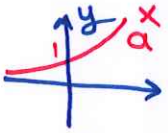
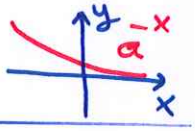
5.3 Solutions of Exponential Equations

Recall that $f(x) = a^x$, $a>1$ represents exponential growth



$f(x) = a^{-x}$, $a>1$ " " " decay



Exp Solve the exponential equation

① $\frac{4}{4}(25)^{2x} = \frac{20}{4} \Rightarrow (25)^{2x} = 5$

or

$(5^2)^{2x} = 5$

$5^{4x} = 5^1$

$4x = 1$

$x = \frac{1}{4}$

$\ln(25)^{2x} = \ln 5$

$2x \ln 25 = \ln 5$

$2x \ln 5^2 = \ln 5$

$4x \ln 5 = \ln 5$

$4x = 1$

$x = \frac{1}{4}$

② $8^{3x} = 216 \Rightarrow 8^{3x} = (8)(27) \Rightarrow \ln 8^{3x} = \ln(8)(27)$

$\Rightarrow 3x \ln 8 = \ln 8 + \ln 27$

$3x \ln 2^3 = \ln 2^3 + \ln 3^3$

$\frac{9x \ln 2}{9 \ln 2} = \frac{3\ln 2 + 3\ln 3}{9\ln 2}$

$x = \frac{1}{3} + \frac{1}{3}\frac{\ln 3}{\ln 2}$

③ $25\,000 = 10,000\ 5^{2x} \Rightarrow \frac{25000}{10000} = 5^{2x} \Rightarrow 5^{2x} = \frac{25}{10}$ (5, 2)

$5^{2x} = \frac{5}{2} \Rightarrow \ln 5^{2x} = \ln \frac{5}{2} \Rightarrow \frac{2x \ln 5}{2 \ln 5} = \frac{\ln 5 - \ln 2}{2\ln 5}$

$x = \frac{\ln 5}{2\ln 5} - \frac{\ln 2}{2\ln 5}$

$= \frac{1}{2} - \frac{\ln 2}{2\ln 5}$

100

(4) $\log_5(x+2) = 2 \Rightarrow 5^2 = x+2$

$25 = x+2$

$\boxed{23 = x}$

---

(5) $\ln(x+2) + \ln x = \ln(x+12)$

$\ln x(x+2) = \ln(x+12)$

$x(x+2) = x+12$

$x^2 + 2x = x+12$

$x^2 + x = 12$

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

Either $x+4=0 \Rightarrow x=-4$ ✗

or $x-3=0 \Rightarrow \boxed{x=3}$ ✓

---

(6) $\log_4 x - \log_4(x+3) = \log_4(x-2)$

$\log_4 \frac{x}{x+3} = \log_4(x-2)$

$\frac{x}{x+3} = x-2$

$x = (x-2)(x+3)$

$x = x^2 + 3x - 2x - 6$

$x^2 + x - 6 = x$

$x^2 - 6 = 0$

$x^2 = 6$

$x = \pm\sqrt{6}$

$\boxed{x=\sqrt{6}}$ ✓ but $x=-\sqrt{6}$ ✗ rejected since $x>0$

---

(7) $\ln x - \ln 2 = 3$

$\ln \frac{x}{2} = \ln e^3$

$\frac{x}{2} = e^3$

$\boxed{x = 2e^3}$

$a>0$ $b>0$

$\ln a = \ln b$ means $a=b$

Exp Suppose the demand for commodity q is given by $p = 40\ 2^{-\frac{q}{2}}$ where p is the price in dollars

(1) At what price per unit will the demand equal 4 units?

when $q = 4 \Rightarrow p = 40\ 2^{-\frac{4}{2}} = 40\ 2^{-2} = 40\ \frac{1}{2^2} = \frac{40}{4}$

$\Rightarrow p = 10$ dollars

(2) How many units will be demanded if the price is \$20

$20 = 40\ 2^{-\frac{q}{2}} \Rightarrow \frac{20}{40} = 2^{-\frac{q}{2}} \Rightarrow \frac{1}{2} = 2^{-\frac{q}{2}}$

$\ln \frac{1}{2} = \ln 2^{-\frac{q}{2}} \Rightarrow \ln 1 - \ln 2 = -\frac{q}{2} \ln 2$

$0 - \ln 2 = -\frac{q}{2} \ln 2$

$\frac{-\ln 2}{\ln 2} = -\frac{q}{2} \Rightarrow \boxed{q = 2}$

Exp Suppose the demand function for a commodity is given by $p = 100\ e^{-\frac{x}{10}}$ where p is the price per unit when x units are sold

(1) Find the total revenue

$R(x) = (p)(x) = \left(100\ e^{-\frac{x}{10}}\right) x = 100x\ e^{-\frac{x}{10}}$

(2) Find the total revenue if 30 units were demanded and supplied

$R(30) = 100\ (30)\ e^{-\frac{30}{10}}$

$= 3000\ e^{-3}$

$\approx (3000)(0.0498)$

$= 149.4$ dollars