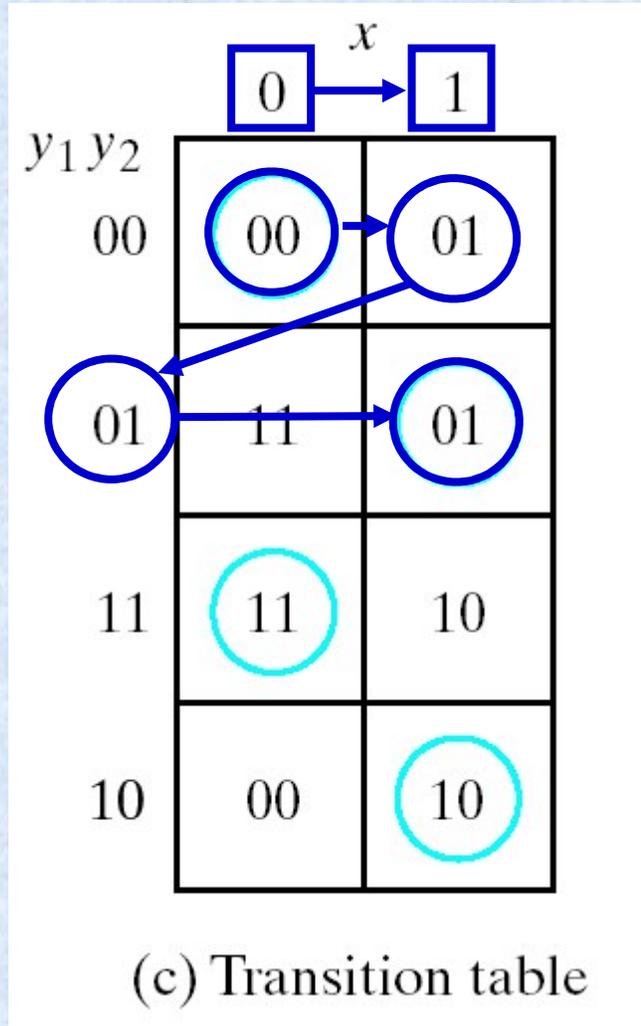
The feedback causes a change in  $y$  to 01. The circuit reaches stable.



# 9-2 Analysis Procedure

## Transition Table

In general, if a change in the input takes the circuit to an unstable state,  $y$  will change until it reaches a stable state.



# 9-2 Analysis Procedure

Flow Table

Flow Table

	$x$	
	0	1
$a$	$a$	$b$
$b$	$c$	$b$
$c$	$c$	$d$
$d$	$a$	$d$

Flow table whose  
 entries are  
 labeled by letter sym  
 bles  
 binary values



	$x$	
	0	1
$y_1 y_2$	$00$	$01$
$01$	$11$	$01$
$11$	$11$	$10$
$10$	$00$	$10$

(c) Transition table



# 9-2 Analysis Procedure

## Flow Table

It is called primitive flow table because it has only one stable state in each row.

	$x$	
	0	1
$a$	$a$	$b$
$b$	$c$	$b$
$c$	$c$	$d$
$d$	$a$	$d$

	$x_1 x_2$			
	00	01	11	10
$a$	$a, 0$	$a, 0$	$a, 0$	$b, 0$
$b$	$a, 0$	$a, 0$	$b, 1$	$b, 0$

(b) Two states with two inputs and one output

It is a flow table with more than one stable state in the same row.



# 9-2 Analysis Procedure

## Flow Table

		x	
		0	1
y <sub>1</sub> y <sub>2</sub>	00	00	01
	01	11	01
	11	11	10
	10	00	10

(c) Transition table

		x <sub>1</sub> x <sub>2</sub>			
		00	01	11	10
y	0	0	0	0	1
	1	0	0	1	1

(a) Transition table  
 $Y = x_1x_2' + x_1y$

		x <sub>1</sub> x <sub>2</sub>			
		00	01	11	10
y	0	0	0	0	0
	1	0	0	1	0

(b) Map for output  
 $z = x_1x_2y$



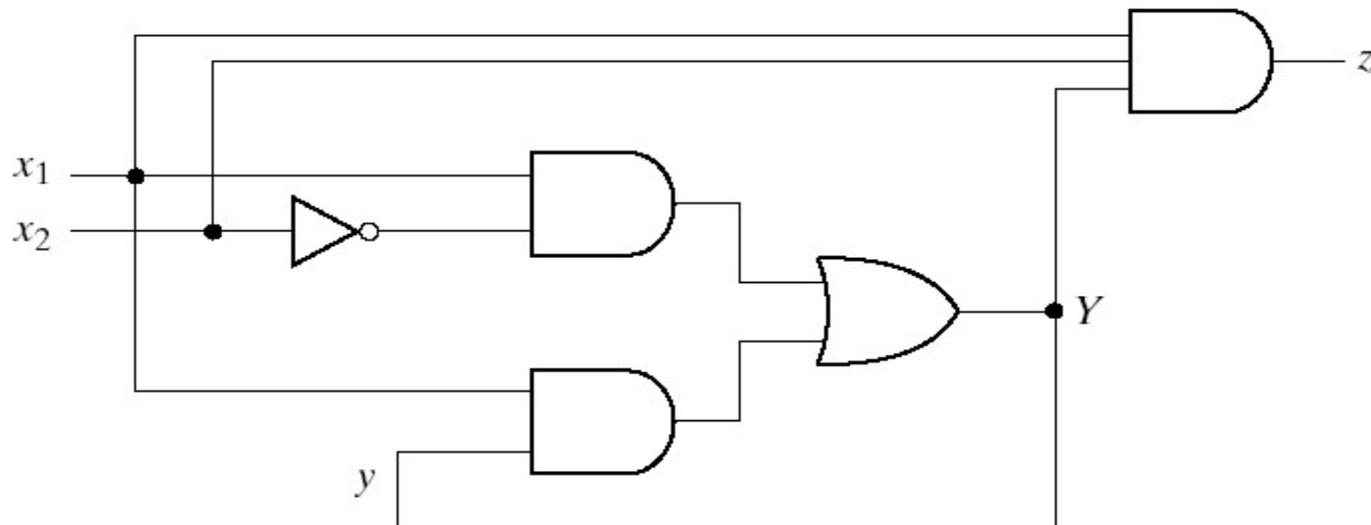
# 9-2 Analysis Procedure

		$x_1 x_2$			
		00	01	11	10
$y$	0	0	0	0	1
	1	0	0	1	1

(a) Transition table  
 $Y = x_1x'_2 + x_1y$

		$x_1 x_2$			
		00	01	11	10
$y$	0	0	0	0	0
	1	0	0	1	0

(b) Map for output  
 $z = x_1x_2y$



# 9-2 Analysis Procedure

## Race Conditions

### *Noncritical Race:*

Two or more binary state variables change value in response to a change in an input. The possible transitions could be

00	11	
00	01	11
00	10	11

It is a **noncritical** race. The final stable state that the circuit reaches does not depend on the order in which the state variables change.



