

ENCS3340 - Artificial Intelligence

Search (Problem Formulation)

Search as Problem-Solving Strategy

- Many problems can be viewed as reaching a **goal** from a given **starting point**
 - often there is an underlying **state space** that defines the problem and its possible **solutions** in a more formal way
 - the space can be traversed by applying a **successor function** (operators, actions, state transitions) to proceed from one state to the next
 - if possible, information about the specific problem or the general domain is used to improve the search
 - experience from previous instances of the problem
 - strategies expressed as heuristics
 - simpler versions of the problem
 - constraints on certain aspects of the problem

Examples

- Loading a moving truck

- **start:** apartment full of boxes and furniture
- **goal:** empty apartment, all boxes and furniture in the truck
- **actions:** select item, carry item from apartment to truck, load item

- Getting settled after moving

- **start:** items randomly distributed over the place
- **goal:** satisfactory arrangement of items
- **actions:** select item, move item

- Repairing a flat tire on your bike

- **start:** bike with a flat tire
- **goal:** bike with two properly inflated tires
- **actions:** remove wheel, remove tire, remove tube, fix tube, return tube, return tire, partially inflate tube, return wheel, fully inflate tube

Problem-Solving Agents

- Agents whose task it is to solve a particular problem
 - problem formulation
 - what are the possible states of the world relevant for solving the problem
 - what information is accessible to the agent
 - how can the agent progress from state to state
 - goal formulation
 - what is the goal state
 - what are important characteristics of the goal state
 - how does the agent know that it has reached the goal
 - are there several possible goal states
 - are they equal or are some more preferable
 - if necessary, a utility function is required to determine priorities among goals

Problem Types

- single-state problems

- accessible world and knowledge of its actions allow the agent to know which state it will be in after a sequence of actions
- Ex: playing chess

- multiple-state problems

- the world is only partially accessible, and the agent has to consider several possible states as the outcome of a sequence of actions
- Ex: walking in a dark room

- contingency problems

- at some points in the sequence of actions, sensing may be required to decide which action to take; this leads to a tree of sequences
- Ex: a new skater in a ring

- exploration problems

- the agent doesn't know the outcome of its actions, and must experiment to discover states of the world and outcomes of actions
- Ex: Mars Exploration Rovers

Well-Defined Problems

- **initial state**
 - starting point from which the agent sets out
- **actions (operators, successor functions)**
 - describe the set of possible actions, and transitions from one state to another
- **state space**
 - set of all states reachable from the initial state by any sequence of actions
- **goal state**
 - terminal state that the agent wants to achieve
- **goal test**
 - determines if a given state is the goal state
- **Path**
 - sequence of actions leading from one state in the state space to another
- **path cost**
 - determines the expenses of the agent for executing the actions in a path
- **Solution**
 - path from the initial state to a goal state

Selecting States and Actions

- states describe distinguishable points or periods during the problem-solving process
 - dependent on the task and domain
- actions move the agent from one state to another one
 - an action is applied to the current state and takes the agent to the successor state
 - dependent on states, capabilities of the agent, and properties of the environment
- choice of suitable states and actions
 - can make the difference between a problem that can or cannot be solved
 - level of abstraction
 - high: smaller state space, complex actions
 - low: simple actions, larger state space

Example Problem: Romania Map

- On vacation in Romania; currently in Arad
- Flight leaves tomorrow from Bucharest

- Initial state

- Arad

- Actions

- Go from one city to another

- Transition model (successor function)

