Induction Motors Suggested Problems

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6-3. A three-phase 60-Hz induction motor runs at 715 r/min at no load and at 670 r/min at full load.

(a) How many poles does this motor have?

(b) What is the slip at rated load?

(c) What is the speed at one-quarter of the rated load?

(d) What is the rotor's electrical frequency at one-quarter of the rated load?

SOLUTION

(a) This machine has 10 poles, which produces a synchronous speed of

$$n_{\text{sync}} = \frac{120 f_e}{P} = \frac{120(60\text{ Hz})}{10} = 720\text{ r/min}$$

(b) The slip at rated load is

$$s = \frac{n_{\text{sync}} - n_m}{n_{\text{sync}}} \times 100\% = \frac{720 - 670}{720} \times 100\% = 6.94\%$$

(c) The motor is operating in the linear region of its torque-speed curve, so the slip at ¼ load will be

$$s = 0.25(0.0694) = 0.0171$$

The resulting speed is

$$n_m = (1 - s)\ n_{\text{sync}} = (1 - 0.0171)(720\text{ r/min}) = 708\text{ r/min}$$

(d) The electrical frequency at ¼ load is

$$f_r = s f_e = (0.0171)(60\text{ Hz}) = 1.03\text{ Hz}$$

6-4. A 50-kW, 460-V, 50-Hz, two-pole induction motor has a slip of 5 percent when operating a full-load conditions. At full-load conditions, the friction and windage losses are 700 W, and the core losses are 600 W. Find the following values for full-load conditions:

(a) The shaft speed n_m

(b) The output power in watts

(c) The load torque τ_{load} in newton-meters

(d) The induced torque τ_{ind} in newton-meters

(e) The rotor frequency in hertz

SOLUTION

(a) The synchronous speed of this machine is

$$n_{\text{sync}} = \frac{120 f_{se}}{P} = \frac{120(50\text{ Hz})}{2} = 3000\text{ r/min}$$

Therefore, the shaft speed is

$$n_m = (1-s)\; n_{\text{sync}} = (1-0.05)(3000\text{ r/min}) = 2850\text{ r/min}$$

(b) The output power in watts is 50 kW (stated in the problem).

(c) The load torque is

$$\tau_{\text{load}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{50 \text{ kW}}{(2850 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 167.5 \text{ N}\cdot\text{m}$$

(d) The induced torque can be found as follows:

$$P_{\text{conv}} = P_{\text{OUT}} + P_{\text{F\&W}} + P_{\text{core}} + P_{\text{misc}} = 50 \text{ kW} + 700 \text{ W} + 600 \text{ W} + 0 \text{ W} = 51.3 \text{ kW}$$

$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m} = \frac{51.3 \text{ kW}}{(2850 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 171.9 \text{ N}\cdot\text{m}$$

(e) The rotor frequency is

$$f_r = sf_e = (0.05)(50 \text{ Hz}) = 2.5 \text{ Hz}$$

6-10. A three-phase 60-Hz two-pole induction motor runs at a no-load speed of 3580 r/min and a full-load speed of 3440 r/min. Calculate the slip and the electrical frequency of the rotor at no-load and full-load conditions. What is the speed regulation of this motor [Equation (3-68)]?

SOLUTION The synchronous speed of this machine is 3600 r/min. The slip and electrical frequency at no-load conditions is

$$s_{\text{nl}} = \frac{n_{\text{sync}} - n_{\text{nl}}}{n_{\text{sync}}} \times 100\% = \frac{3600 - 3580}{3600} \times 100\% = 0.56\%$$

$$f_{r,\text{nl}} = sf_e = (0.0056)(60 \text{ Hz}) = 0.33 \text{ Hz}$$

The slip and electrical frequency at full load conditions is

$$s_{\text{fl}} = \frac{n_{\text{sync}} - n_{\text{nl}}}{n_{\text{sync}}} \times 100\% = \frac{3600 - 3440}{3600} \times 100\% = 4.44\%$$

$$f_{r,\text{fl}} = sf_e = (0.0444)(60 \text{ Hz}) = 2.67 \text{ Hz}$$

The speed regulation is

$$\text{SR} = \frac{n_{\text{nl}} - n_{\text{fl}}}{n_{\text{fl}}} \times 100\% = \frac{3580 - 3440}{3440} \times 100\% = 4.07\%$$

6-15. A 460-V 60-Hz four-pole Y-connected induction motor is rated at 25 hp. The equivalent circuit parameters are