# 9-2 Analysis Procedure

## Race Conditions

*Critical Race:*

State variables change from 00 to 11. The possible transition could be

00 11  
00 01 11  
00 10

It is a **critical** race. The final stable state depends on the order in which the state variables change.

		x	
		0	1
y <sub>1</sub> y <sub>2</sub>	00	00	11
	01		11

(b) Possible transitions:



# 9-2 Analysis Procedure

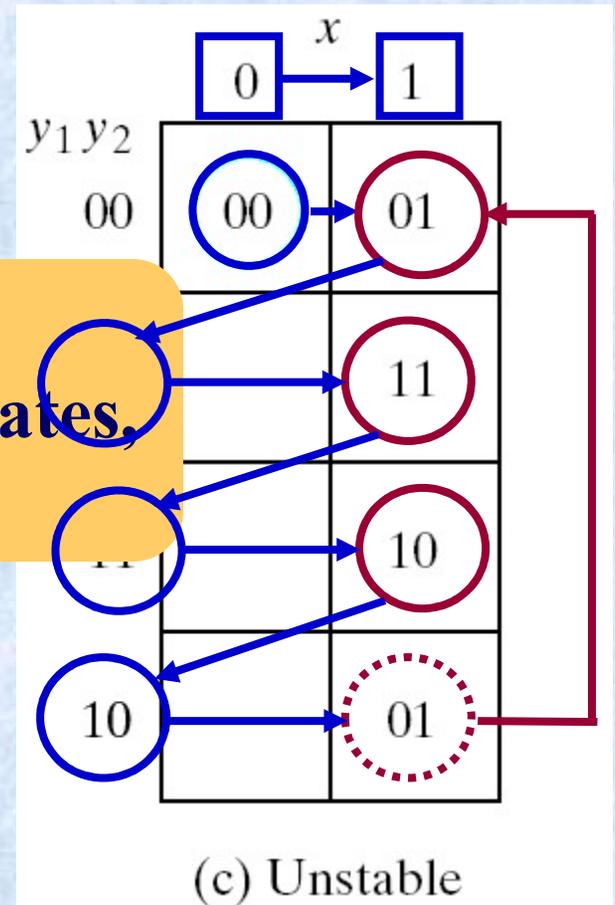
## Race Conditions

### Cycle

It starts at state 00  
the  
0 to

When a circuit goes through a unique sequence of unstable states, it is said to have a *cycle*.

The sequence is as follows,

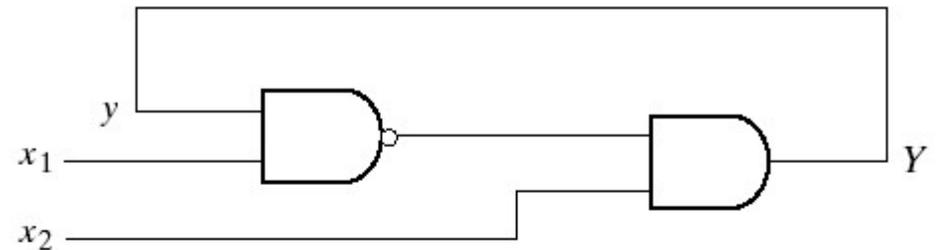


# 9-2 Analysis Procedure

## Stability Consideration

Column 11 has no stable state. With input  $x_1 x_2 = 11$ ,  $Y$  and  $y$  are never the same.

This will cause instability.



	$x_1 x_2$			
$y$	00	01	11	10
0	0	1	1	0
1	0	1	0	0

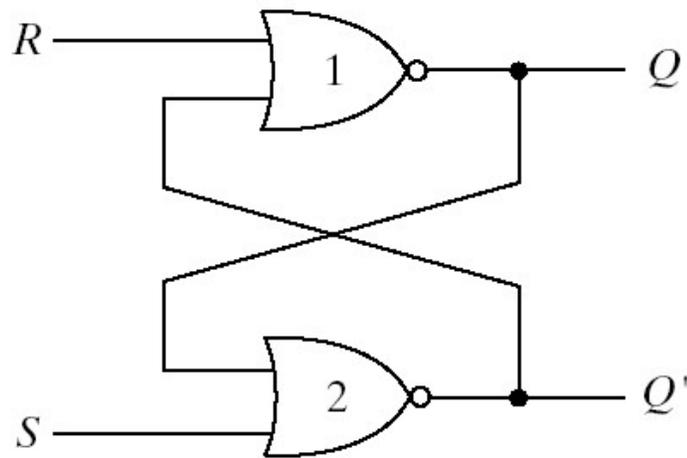
(b) Transition table



# 9-3 Circuits with Latches

## SR Latch

The circuit diagram and truth table of the *SR* latch are shown as follows,



(a) Crossed-coupled circuit

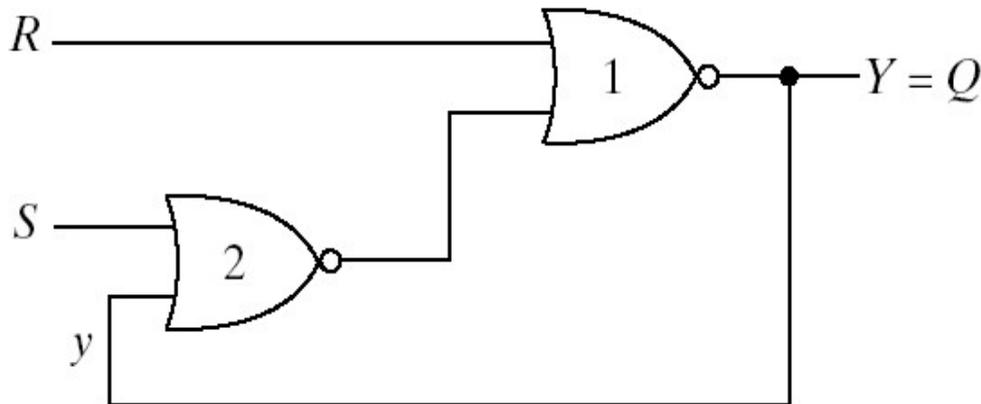
$S$	$R$	$Q$	$Q'$	
1	0	1	0	
0	0	1	0	(After $SR = 10$ )
0	1	0	1	
0	0	0	1	(After $SR = 01$ )
1	1	0	0	

(b) Truth table

# 9-3 Circuits with Latches

## SR Latch

The circuit diagram of the SR latch can be redrawn as follows,



(c) Circuit showing feedback

	00	01	11	10
0	0	0	0	1
1	1	0	0	1

$$Y = SR' + R'y$$

$$Y = S + R'y \text{ when } SR = 0$$

(d) Transition table

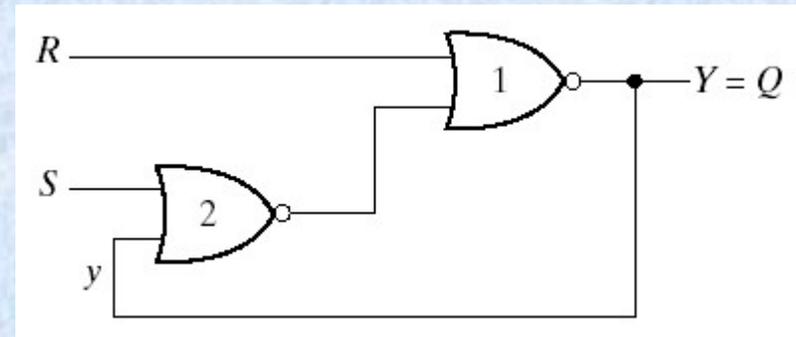
# 9-3 Circuits with Latches

## SR Latch

$$Y = [(S + y)' + R]'$$
$$= (S + y)R' = SR' + R'y$$

$$\left. \begin{array}{l} SR' + SR = S(R' + R) = S \\ SR = 0 \end{array} \right\} SR' = S$$