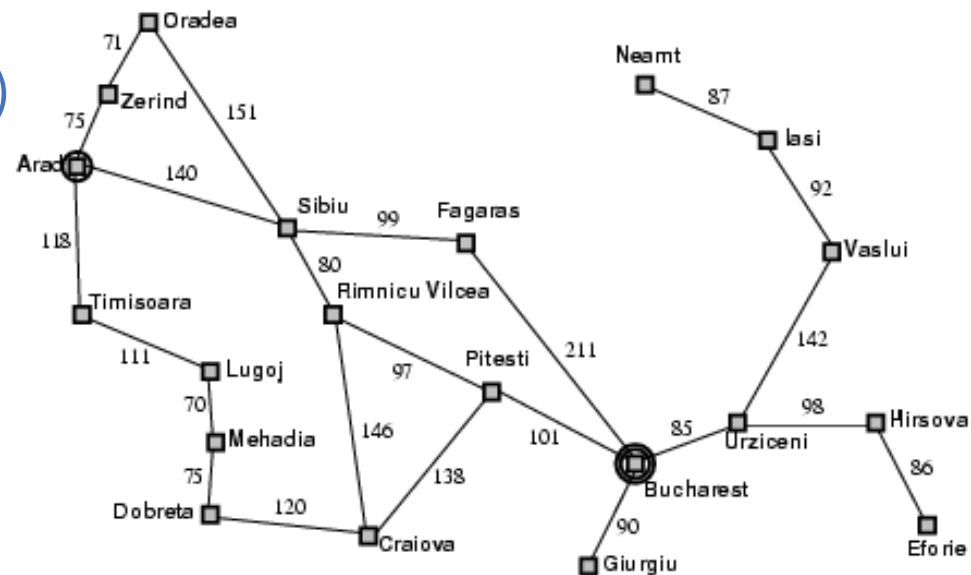
- If you go from city A to city B, you end up in city B

- Goal state

- Bucharest

- Path cost

- Sum of edge costs



Example Problem: Vacuum world

- States

- Agent location and dirt location
- How many possible states?
- What if there are n possible locations?