$R_1 = 0.15\ \Omega$ $R_2 = 0.154\ \Omega$ $X_M = 20\ \Omega$

$X_1 = 0.852\ \Omega$ $X_2 = 1.066\ \Omega$

$P_{\text{F\&W}} = 400\text{ W}$ $P_{\text{misc}} = 150\text{ W}$ $P_{\text{core}} = 400\text{ W}$

For a slip of 0.02, find

(a) The line current I_L

(b) The stator power factor

(c) The rotor power factor

(d) The rotor frequency

(e) The stator copper losses P_{SCL}

(f) The air-gap power P_{AG}

(g) The power converted from electrical to mechanical form P_{conv}

(h) The induced torque τ_{ind}

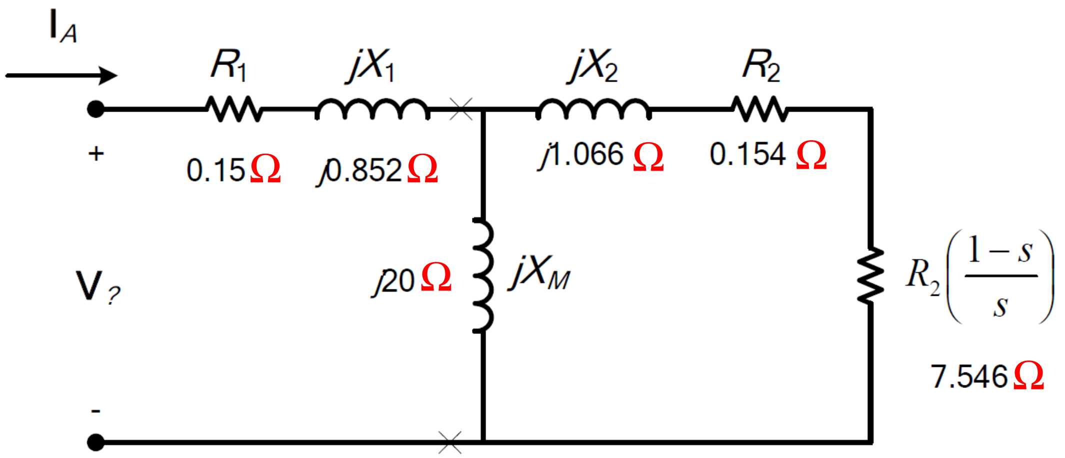
(i) The load torque τ_{load}

(j) The overall machine efficiency η

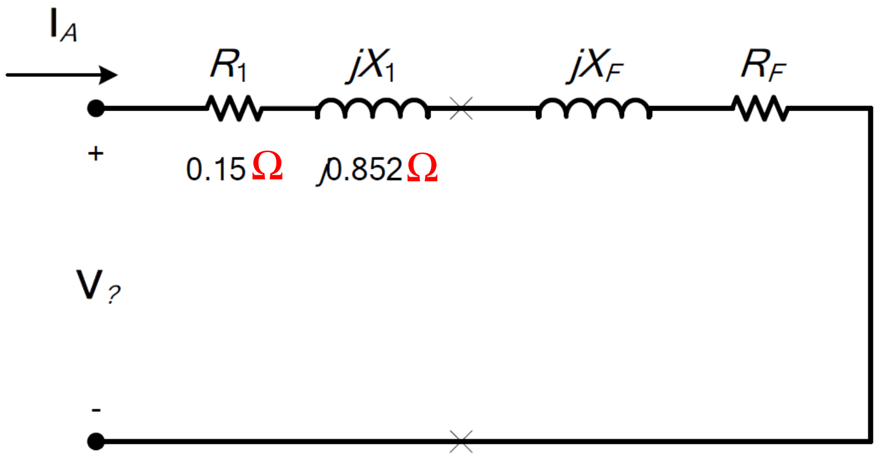
(k) The motor speed in revolutions per minute and radians per second

(l) What is the starting code letter for this motor?

SOLUTION The equivalent circuit of this induction motor is shown below:



(a) The easiest way to find the line current (or armature current) is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M, and then calculate the current as the phase voltage divided by the sum of the series impedances, as shown below.