$$\Rightarrow Y = SR' + R'y = S + R'y \quad \text{when } SR=0$$



# 9-3 Circuits with Latches

## Analysis Example

$$S_1 = x_1 y_2$$

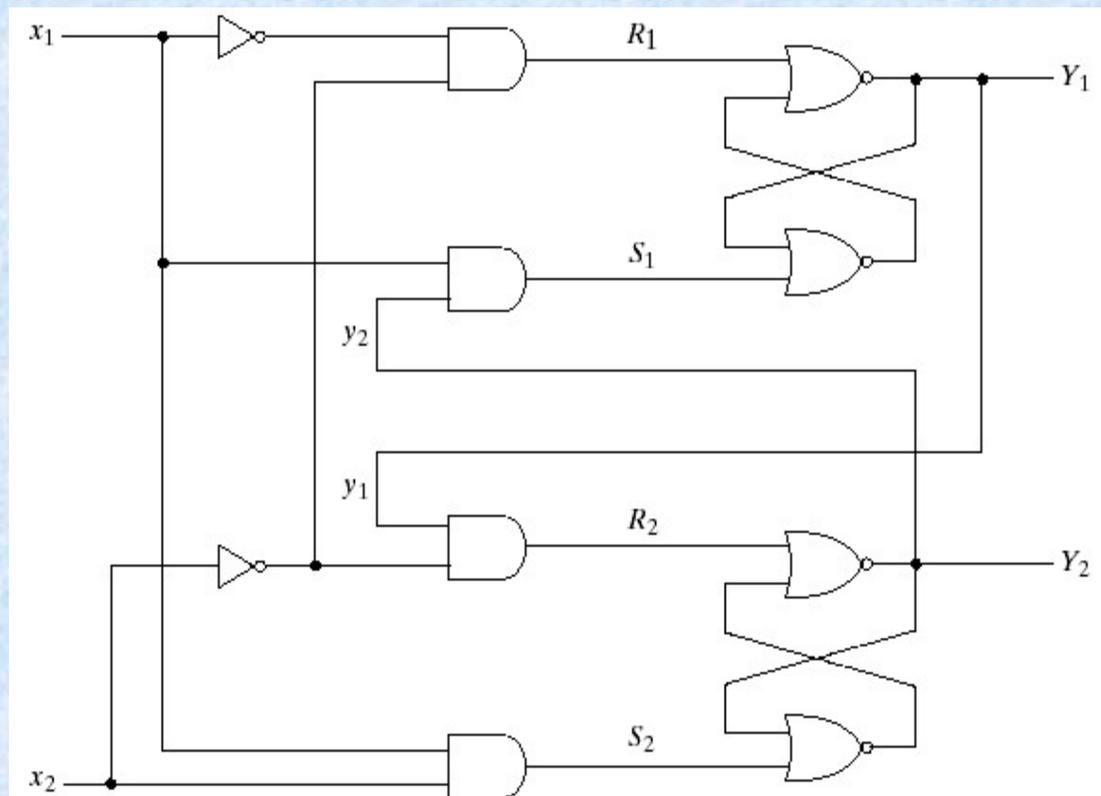
$$S_2 = x_1 x_2$$

$$R_1 = x'_1 x'_2$$

$$R_2 = x'_2 y_1$$

$$S_1 R_1 = x_1 y_2 x'_1 x'_2 = 0$$

$$S_2 R_2 = x_1 x_2 x'_2 y_1 = 0$$



# 9-3 Circuits with Latches

## Analysis Example

$$Y_1 = S_1 + R'_1 y_1 = x_1 y_2 + (x_1 + x_2) y_1$$

$$Y_2 = S_2 + R'_2 y_2 = x_1 x_2 + (x_2 + y'_1) y_2$$

There is a critical race condition

		$x_1 x_2$			
		00	01	11	10
$y_1 y_2$	00	00	00	01	00
	01	01	01	11	11
	11	00	11	11	10
	10	00	10	11	10



# 9-3 Circuits with Latches

## Latch Excitation Table

A table that lists the required inputs  $S$  and  $R$  for each of the possible transitions from  $y$  to  $Y$

The first two columns list the four possible transitions from  $y$  to  $Y$ .

The next two columns specify the required input values that will result in the specified transition.

$y$	$Y$	$S$	$R$
0	0	0	$X$
0	1	1	0
1	0	0	1
1	1	$X$	1

(b) Latch excitation table



# 9-3 Circuits with Latches

## Implementation Example

	$x_1x_2$			
	00	01	11	10
$y$				
0	0	0	0	1
1	0	0	1	1

(a) Transition table

$$Y = x_1x'_2 + x_1y$$

$y$	$Y$	$S$	$R$
0	0	0	$X$
0	1	1	0
1	0	0	1
1	1	$X$	1

(b) Latch excitation table

	$x_1x_2$			
	00	01	11	10
$y$				
0	0	0	0	1
1	0	0	$X$	$X$

(c) Map for  $S = x_1x'_2$

	$x_1x_2$			
	00	01	11	10
$y$				
0	$X$	$X$	$X$	0
1	1	1	0	0

(d) Map for  $R = x'_1$

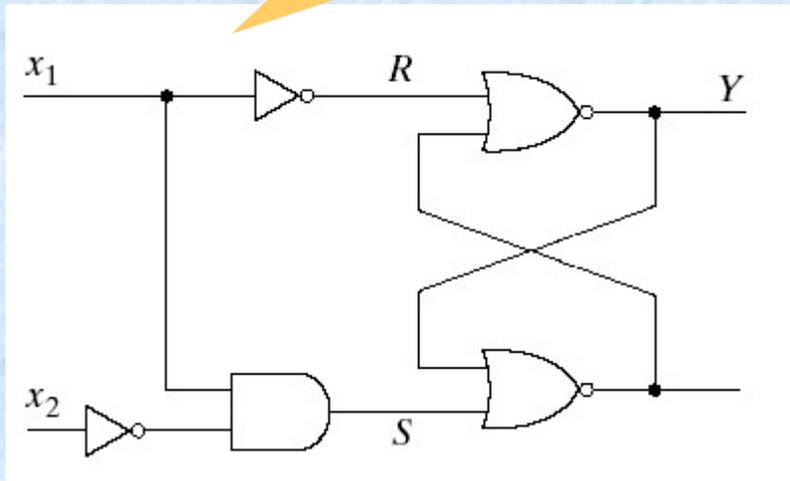


# 9-3 Circuits with Latches

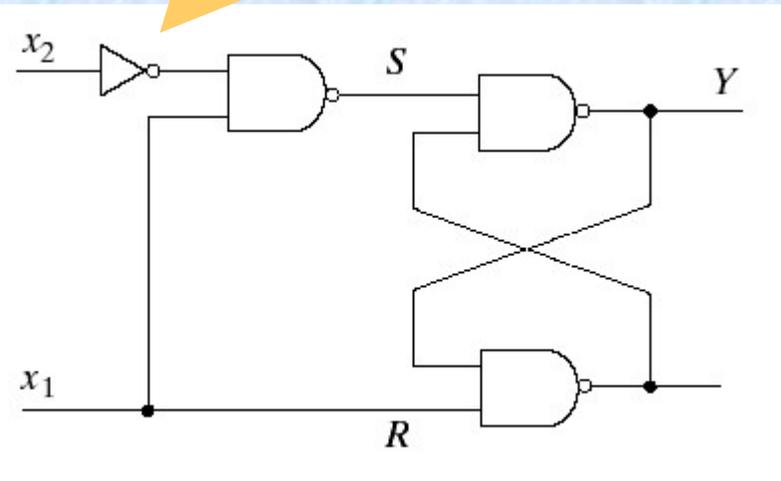
## Implementation Example

$$S = x_1x'_2 \quad R = x'_1$$

Circuit with  
NOR latch



Circuit with  
NAND latch



# 9-4 Design Procedure

## Design Example

Design a gated latch circuit with two inputs  $G$  (gate) and  $D$  (data), and one output  $Q$ .