- Actions

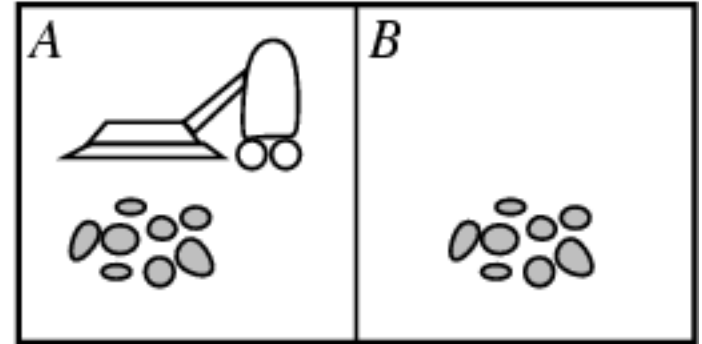
- Left, right, suck

- goal test

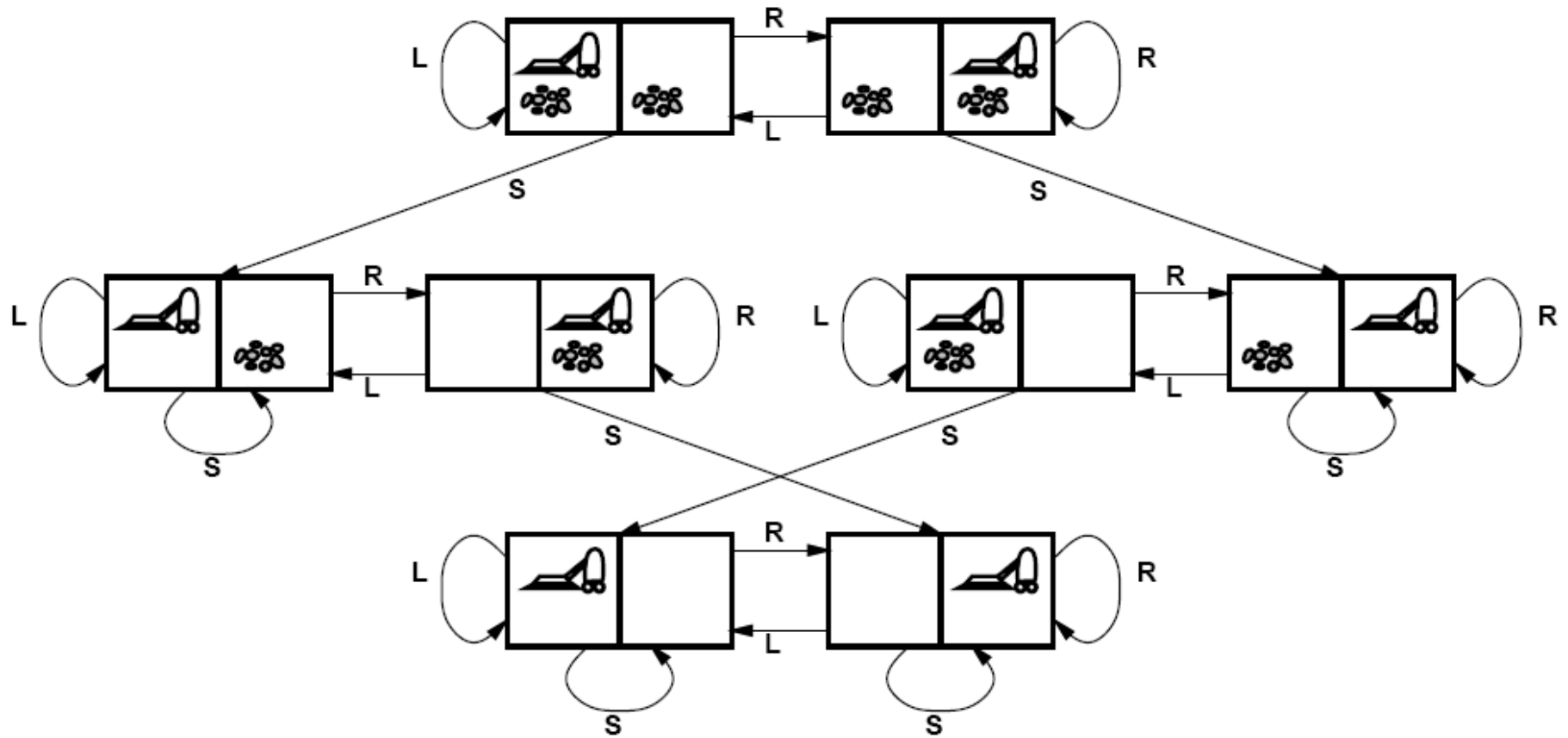
- all squares clean

- path cost

- one unit per action



Vacuum world state space graph



Example Problem: The 8-puzzle

- States

- location of tiles (including blank tile)
- $9!/2 = 181,440$ reachable states

- Actions

- Move blank left, right, up, down

- Path cost

- 1 per move

- Finding the optimal solution of n-Puzzle is NP-hard

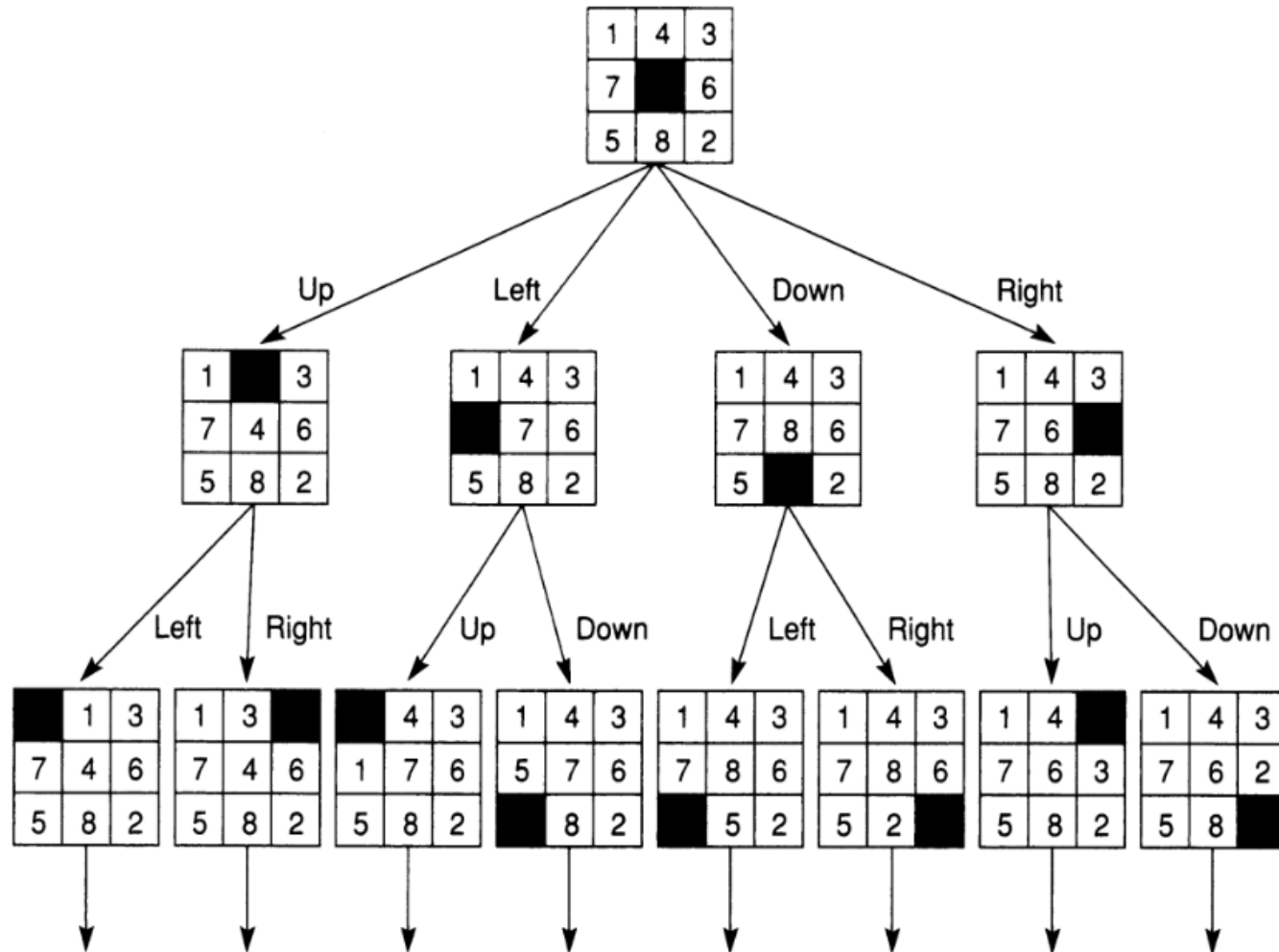
7	2	4
5		6
8	3	1