The equivalent impedance of the rotor circuit in parallel with jX_M is:

$$Z_F = \frac{1}{\frac{1}{jX_M} + \frac{1}{Z_2}} = \frac{1}{\frac{1}{j20\ \Omega} + \frac{1}{7.70 + j1.066}} = 6.123 + j3.25 = 6.932\angle 28.0°\ \Omega$$

The phase voltage is $460/\sqrt{3} = 266$ V, so line current I_L is

$$I_L = I_A = \frac{V_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{266\angle 0° \text{ V}}{0.15\ \Omega\ + j0.852\ \Omega + 6.123\ \Omega + j3.25\ \Omega}$$

$$I_L = I_A = 35.5\angle -33.2° \text{ A}$$

(b) The stator power factor is

$$\text{PF} = \cos(33.2°) = 0.837 \text{ lagging}$$

(c) To find the rotor power factor, we must find the impedance angle of the rotor

$$\theta_R = \tan^{-1}\frac{X_2}{R_2 / s} = \tan^{-1}\frac{1.066}{7.70} = 7.88°$$

(d) The rotor frequency is

$$f_r = sf_s = (0.02)(60 \text{ Hz}) = 1.2 \text{ Hz}$$

Therefore the rotor power factor is

$$\text{PF}_R = \cos 7.88° = 0.991 \text{ lagging}$$

(e) The stator copper losses are

$$P_{\text{SCL}} = 3I_A{}^2 R_1 = 3(35.5 \text{ A})^2 (0.15\ \Omega) = 567 \text{ W}$$

(f) The air gap power is $P_{AG} = 3I_2^2 \frac{R_2}{s} = 3I_A^2 R_F$

(Note that $3I_A^2 R_F$ is equal to $3I_2^2 \frac{R_2}{s}$, since the only resistance in the original rotor circuit was R_2/s, and the resistance in the Thevenin equivalent circuit is R_F. The power consumed by the Thevenin equivalent circuit must be the same as the power consumed by the original circuit.)

$$P_{AG} = 3I_2^2 \frac{R_2}{s} = 3I_A^2 R_F = 3(35.5 \text{ A})^2 (6.123 \ \Omega) = 23.15 \text{ kW}$$

(g) The power converted from electrical to mechanical form is

$$P_{conv} = (1-s) P_{AG} = (1-0.02) \ (23.15 \text{ kW}) = 22.69 \text{ kW}$$

(h) The synchronous speed of this motor is

$$n_{sync} = \frac{120 f_{se}}{P} = \frac{120(60 \text{ Hz})}{4} = 1800 \text{ r/min}$$

$$\omega_{sync} = (1800 \text{ r/min}) \left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 188.5 \text{ rad/s}$$

Therefore the induced torque in the motor is

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}} = \frac{23.15 \text{ kW}}{(1800 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 122.8 \text{ N} \cdot \text{m}$$

(i) The output power of this motor is

$$P_{\text{OUT}} = P_{\text{conv}} - P_{\text{mech}} - P_{\text{core}} - P_{\text{misc}} = 22.69 \text{ kW} - 400 \text{ W} - 400 \text{ W} - 150 \text{ W} = 21.74 \text{ kW}$$

The output speed is

$$n_m = (1-s)\ n_{\text{sync}} = (1-0.02)\ (1800 \text{ r/min}) = 1764 \text{ r/min}$$

Therefore the load torque is

$$\tau_{\text{load}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{21.74 \text{ kW}}{(1764 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 117.7 \text{ N}\cdot\text{m}$$

(j) The overall efficiency is

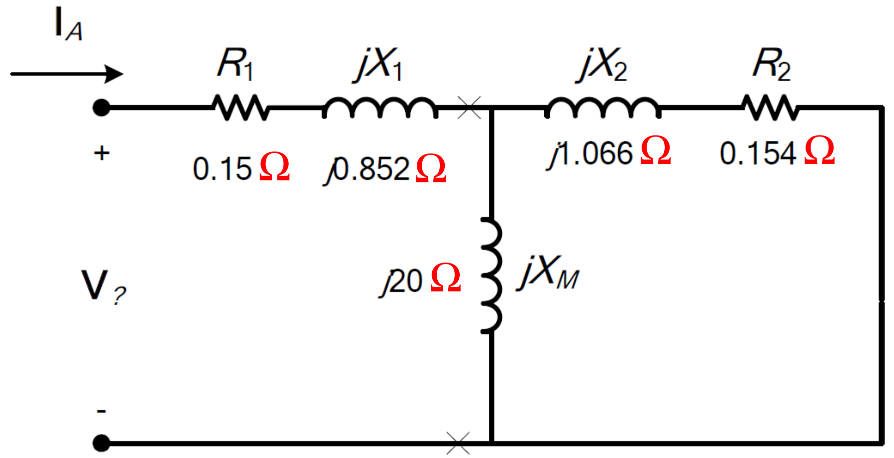
$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{P_{\text{OUT}}}{3V_\phi I_A \cos\theta} \times 100\%$$

$$\eta = \frac{21.74 \text{ kW}}{3(266 \text{ V})(35.5 \text{ A})\cos(33.2°)} \times 100\% = 91.7\%$$