Gated-Latch  
Total States

State	Inputs		Output	comments
	$D$	$G$	$Q$	
$a$	0	1	0	$D = Q$ because $G = 1$
$b$	1	1	1	$D = Q$ because $G = 1$
$c$	0	0	0	After state $a$ or $d$
$d$	1	0	0	After state $c$
$e$	1	0	1	After state $b$ or $f$
$f$	0	0	1	After state $e$



# 9-4 Design Procedure

## Design Example

State	Inputs		Output
	$D$	$G$	$Q$
$a$	0	1	0
$b$	1	1	1
$c$	0	0	0
$d$	1	0	0
$e$	1	0	1
$f$	0	0	1



	$DG$			
	00	01	11	10
$a$	$c, -$	$a, 0$	$b, -$	$-, -$
$b$	$-, -$	$a, -$	$b, 1$	$e, -$
$c$	$c, 0$	$a, -$	$-, -$	$d, -$
$d$	$c, -$	$-, -$	$b, -$	$d, 0$
$e$	$f, -$	$-, -$	$b, -$	$e, 1$
$f$	$f, 1$	$a, -$	$-, -$	$e, -$



# 9-4

Two or more rows in the primitive flow table can be merged into one row if there are **non-conflicting** states and outputs in each of the columns.

## Reduction of Primitive Flow Table

		DG			
		00	01	11	10
a	c, -	a, 0	b, -	- , -	
b	- , -	a , -	b , 1	e , -	
c	c , 0	a , -	- , -	d , -	
d	c , -	- , -	b , -	d , 0	
e	f , -	- , -	b , -	e , 1	
f	f , 1	a , -	- , -	e , -	



		DG			
		00	01	11	10
a	c, -	a, 0	b, -	- , -	
c	c , 0	a , -	- , -	d , -	
d	c , -	- , -	b , -	d , 0	

		DG			
		00	01	11	10
b	- , -	a , -	b , 1	e , -	
e	f , -	- , -	b , -	e , 1	
f	f , 1	a , -	- , -	e , -	

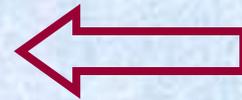


# 9-4 Design Procedure

## Reduction of the Primitive Flow Table

*DG*

	00	01	11	10
<i>a, c, d</i>	<b>c</b> , 0	<b>a</b> , 0	<i>b</i> , -	<b>d</b> , 0
<i>b, e, f</i>	<b>f</b> , 1	<i>a</i> , -	<b>b</b> , 1	<b>e</b> , 1



*DG*

	00	01	11	10
<i>a</i>	<i>c</i> , -	<b>a</b> , 0	<i>b</i> , -	- , -
<i>c</i>	<b>c</b> , 0	<i>a</i> , -	- , -	<i>d</i> , -
<i>d</i>	<i>c</i> , -	- , -	<i>b</i> , -	<b>d</b> , 0



*DG*

	00	01	11	10
<i>a</i>	<b>a</b> , 0	<b>a</b> , 0	<i>b</i> , -	<b>a</b> , 0
<i>b</i>	<b>b</b> , 1	<i>a</i> , -	<b>b</b> , 1	<b>b</b> , 1

*DG*

	00	01	11	10
<i>b</i>	- , -	<i>a</i> , -	<b>b</b> , 1	<i>e</i> , -
<i>e</i>	<i>f</i> , -	- , -	<i>b</i> , -	<b>e</b> , 1
<i>f</i>	<b>f</b> , 1	<i>a</i> , -	- , -	<i>e</i> , -



# 9-4 Design Procedure

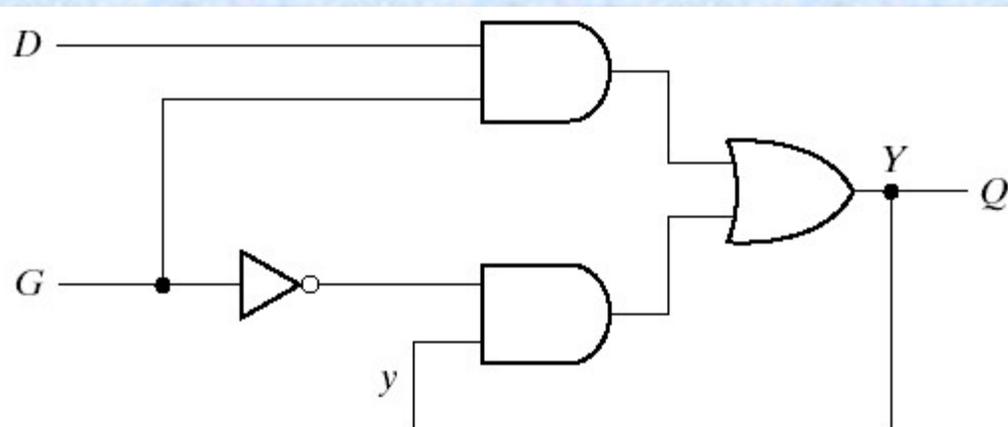
## Transition Table and Logic Diagram

	<i>DG</i>			
<i>y</i>	00	01	11	10
0	0	0	1	0
1	1	0	1	1

(a)  $Y = DG + G'y$

	<i>DG</i>			
<i>y</i>	00	01	11	10
0	0	0	1	0
1	1	0	1	1

(b)  $Q = Y$



# 9-4 Design Procedure

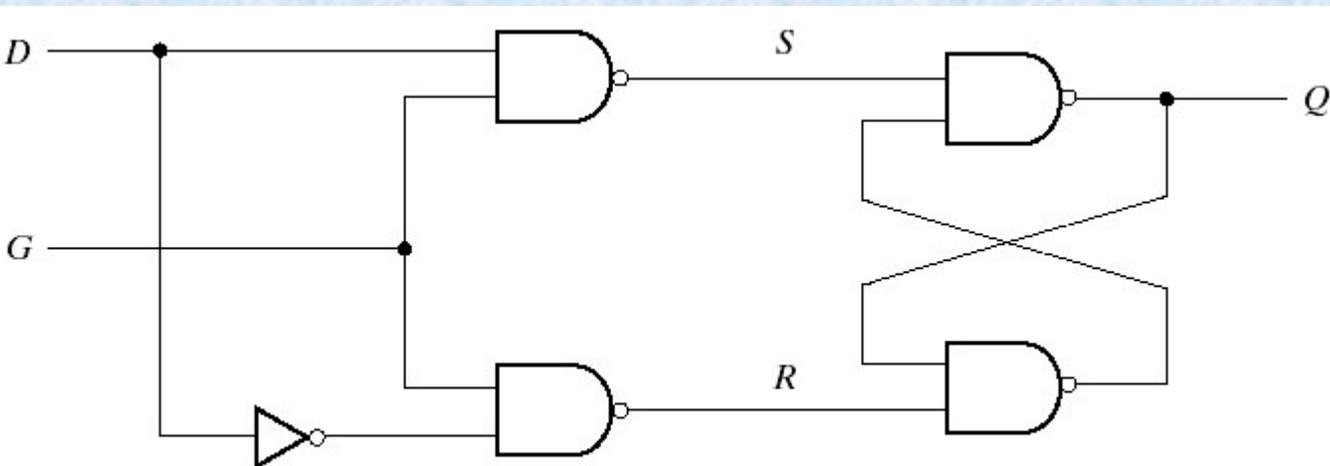
## Circuit With *SR* Latch

		<i>DG</i>			
<i>y</i>		00	01	11	10
0	0	0	0	1	0
1	<i>X</i>	0	<i>X</i>	<i>X</i>	<i>X</i>

(a)  $S = DG$

		<i>DG</i>			
<i>y</i>		00	01	11	10
0	<i>X</i>	<i>X</i>	<i>X</i>	0	<i>X</i>
1	0	1	0	0	0

$R = D'G$



# 9-4 Design Procedure

## Assigning Output to Unstable States

1. Assign a **0** to an output variable associated with an unstable state that is a transient state between two stable states that have a **0** in the corresponding output variable.
2. Assign a **1** to an output variable associated with an unstable state that is a transient state between two stable states that have a **1** in the corresponding output variable.
3. Assign a **don't-care** condition to an output variable associated with an unstable state that is a transient state between two stable states that have **different values** in the corresponding output variable.