Start State

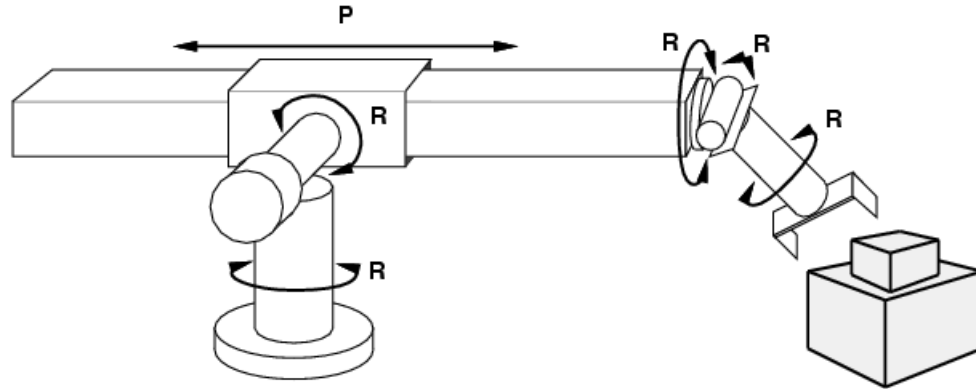
	1	2
3	4	5
6	7	8

Goal State

State Space for the 8-puzzle



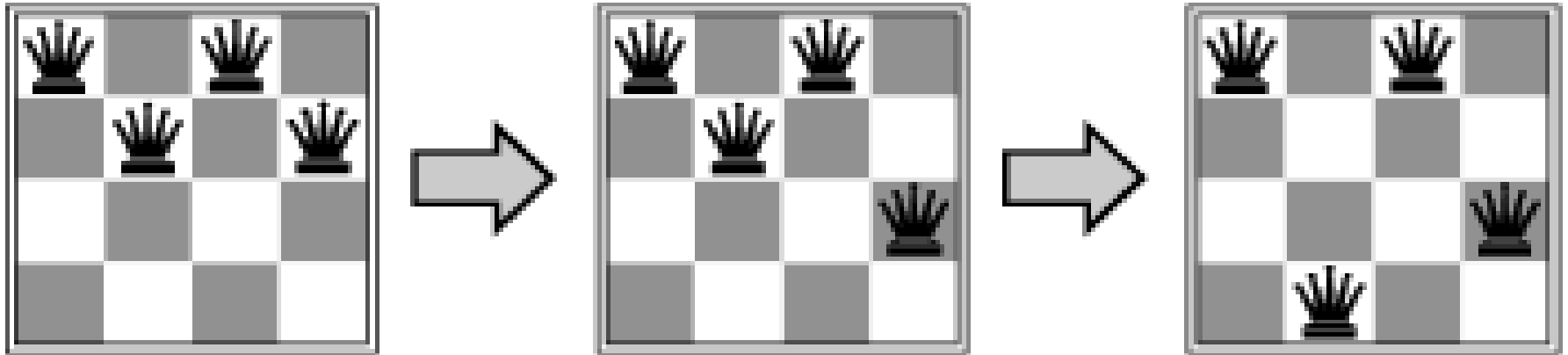
Example Problem: Robot motion planning



- **States**
 - Real-valued coordinates of robot joint angles
- **Actions**
 - Continuous motions of robot joints
- **Goal state**
 - Desired final configuration (e.g., object is grasped)
- **Path cost**
 - Time to execute, smoothness of path, etc.

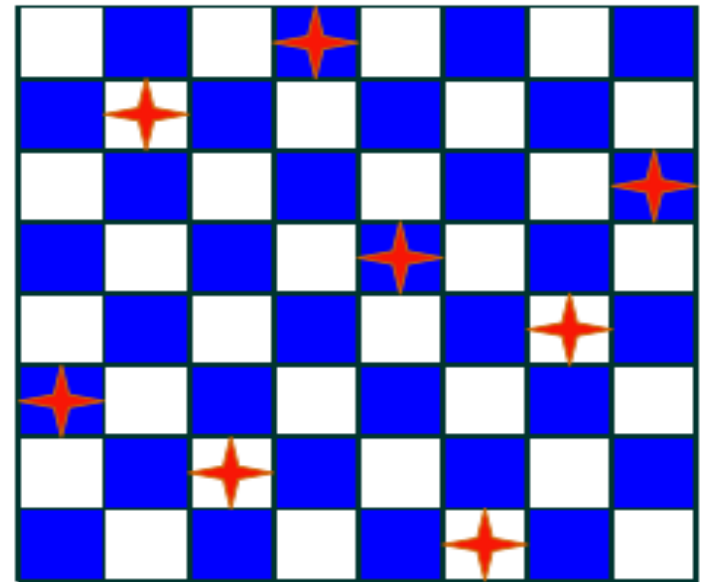
Example: n-queens

