

Using F table (2 degrees of freedom numerator and 44 denominator), p -value is less than 0.01

Actual p -value = 0.0000 (to 4 decimal places)

Because p -value $\leq \alpha = 0.05$, we reject the hypothesis that the treatment means are equal.

26 a.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Treatments	2	4 560	2280	9.87
Error	27	6 240	231.11	
Total	29	10 800		

b. Using F table (2 degrees of freedom numerator and 27 denominator), p -value is less than 0.01
Actual p -value = 0.0006
Because p -value $\leq \alpha = 0.05$, we reject the null hypothesis that the means of the three assembly methods are equal.

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	50°	60°	70°
Sample Mean	33	29	28
Sample Variance	32	17.5	9.5

$$\bar{x} = (33 + 29 + 28)/3 = 30$$

$$SSTR = 70$$

$$MSTR = SSTR/(k - 1) = 70/2 = 35$$

$$SSE = 236$$

$$MSE = SSE/(n_T - k) = 236/(15 - 3) = 19.67$$

$$F = MSTR/MSE = 35/19.67 = 1.78$$

Using F table (2 degrees of freedom numerator and 12 denominator), p -value is greater than 0.10

Actual p -value = 0.2104

Because p -value $> \alpha = 0.05$, we cannot reject the null hypothesis that the mean yields for the three temperatures are equal.

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	Paint 1	Paint 2	Paint 3	Paint 4
Sample Mean	13.3	139	136	144
Sample Variance	47.5	0.50	21	54.5

$$\bar{x} = (133 + 139 + 136 + 144)/3 = 138$$

$$SSTR = \sum_{j=1}^k n_j(x_j - \bar{x})^2 = 5(133 - 138)^2$$

$$+ 5(139 - 138)^2 + 5(136 - 138)^2$$

$$+ 5(144 - 138)^2 = 338$$

$$MSTR = SSTR/(k - 1) = 330/3 = 110$$

$$SSE = \sum_{j=1}^k (n_j - 1)s_j^2 = 4(47.5) + 4(50) \\ + 4(21) + 4(54.5) = 692$$

$$MSE = SSE/(n_T - k) = 692/(20 - 4) = 43.25$$

$$F = MSTR/MSE = 110/43.25 = 2.54$$

Using F table (3 degrees of freedom numerator and 16 denominator), p -value is between 0.05 and 0.10

Actual p -value = 0.0931

Because p -value $> \alpha = 0.05$, we cannot reject the null hypothesis that the mean drying times for the four paints are equal.

32 Note: degrees of freedom for $t_{\alpha/2}$ are 18

$$LSD = t_{\alpha/2} \sqrt{MSE(\frac{1}{n_i} + \frac{1}{n_j})} = t_{0.025} \sqrt{5.09(\frac{1}{7} + \frac{1}{7})}$$

$$= 2.101 \sqrt{1.4543} = 2.53$$

$$|\bar{x}_1 - \bar{x}_2| = |117.0 - 20.4| = 3.4 > 2.53;$$

significant difference

$$|\bar{x}_1 - \bar{x}_3| = |117.0 - 25.0| = 8 > 2.53;$$

significant difference

$$|\bar{x}_2 - \bar{x}_3| = |20.4 - 25| = 4.6 > 2.53;$$

significant difference

34 Treatment Means:

$$\bar{x}_1 = 13.6 \quad \bar{x}_2 = 11.0 \quad \bar{x}_3 = 10.6$$

Block Means:

$$\bar{x}_1 = 9 \quad \bar{x}_2 = 7.67 \quad \bar{x}_3 = 15.67 \quad \bar{x}_4 = 18.67 \quad \bar{x}_5 = 7.67$$

Overall Mean:

$$\bar{x} = 176/15 = 11.73$$

Step 1

$$SST = \sum_{j=1}^k (x_{ij} - \bar{x})^2 = (10 - 11.73)^2 + (9 - 11.73)^2 \\ + \dots + (8 - 11.73)^2 = 354.93$$

Step 2

$$SSTR = b \sum_{j=1}^k (\bar{x}_j - \bar{x})^2 = 5[(13.6 - 11.73)^2 \\ + (11.0 - 11.73)^2 + (10.6 - 11.73)^2] = 26.53$$