## 9-5 Reduction of State and Flow Tables

### Equivalent States

Two states are equivalent if for each possible input, they give exactly the same output and go to the same next states or to equivalent next states.

The characteristic of equivalent states is that if  $(a,b)$  imply  $(c,d)$  and  $(c,d)$  imply  $(a,b)$ , then both pairs of states are equivalent.



## 9-5 Reduction of State and Flow Tables

### Implication Table

Two states are equivalent if for each possible input, they give exactly the same output and go to the same next states or to equivalent next states.

The characteristic of equivalent states is that if  $(a,b)$  imply  $(c,d)$  and  $(c,d)$  imply  $(a,b)$ , then both pairs of states are equivalent.



# 9-5 Reduction of State and Flow Tables

## Implication

Without the first

### Example

$S_n \backslash X_1 X_2$	00	01	11	10
A	D/0	D/0	F/0	A/0
B	C/1	D/0	E/1	F/0
C	C/1	D/0	E/1	A/0
D	D/0	B/0	A/0	F/0
E	C/1	F/0	E/1	A/0
F	D/0	D/0	A/0	F/0
G	G/0	G/0	A/0	A/0
H	B/1	D/0	E/1	A/0

$S_{n+1}/Z_n$

B	×						
C	×	AF					
D	BD AF	×	×				
E	×	DF AF	DF	×			
F	√	×	×	BD	×		
G	DG AF	×	×	BG AF	×	DG AF	
H	×	BC AF	BC	×	BC DF	×	×
	A	B	C	D	E	F	G

Without the last



# 9-5 Reduction of State and Flow Tables

## Implied Pairs

example

$S_n \backslash X_1 X_2$	00	01	11	10
A	D/0	D/0	F/0	A/0
B	C/1	D/0	E/1	F/0
C	C/1	D/0	E/1	A/0
D	D/0	B/0	A/0	F/0
E	C/1	F/0	E/1	A/0
F	D/0	D/0	A/0	F/0
G	G/0	G/0	A/0	A/0
H	B/1	D/0	E/1	A/0

$S_{n+1}/Z_n$

B	×						
C	×	AF					
D	BD AF	×	×				
E	×	DF AF	DF	×			
F	√	×	×	BD	×		
G	DG AF	×	×	BG AF	×	DG AF	
H	×	BC AF	BC	×	BC DF	×	×
	A	B	C	D	E	F	G



# 9-5 Reduction of State and Flow Tables

The equivalent states:  $[A, F]$ ,  $[B, H]$ ,  $[B, C]$ ,  $[C, H]$ .

Denoted by A

Denoted by B

equivalent states  
 $[A, F]$ ,  $[B, C, H]$ ,  
 $[D]$ ,  $[E]$ ,  $[G]$

$S_n \backslash X_1 X_2$	00	01	11	10
A	D/0	D/0	A/0	A/0
B	C/1	D/0	E/1	A/0
D	D/0	B/0	A/0	A/0
E	B/1	A/0	E/1	A/0
G	G/0	G/0	A/0	A/0

$S_{n+1}/Z_n$

B	×						
C	×	AF					
D	BD AF	×	×				
E	×	DF AF	DF	×			
F	√	×	×	BD	×		
G	DG AF	×	×	BG AF	×	×	DG AF
H	×	BC AF	BC	×	BC DF	×	×
	A	B	C	D	E	F	G

Combined into  $[B,C,H]$



## 9-6 Race-Free State Assignment

The primary objective in choosing a proper binary state assignment is the **prevention of critical races**.

Critical races can be avoided by making a binary state assignment in such a way that **only one variable changes** at any given time when a state transition occurs in the flow table.

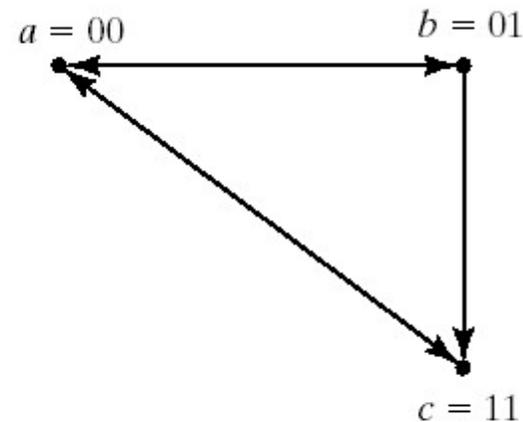


# 9-6 Race-Free State Assignment

## Three-Row Flow-Table Example

		$x_1 x_2$			
		00	01	11	10
00	<i>a</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>a</i>
01	<i>b</i>	<i>a</i>	<i>b</i>	<i>b</i>	<i>c</i>
11	<i>c</i>	<i>a</i>	<i>c</i>	<i>c</i>	<i>c</i>

(a) Flow table



(b) Transition diagram

**This assignment will cause a critical race during the transition from  $a$  to  $c$ .**

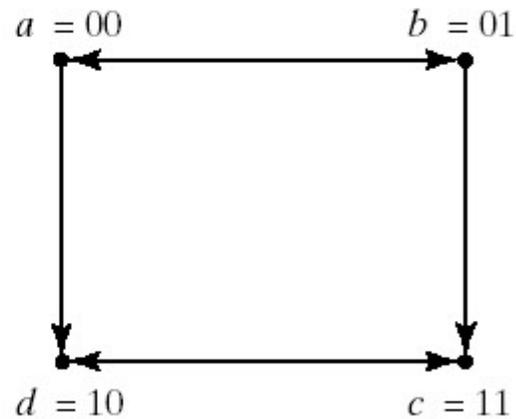


# 9-6 Race-Free State Assignment

## Three-Row Flow-Table Example

		$x_1x_2$			
		00	01	11	10
00	$a$	$a$	$b$	$d$	$a$
01	$b$	$a$	$b$	$b$	$c$
11	$c$	$d$	$c$	$c$	$c$
10	$d$	$a$	-	$c$	-

(a) Flow table



**The transition from  $a$  to  $c$  must now go through  $d$ , thus avoiding a critical race.**

# 9-6 Race-Free State Assignment

## Multiple-Row Method

In the multiple-row assignment, each state in the original flow table is replaced by two or more combinations of state variables.

		$y_2 y_3$			
		00	01	11	10
$y_1$	0	$a_1$	$b_1$	$c_1$	$d_1$
	1	$c_2$	$d_2$	$a_2$	$b_2$

(a) Binary assignment

Note that  $a_2$  is adjacent to  $d_2$ ,  $c_1$ ,  $b_2$ .