- Put n queens on a $n \times n$ board with no two queens on the same row, column, or diagonal



8-Queens: Incremental Approach

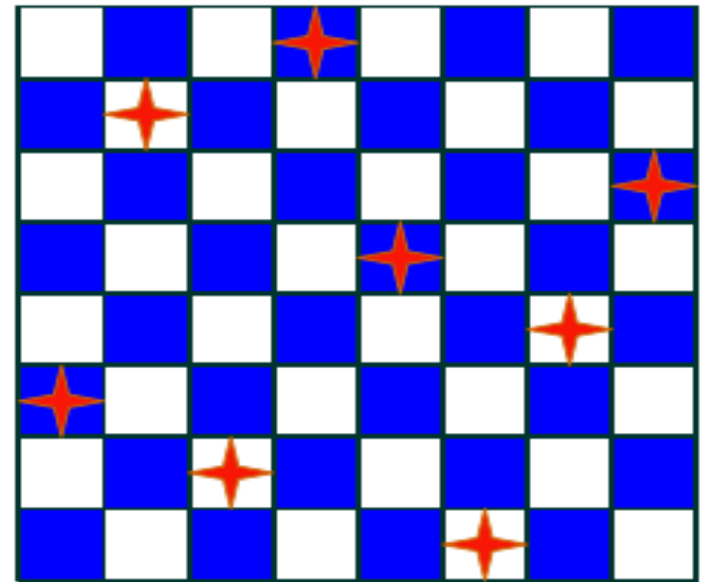
- start with an empty board
- add queens one by one (no violation of constraints)
- incremental formulation
 - **states**
 - arrangement of up to 8 queens on the board
 - **initial state**
 - empty board
 - **successor function (actions)**
 - add a queen to any square
 - **goal test**
 - all queens on board
 - no queen attacked
 - **path cost**
 - irrelevant (all solutions equally valid)