Step 3

$$SSBL = k \sum_{i=1}^5 (\bar{x}_i - \bar{x})^2 = 3[(9 - 11.73)^2 \\ + (7.67 - 11.73)^2 + (15.67 - 11.73)^2 \\ + (18.67 - 11.73)^2 + (7.67 - 11.73)^2] \\ = 312.32$$

Step 4

$$SSE = SST - SSTR - SSBL = 354.93 - 26.53 \\ - 312.32 = 16.08$$

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Treatments	2	26.53	13.27	6.60
Blocks	4	312.32	78.08	
Error	8	16.08	2.01	
Total	14	354.93		

Using F table (2 degrees of freedom numerator and 8 denominator), p -value is between 0.01 and 0.025

Actual p -value = 0.0203

Because p -value $\leq \alpha = 0.05$, we reject the null hypothesis that the means of the three treatments are equal.

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Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Treatments	3	900	300	12.60
Blocks	7	400	57.14	
Error	21	500	23.81	
Total	31	1800		

Using F table (3 degrees of freedom numerator and 21 denominator), p -value is less than 0.01

Actual p -value = 0.0001

Because p -value $\leq \alpha = 0.05$, we reject the null hypothesis that the means of the treatments are equal.

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Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Cloth(block)	2	0.389	0.195	9.00
Silicone	4	16.103	4.026	
Error	8	3.577	0.447	
Total	14	20.069		

Using F table (4 degrees of freedom numerator and 8 denominator), p -value is less than 0.01

Actual p -value = 0.005

Because p -value $\leq \alpha = 0.05$, we reject the null hypothesis that the mean indices corresponding to the five silicone solution strengths are equal. Note that the highest mean index is for the 15 per cent solution.

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		Factor B			Factor A
		Level 1	Level 2	Level 3	Means
Factor A	Level 1	$\bar{x}_{11} = 150$	$\bar{x}_{12} = 78$	$\bar{x}_{13} = 84$	$\bar{x}_{1.} = 104$
	Level 2	$\bar{x}_{21} = 110$	$\bar{x}_{22} = 116$	$\bar{x}_{23} = 128$	$\bar{x}_{2.} = 118$
Factor B Means		$\bar{x}_{.1} = 130$	$\bar{x}_{.2} = 97$	$\bar{x}_{.3} = 106$	$\bar{x} = 111$

Step 1

$$SST = \sum_i \sum_j \sum_k (x_{ijk} - \bar{x})^2 \\ = (135 - 111)^2 + (165 - 111)^2 \\ + \dots + (136 - 111)^2 = 9028$$

Step 2

$$SSA = br \sum_i (\bar{x}_i - \bar{x})^2 = 3(2)[(104 - 111)^2 \\ + (118 - 111)^2] = 588$$

Step 3

$$SSB = ar \sum_j (\bar{x}_j - \bar{x})^2 = 2(2)[(130 - 111)^2 \\ + (97 - 111)^2 + (106 - 111)^2] \\ = 2328$$

Step 4

$$SSAB = r \sum_i \sum_j (\bar{x}_{ij} - \bar{x}_i - \bar{x}_j + \bar{x})^2 \\ = 2[(150 - 104 - 130 - 111)^2 \\ + (78 - 104 - 97 + 111)^2] + \\ + (128 - 118 - 106 + 111)^2 = 4392$$

Step 5

$$SSE = SST - SSA - SSB - SSAB = \\ 9028 - 588 - 2328 - 4392 = 1720$$

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Factor A	1	588	588	2.05
Factor B	2	2328	1164	4.06
Interaction	2	4392	2196	7.66
Error	6	1720	286.67	
Total	11	9028		

Factor A: $F = 2.05$

Using F table (1 degree of freedom numerator and 6 denominator), p -value is greater than 0.10

Actual p -value = 0.2022

Because p -value $> \alpha = 0.05$, Factor A is not significant

Factor B: $F = 4.06$

Using F table (2 degrees of freedom numerator and 6 denominator), p -value is between 0.05 and 0.10

Actual p -value = 0.0767

Because p -value $> \alpha = 0.05$, Factor B is not significant

Interaction: $F = 7.66$

Using F table (2 degrees of freedom numerator and 6 denominator), p -value is between 0.01 and 0.025

Actual p -value = 0.0223

Because p -value $\leq \alpha = 0.05$, Interaction is significant