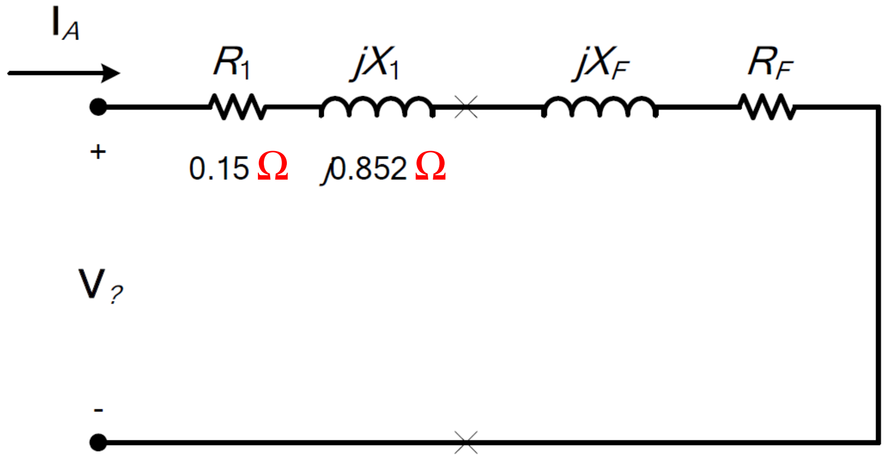
(k) The motor speed in revolutions per minute is 1764 r/min. The motor speed in radians per second is

$$\omega_m = (1764 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 184.7 \text{ rad/s}$$

(l) The equivalent circuit of this induction motor *at starting conditions* is shown below:



The easiest way to find the line current (or armature current) is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M, and then calculate the current as the phase voltage divided by the sum of the series impedances, as shown below.



The equivalent impedance of the rotor circuit in parallel with jX_M is:

$$Z_F = \frac{1}{\frac{1}{jX_M} + \frac{1}{Z_2}} = \frac{1}{\frac{1}{j20\ \Omega} + \frac{1}{0.154 + j1.066}} = 0.139 + j1.013 = 1.023\angle 82.2°\ \Omega$$

The equivalent impedance of the rotor circuit in parallel with jX_M is:

$$Z_F = \frac{1}{\frac{1}{jX_M} + \frac{1}{Z_2}} = \frac{1}{\frac{1}{j20\ \Omega} + \frac{1}{0.154 + j1.066}} = 0.139 + j1.013 = 1.023\angle 82.2°\ \Omega$$

The phase voltage is $460/\sqrt{3} = 266$ V, so line current I_L is

$$I_L = I_A = \frac{V_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{266\angle 0°\ \text{V}}{0.15\ \Omega + j0.852\ \Omega + 0.139\ \Omega + j1.023\ \Omega}$$

$$I_L = I_A = 140.2\angle -81.2°\ \text{A}$$

The starting kVA of the motor is

$$S_{\text{start}} = 3V_\phi I_A = 3(266\ \text{V})(140\ \text{A}) = 111.7\ \text{kVA}$$