# 9-6 Race-Free State Assignment

## Multiple-Row Method

	00	01	11	10
<i>a</i>	<i>b</i>	<i>a</i>	<i>d</i>	<i>a</i>
<i>b</i>	<i>b</i>	<i>d</i>	<i>b</i>	<i>a</i>
<i>c</i>	<i>c</i>	<i>a</i>	<i>b</i>	<i>c</i>
<i>d</i>	<i>c</i>	<i>d</i>	<i>d</i>	<i>c</i>

(a) Flow table

The original flow table



	00	01	11	10
000 = <i>a</i> <sub>1</sub>	<i>b</i> <sub>1</sub>	<i>a</i> <sub>1</sub>	<i>d</i> <sub>1</sub>	<i>a</i> <sub>1</sub>
111 = <i>a</i> <sub>2</sub>	<i>b</i> <sub>2</sub>	<i>a</i> <sub>2</sub>	<i>d</i> <sub>2</sub>	<i>a</i> <sub>2</sub>
001 = <i>b</i> <sub>1</sub>	<i>b</i> <sub>1</sub>	<i>d</i> <sub>2</sub>	<i>b</i> <sub>1</sub>	<i>a</i> <sub>1</sub>
110 = <i>b</i> <sub>2</sub>	<i>b</i> <sub>2</sub>	<i>d</i> <sub>1</sub>	<i>b</i> <sub>2</sub>	<i>a</i> <sub>2</sub>
011 = <i>c</i> <sub>1</sub>	<i>c</i> <sub>1</sub>	<i>a</i> <sub>2</sub>	<i>b</i> <sub>1</sub>	<i>c</i> <sub>1</sub>
100 = <i>c</i> <sub>2</sub>	<i>c</i> <sub>2</sub>	<i>a</i> <sub>1</sub>	<i>b</i> <sub>2</sub>	<i>c</i> <sub>2</sub>
010 = <i>d</i> <sub>1</sub>	<i>c</i> <sub>1</sub>	<i>d</i> <sub>1</sub>	<i>d</i> <sub>1</sub>	<i>c</i> <sub>1</sub>
101 = <i>d</i> <sub>2</sub>	<i>c</i> <sub>2</sub>	<i>d</i> <sub>2</sub>	<i>d</i> <sub>2</sub>	<i>c</i> <sub>2</sub>

(b) Flow table



## 9-7 Hazards

**Hazards are unwanted switching transients that may appear at the output of a circuit because different paths exhibit different propagation delays.**

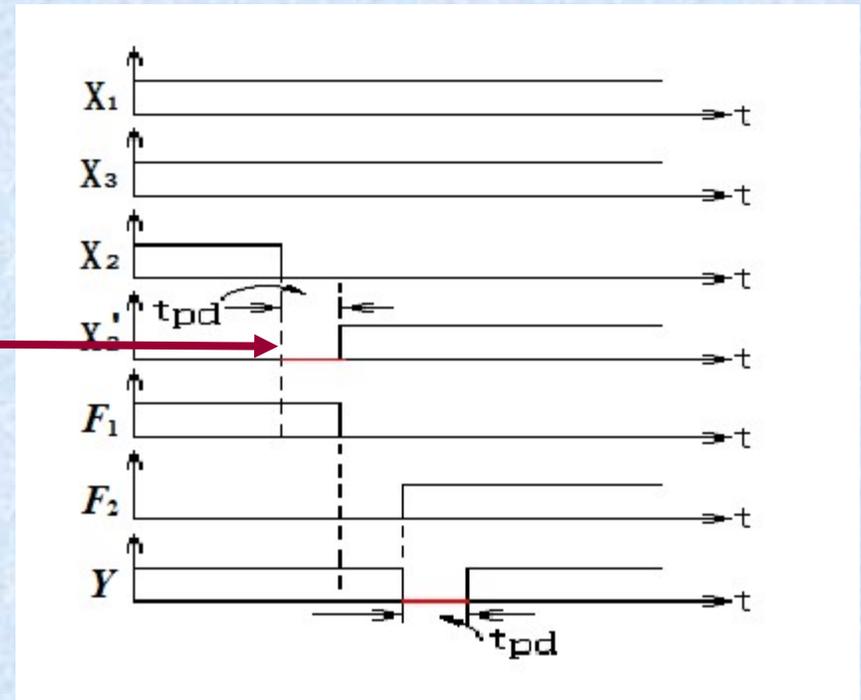
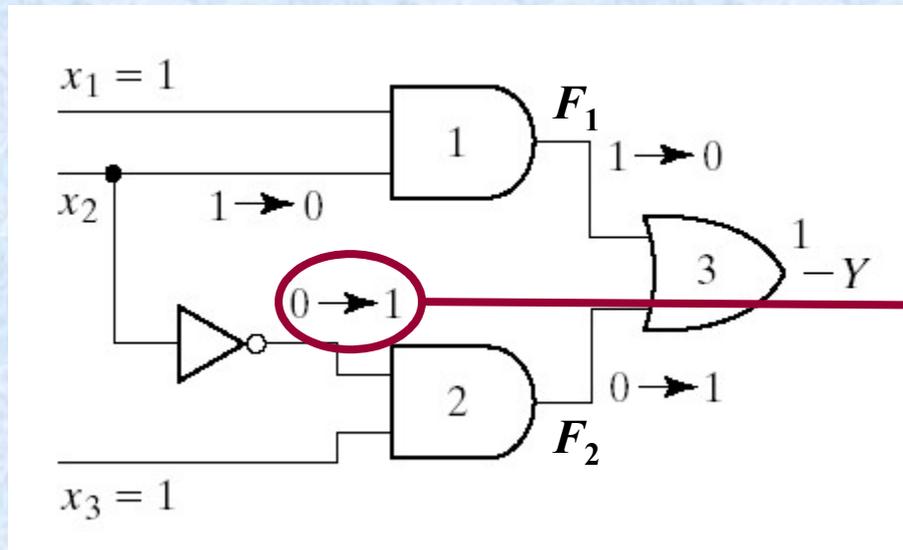
**Hazards occur in combinational circuits, where they may cause a temporary false-output value.**

**When hazards occur in sequential circuits, it may result in a transition to a wrong stable state.**



# 9-7 Hazards

## Hazards in Combinational Circuits



## 9-7 Hazards

### Hazards in Combinational Circuits



(a) Static 1-hazard



(b) Static 0-hazard

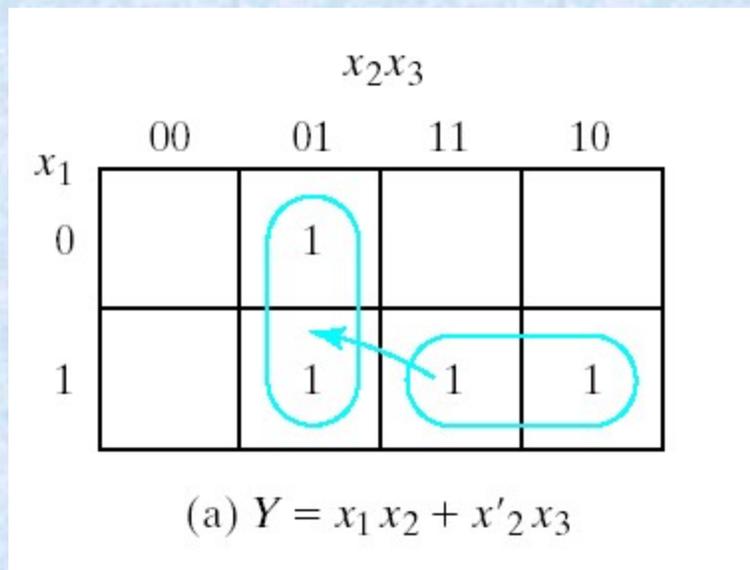


(c) Dynamic hazard



## 9-7 Hazards

### Hazards in Combinational Circuits

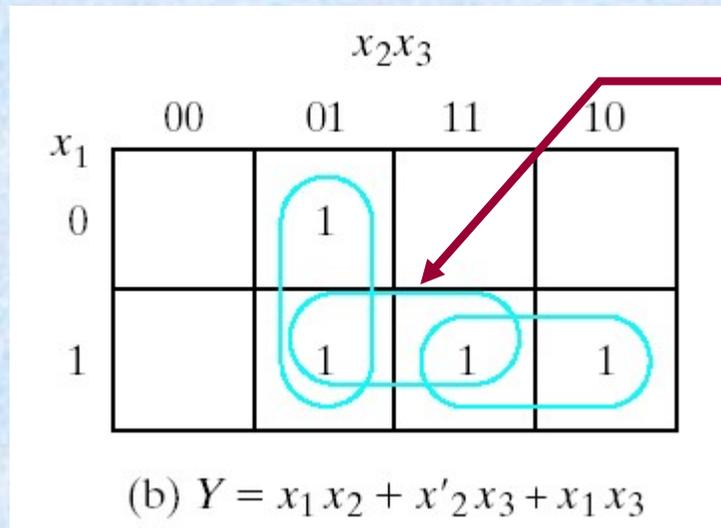


The hazard exists because the change of input results in a different product term covering the two minterms.



