0.3 Integral Exponents

- For any positive integer $n \Rightarrow a^n = a \cdot a \cdot a \cdot \dots \cdot a$ (n times)

  n is called exponent

  a is called base
- Exponent is an easy way to denote certain multiplication

Exp $2^3 = 2 \cdot 2 \cdot 2 = 8$

$-3^2 = -3 \cdot 3 = -9$

$(-4)^3 = (-4) \cdot (-4) \cdot (-4) = -64$

Rules of Exponents: For any real numbers a and b and integers m and n

1. $a^m \cdot a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$ where $a \neq 0$ " denominator is non zero"
3. $(ab)^m = a^m b^m$
4. $(a^m)^n = a^{mn}$
5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ where $b \neq 0$ " denominator is non zero"
6. $a^0 = 1$ where $a \neq 0$ " since 0^0 is undefined"

7. $a^{-n} = \frac{1}{a^n}$ where $a \neq 0$ " denominator is nonzero"
8. $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ where $a \neq 0$ and $b \neq 0$

Exp write the following expressions without exponents

(1) $3^{-2} = \frac{1}{3^2} = \frac{1}{3 \cdot 3} = \frac{1}{9}$

(2) $\left(\frac{2}{5}\right)^{-1} = \left(\frac{5}{2}\right)^{1} = \frac{5^1}{2^1} = \frac{5}{2}$

(3) $4 \cdot 7^0 = 4 \cdot 1 = 4$

(4) $-5^2 = -5 \cdot 5 = -25$

(5) $(-3)^{-3} = \frac{1}{(-3)^3} = \frac{1}{(-3)(-3)(-3)} = \frac{1}{-27}$

(6) $(-4)^2 = (-4) \cdot (-4) = 16$

(7) $\left(\frac{1}{2}\right)^{-2} = (2)^2 = 2 \cdot 2 = 4$

(8) $0^5 = 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 = 0$

(9) $5^0 = 1$

(10) 0^0 is undefined "Meaningless"

(11) 0^x, $x > 0 \Rightarrow 0^x = 0$ since $x > 0$

(12) 0^{-3} is undefined "Meaningless" since $0^{-3} = \frac{1}{0^3} = \frac{1}{0 \cdot 0 \cdot 0} = \frac{1}{0}$

(13) $-5^{-2} = -\frac{1}{5^2} = -\frac{1}{5 \cdot 5} = -\frac{1}{25}$

(14) $(-2)^5 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) = -32$

(15) $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 3} = \frac{8}{27}$

Exp Simplify the following expressions so that the positive exponents remain:

(1) $3(y^2)^{-3} = 3\,y^{-6} = \frac{3}{y^6}$

(2) $x^{-4} \cdot x^{-5} = x^{-4-5} = x^{-9} = \frac{1}{x^9}$

(3) $\frac{x^{-9}}{x^{-3}} = x^{-9+3} = x^{-6} = \frac{1}{x^6}$

(4) $\left(\frac{2x^2}{3x^{-4}}\right)^{-3} = \left(\frac{2x^6}{3}\right)^{-3} = \left(\frac{3}{2x^6}\right)^3 = \frac{3\cdot3\cdot3}{2\cdot2x^6\cdot2x^6} = \frac{27}{8x^{18}}$

(5) $x^{18} \div x^{-6} = \frac{x^{18}}{x^{-6}} = x^{18+6} = x^{24}$

(6) $(2x^3y)^4 = 2^4\,x^{3\cdot4}\,y^4 = 2\cdot2\cdot2\cdot2 \cdot x^{12} \cdot y^4 = 16x^{12}y^4$

(7) $\left(\frac{a^{-2}\,b^{-1}\,c^{-4}}{a^4\,b^{-3}\,c^0}\right)^{-3} = \left(a^{-2-4}\,b^{-1+3}\,c^{-4-0}\right)^{-3} = \left(a^{-6}\,b^2\,c^{-4}\right)^{-3}$

$= \left(\frac{b^2}{a^6\,c^4}\right)^{-3} = \left(\frac{a^6\,c^4}{b^2}\right)^3 = \frac{a^{18}\,c^{12}}{b^6}$

Exp Write (1) $\frac{x^2y}{9wz^3}$ with all factors in the numerator

$x^2\,y^{-1}\,9^{-1}\,w^{-1}\,z^{-3}$

(2) $\frac{3x^5(x^2y^3)^0}{7x^{-2}y^4} = \frac{3x^5}{7x^{-2}y^4} = 3\cdot7^{-1}\cdot x^{5+2}\cdot y^{-4}$

$= 3\cdot7^{-1}\cdot x^7\cdot y^{-4}$