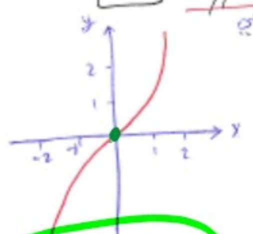
7.7

Saturday, May 29, 2021 9:52 AM

$\boxed{7.7}$ Hyperbolic Functions

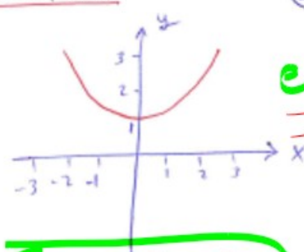
(24)

odd



$\sinh x = \dfrac{e^{x} - e^{-x}}{2}$

even +



$\cosh x = \dfrac{e^{x} + e^{-x}}{2}$

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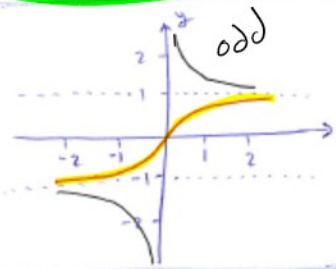
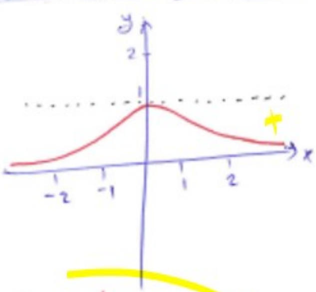
odd



$\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$

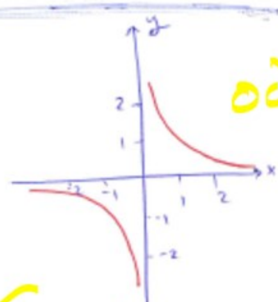
$\coth x = \dfrac{\cosh x}{\sinh x} = \dfrac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$

Even



$\operatorname{sech} x = \dfrac{1}{\cosh x} = \dfrac{2}{e^{x} + e^{-x}}$

odd



$\operatorname{csch} x = \dfrac{1}{\sinh x} = \dfrac{2}{e^{x} - e^{-x}}$

* Identities for Hyperbolic Functions

✓ • $\cosh^2 x - \sinh^2 x = 1$ $\Rightarrow$ f

• $\sinh 2x = 2 \sinh x \cosh x$

• $\cosh 2x = \cosh^2 x + \sinh^2 x$

$= 2\cosh^2 x - 1$

$= 2\sinh^2 x + 1$

• $\tanh^2 x + \operatorname{sech}^2 x = 1$

• $\coth^2 x - \operatorname{csch}^2 x = 1$

* Derivatives of Hyperbolic Functions

- $\frac{d}{dx}(\sinh u) = \cosh u \frac{du}{dx}$
- $\frac{d}{dx}(\cosh u) = \sinh u \frac{du}{dx}$ → Proof
- $\frac{d}{dx}(\tanh u) = \operatorname{sech}^2 u \frac{du}{dx}$
- $\frac{d}{dx}(\coth u) = -\operatorname{csch}^2 u \frac{du}{dx}$
- $\frac{d}{dx}(\operatorname{sech} u) = -\operatorname{sech} u \tanh u \frac{du}{dx}$
- $\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} u \coth u \frac{du}{dx}$

cosh $\frac{d}{dx}$(cosh (cot

* Integrals of Hyperbolic Functions

- $\int \sinh u \, du = \cosh u + C$
- $\int \cosh u \, du = \sinh u + C$
- $\int \operatorname{sech}^2 u \, du = \tanh u + C$
- $\int \operatorname{csch}^2 u \, du = -\coth u + C$
- $\int \operatorname{sech} u \tanh u \, du = -\operatorname{sech} u + C$
- $\int \operatorname{csch} u \coth u \, du = -\operatorname{csch} u + C$

7.7	17, 52,	1, 5, 10, 15, 22, 32, 47, 49, 54, 57

1) $\sinh x = -\frac{3}{4}$ Find the remaining values of the other hyperbolic functions

$\cosh^2 x - \sinh^2 x = 1$

$\cosh^2 x - \left(-\frac{3}{4}\right)^2 = 1 \Rightarrow \cosh^2 x - \frac{9}{16} = 1 \Rightarrow \cosh^2 x = 1 + \frac{9}{16}$

$\cosh^2 x = \frac{25}{16} \Rightarrow \boxed{\cosh x = \frac{5}{4}}$

$\tanh x = \frac{-\frac{3}{4}}{\frac{5}{4}} = -\frac{3}{5} \Rightarrow \coth x = -\frac{5}{3}$

$\operatorname{sech} x = \frac{4}{5}$

$\operatorname{csch} x = -\frac{4}{3}$

10) Simplify as much as possible

$\ln(\cosh x + \sinh x) + \ln(\cosh x - \sinh x)$

$\ln(\cosh x + \sinh x)(\cosh x - \sinh x) = \ln(\underbrace{\cosh^2 x - \sinh^2 x}_{1}) = \ln 1 = 0$

22) $y = \ln(\sinh v) - \frac{1}{2}\coth^2 v$

$-\coth v + \coth v \operatorname{csch}^2 v$

(22) $y = \ln(\sinh v) - \frac{1}{2} \ldots$

$$y' = \frac{\cosh v}{\sinh v} - \cancel{\tfrac{1}{2}(2)} \coth v \left(-\operatorname{csch}^2 v\right) = \coth v + \coth v \operatorname{csch}^2 v$$

$$= \coth v \left[1 + \operatorname{csch}^2 v\right] = \coth v \coth^2 v = \coth^3 v$$

(47) $\int \operatorname{sech}^2\left(x - \frac{1}{2}\right) dx$

$u = x - \frac{1}{2}$

$du = dx$

$$\int \operatorname{sech}^2 u \, du$$

$$= \tanh u + C = \tanh\left(x - \tfrac{1}{2}\right) + C$$

(57) $\displaystyle\int_1^2 \frac{\cosh(\ln t)}{t} \, dt$

$u = \ln t$

$du = \frac{dt}{t}$

$t = 1 \Rightarrow u = \ln 1 = 0$

$t = 2 \Rightarrow u = \ln 2$

$$\int_0^{\ln 2} \cosh u \, du = \sinh u \Big|_0^{\ln 2} = \sinh(\ln 2) - \cancel{\sinh 0} = \sinh \ln 2$$

$\sinh x = \frac{e^x - e^{-x}}{2}$

$$= \frac{e^{\ln 2} - e^{-\ln 2}}{2}$$

$$= \frac{2 - 2^{-1}}{2} = 1 - \frac{1}{4} = \frac{3}{4}$$