## 9-7 Hazards

### Hazards in Combinational Circuits



The remedy for eliminating a hazard is to enclose the two minterms in question with another product term that overlap both grouping.



## 9-7 Hazards

### Hazards in Combinational Circuits

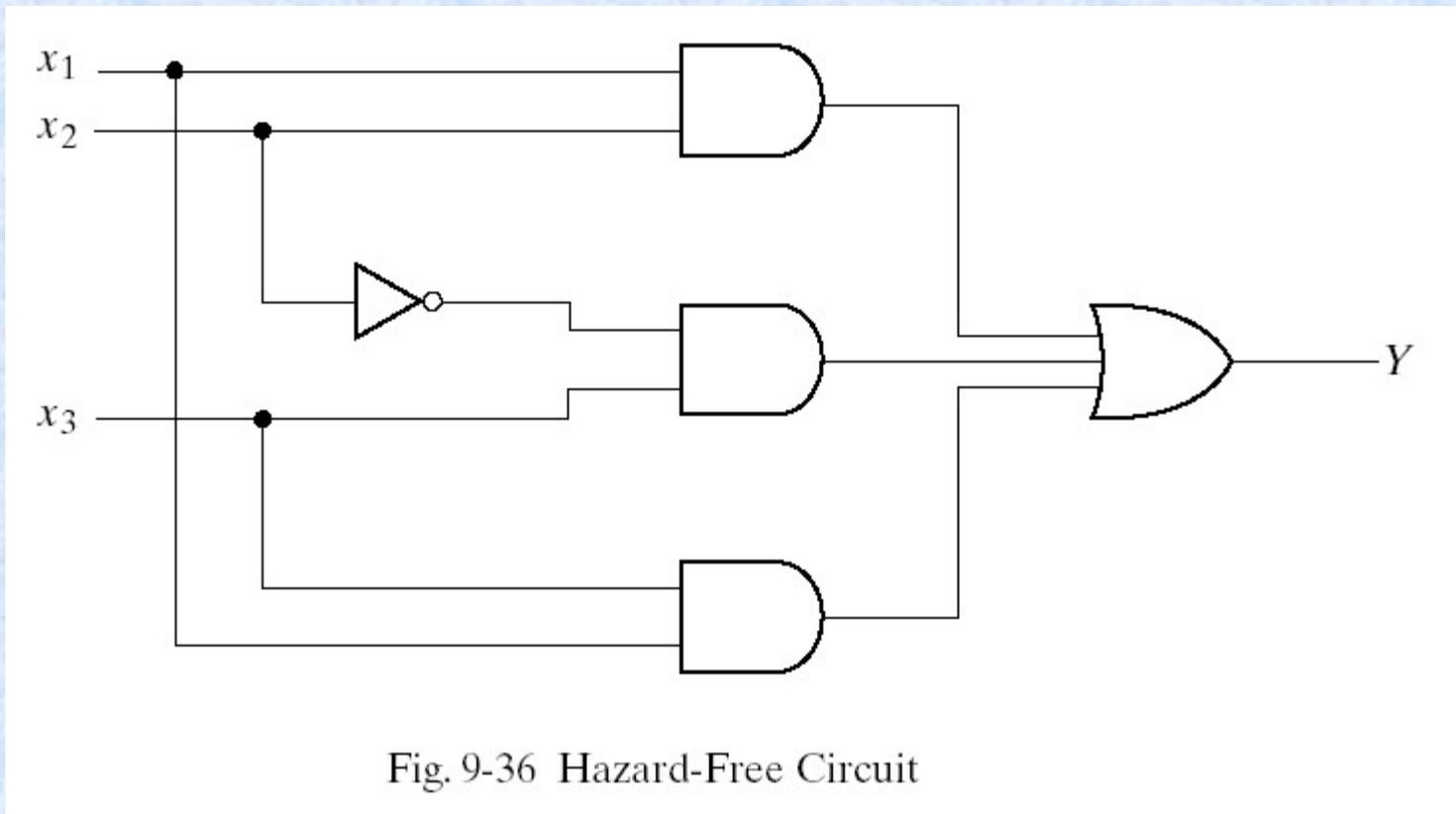


Fig. 9-36 Hazard-Free Circuit



## 9-7 Hazards

### Implementation with SR Latches

Consider a NAND SR Latch  
Boolean functions for

$$S = AB + CD$$

$$R = A'C$$

$$S = (AB + CD)' = (AB)' (CD)'$$

$$R = (A'C)'$$

$$Q = (Q'S)' = [Q' (AB)' (CD)' ]'$$

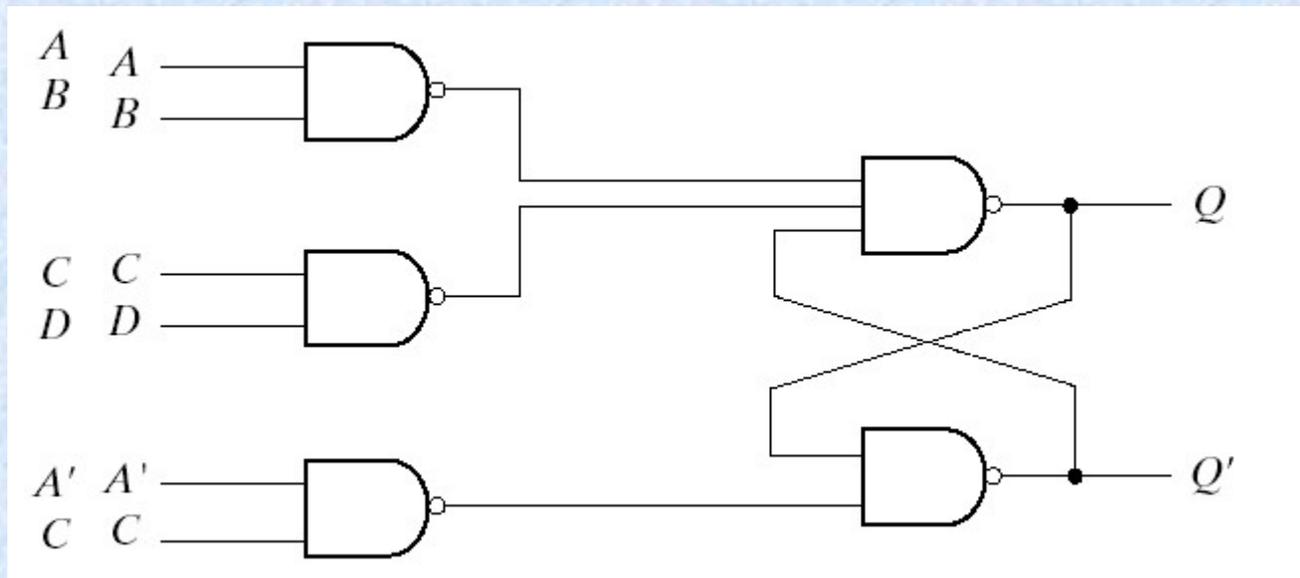
Since this is a NAND latch,  
we apply the complemented  
values to the inputs:



## 9-7 Hazards

### Implementation with SR Latches

$$Q = (Q'S)' = [Q' (AB)' (CD)']'$$



## 9-8 Design Example

### The Recommended Procedure

1. State the design specifications
2. Derive a primitive flow table
3. Reduce the flow table by merging the rows
4. Make a race-free binary state assignment
5. Obtain the transition table and output map
6. Obtain the logic diagram using *SR* latch



## 9-8 Design Example

### Design Specifications

It is necessary to design a negative-edge-triggered flip-flop. The circuit has two inputs,  $T$  (toggle) and  $C$  (clock), and one output,  $Q$ .