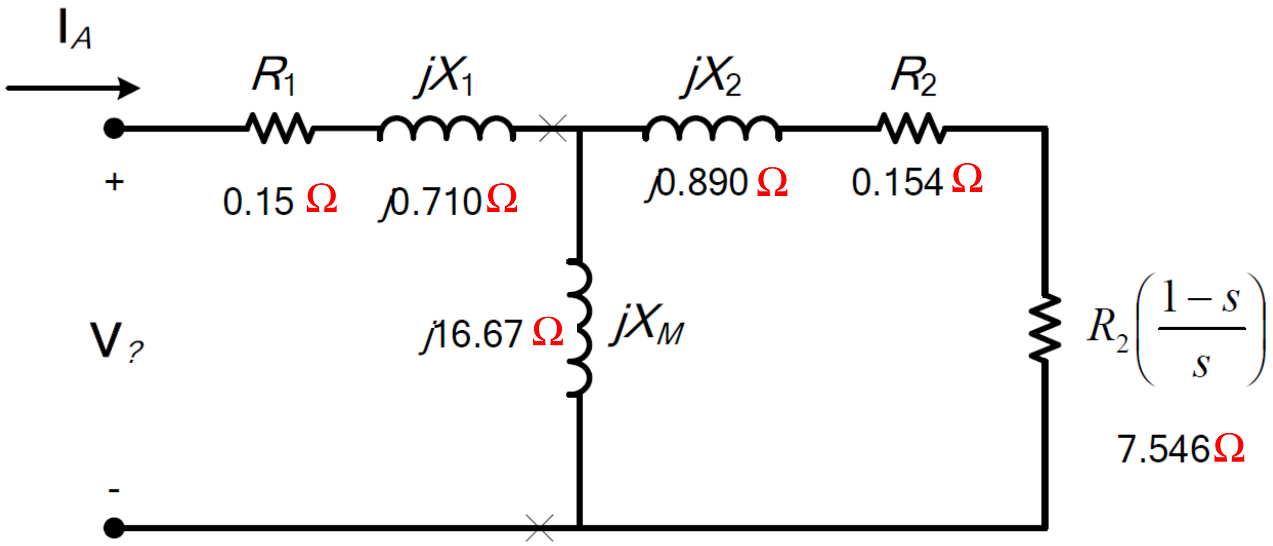
The locked rotor kVA/hp is

$$\text{kVA/hp} = \frac{111.7\ \text{kVA}}{25\ \text{hp}} = 4.47$$

Therefore this motor is Starting Code Letter D. To be obtained from NEMA Table

6-17. If the motor in Problem 6-15 is to be driven from a 460-V 50-Hz power supply, what will the pullout torque be? What will the slip be at pullout?

SOLUTION If this motor is driven from a 50 Hz source, the resistances will be unchanged and the reactances will be increased by a ratio of 5/6. The resulting equivalent circuit is shown below.



The phase voltage must be derated by 5/6, so $V_\phi = 212.5\text{ V}$.

The slip at pullout torque is found by calculating the Thevenin equivalent of the input circuit from the rotor back to the power supply, and then using that with the rotor circuit model.

$$Z_{TH} = \frac{jX_M(R_1 + jX_1)}{R_1 + j(X_1 + X_M)} = \frac{(j16.67\ \Omega)(0.15\ \Omega + j0.710\ \Omega)}{0.15\ \Omega + j(0.710\ \Omega + 16.67\ \Omega)} = 0.138 + j0.6822\ \Omega = 0.696\angle 78.6^\circ\ \Omega$$

$$\mathbf{V}_{TH} = \frac{jX_M}{R_1 + j(X_1 + X_M)}\mathbf{V}_\phi = \frac{(j16.67\ \Omega)}{0.15\ \Omega + j(0.710\ \Omega + 16.67\ \Omega)}(212.5\angle 0^\circ\text{ V}) = 203\angle 0.49^\circ\text{ V}$$

The slip at pullout torque is

$$s_{\max} = \frac{R_2}{\sqrt{R_{TH}{}^2 + (X_{TH} + X_2)^2}}$$

$$s_{\max} = \frac{0.154\ \Omega}{\sqrt{(0.138\ \Omega)^2 + (0.6822\ \Omega\ + 0.890\ \Omega)^2}} = 0.0976$$

The synchronous speed of this motor is

$$n_{sync} = \frac{120 f_e}{P} = \frac{120(50\text{ Hz})}{4} = 1500\text{ r/min}$$

$$\omega_{sync} = (1500\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 157.1\text{ rad/s}$$

This corresponds to a rotor speed of

$$n_{\max} = (1 - s_{\max})n_{sync} = (1 - 0.0976)(1500\text{ r/min}) = 1354\text{ r/min}$$

The pullout torque of the motor is

$$\tau_{\max} = \frac{3V_{TH}^2}{2\omega_{sync}\left[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}\right]}$$

$$\tau_{\max} = \frac{3(203\text{ V})^2}{2(157.1\text{ rad/s})\left[0.138\ \Omega + \sqrt{(0.138\ \Omega)^2 + (0.6822\ \Omega\ + 0.890\ \Omega)^2}\right]}$$

$$\tau_{\max} = 229.3\text{ N}\cdot\text{m}$$

Many Thanks

for

Your Attention!

Reference

- Instructor's Solutions Manual to accompany Electric Machinery Fundamentals by Stephen Chapman, 5th Ed., McGraw-Hill, Inc., 2012.