## 9-8 Design Example

### Primitive Flow Table

Specification of  
Total States

State	Inputs		Output	Comments
	$T$	$C$	$Q$	
$a$	1	1	0	Initial input is 0
$b$	1	0	1	After state $a$
$c$	1	1	1	Initial input is 1
$d$	1	0	0	After state $c$
$e$	0	0	0	After state $d$ or $f$
$f$	0	1	0	After state $e$ or $a$
$g$	0	0	1	After state $b$ or $h$
$h$	0	1	1	After state $g$ or $c$



## Primitive Flow Table

## 9-8 Design Example

## Primitive Flow Table

	TC			
	00	01	11	10
a	- , -	f , -	a , 0	b , -
b	g , -	- , -	c , -	b , 1
c	- , -	h , -	c , 1	d , -
d	e , -	- , -	a , -	d , 0
e	e , 0	f , -	- , -	d , -
f	e , -	f , 0	a , -	- , -
g	g , 1	h , -	- , -	b , -
h	g , -	h , 1	c , -	- , -



## 9-8 Design Example

### Implication Table

<i>b</i>	<i>a, c</i> ×						
<i>c</i>	×	<i>b, d</i> ×					
<i>d</i>	<i>b, d</i> ×		×	<i>a, c</i> ×			
<i>e</i>	<i>b, d</i> ×	<i>e, g</i> × <i>b, d</i> ×	<i>f, h</i> ×	✓			
<i>f</i>	✓	<i>e, g</i> × <i>a, c</i> ×	<i>f, h</i> × <i>a, c</i> ×	✓	✓		
<i>g</i>	<i>f, h</i> ×	✓	<i>b, d</i> ×	<i>e, g</i> × <i>b, d</i> ×	×	<i>e, g</i> × <i>f, h</i> ×	
<i>h</i>	<i>f, h</i> × <i>a, c</i> ×	✓	✓	<i>d, e</i> × <i>c, f</i> ×	<i>e, g</i> × <i>f, h</i> ×	×	✓
	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>

### Merging the Flow Table

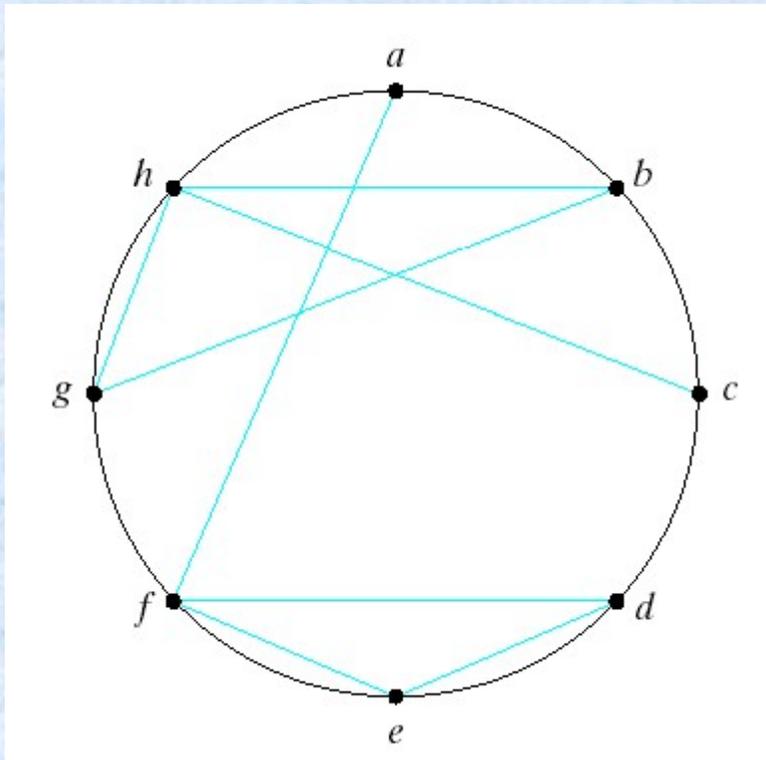
The compatible pairs:

$(a, f)$   $(b, g)$   $(b, h)$   
 $(c, h)$   $(d, e)$   $(d, f)$   
 $(e, f)$   $(g, h)$



## 9-8 Design Example

### Merging the Flow Table



The maximal compatible set:

$(a, f)$   $(b, g, h)$   $(c, h)$   
 $(d, e, f)$



# 9-8 Design Example

## Merging the Flow Table

Reduced flow table

	TC			
	00	01	11	10
<i>a, f</i>	<i>e</i> , -	<i>f</i> , 0	<i>a</i> , 0	<i>b</i> , -
<i>b, g, h</i>	<i>g</i> , 1	<i>h</i> , 1	<i>c</i> , -	<i>b</i> , 1
<i>c, h</i>	<i>g</i> , 1	<i>h</i> , 1	<i>c</i> , 1	<i>d</i> , -
<i>d, e, f</i>	<i>e</i> , 0	<i>f</i> , 0	<i>a</i> , -	<i>d</i> , 0

(a)

	TC			
	00	01	11	10
<i>a</i>	<i>d</i> , -	<i>a</i> , 0	<i>a</i> , 0	<i>b</i> , -
<i>b</i>	<i>b</i> , 1	<i>b</i> , 1	<i>c</i> , -	<i>b</i> , 1
<i>c</i>	<i>b</i> , -	<i>c</i> , 1	<i>c</i> , 1	<i>d</i> , -
<i>d</i>	<i>d</i> , 0	<i>d</i> , 0	<i>a</i> , -	<i>d</i> , 0

(b)



# 9-8 Design Example

## State Assignment and Transition Table

		<i>TC</i>			
		00	01	11	10
<i>a</i>	<i>y</i> <sub>1</sub> <i>y</i> <sub>2</sub>				
	<i>a</i> = 00	10	00	00	01
	<i>b</i> = 01	01	01	11	01
	<i>c</i> = 11	01	11	11	10

		<i>TC</i>			
		00	01	11	10
<i>d</i>	<i>y</i> <sub>1</sub> <i>y</i> <sub>2</sub>				
	00	0	0	0	<i>X</i>
	01	1	1	1	1
	11	1	1	1	<i>X</i>
	10	0	0	0	0

(a) Transition table

(b) Output map  $Q = y_2$



# 9-8 Design Example

## Logic Diagram

		<i>TC</i>			
		00	01	11	10
<i>y</i> <sub>1</sub> <i>y</i> <sub>2</sub>	00	1	0	0	0
	01	0	0	1	0
	11	0	X	X	X
	10	X	X	0	X

(a)  $S_1 = y_2 TC + y_2' TC'$

		<i>TC</i>			
		00	01	11	10
<i>y</i> <sub>1</sub> <i>y</i> <sub>2</sub>	00	0	X	X	X
	01	X	X	0	X
	11	1	0	0	0
	10	0	0	1	0

(b)  $R_1 = y_2 TC' + y_2' TC$

		<i>TC</i>			
		00	01	11	10
<i>y</i> <sub>1</sub> <i>y</i> <sub>2</sub>	00	0	0	0	1
	01	X	X	X	X
	11	X	X	X	0
	10	0	0	0	0

(c)  $S_2 = y_1' TC'$

		<i>TC</i>			
		00	01	11	10
<i>y</i> <sub>1</sub> <i>y</i> <sub>2</sub>	00	X	X	X	0
	01	0	0	0	0
	11	0	0	0	1
	10	X	X	X	X

(d)  $R_2 = y_1 TC'$



# 9-8 Design Example

## Logic Diagram