A solution

8-Queens: Complete-State Approach

- start with a full board (all n queens placed on the board, conflicts are to be expected)
- try to find a better configuration (reduced number of conflicts)
- Complete-state formulation
 - **states**
 - arrangement of the 8 queens on the board
 - **initial state**
 - all 8 queens on board
 - **successor function (actions)**
 - move a queen to a different square
 - **goal test**
 - no queen attacked
 - **path cost**
 - irrelevant (all solutions equally valid)



A solution

Example Problem: VLSI Layout

- States

- positions of components, wires on a chip

- Initial state

- incremental: no components placed
- complete-state: all components placed (e.g. randomly, manually)

- Successor function (actions)

- incremental: place components, route wire
- complete-state: move component, move wire

- Goal test

- all components placed
- components connected as specified

- Path cost

- may be complex: distance, capacity, number of connections per component

Searching for Solutions

- Given
 - Initial state
 - Actions
 - Transition model
 - Goal state
 - Path cost
- How do we find the optimal solution?
 - How about building the state space and then using Dijkstra's shortest path algorithm?
 - The state space may be huge!
 - Complexity of Dijkstra's is $O(E + V \log V)$, where V is the size of the state space

Searching for Solutions

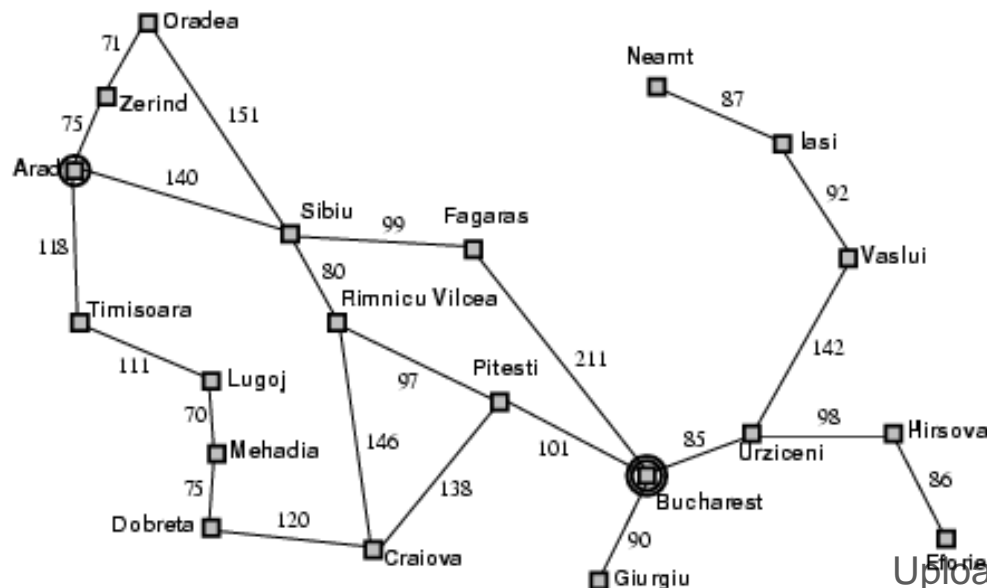
- traversal of the search space
 - from the initial state to a goal state
 - legal sequence of actions as defined by successor function (actions, operators, state transitions)
- general procedure
 - check for goal state
 - expand the current state
 - determine the set of reachable states
 - return “failure” if the set is empty
 - select one from the set of reachable states
 - move to the selected state
- a search tree is generated
 - nodes are added as more states are visited

Search Terminology

- search tree
 - generated as the search space is traversed
 - the search space itself is not necessarily a tree, frequently it is a graph
 - the tree specifies possible paths through the search space
- expansion of nodes
 - as states are explored, the corresponding nodes are expanded by applying the successor function
 - this generates a new set of (child) nodes
 - the fringe (frontier) is the set of nodes not yet visited
 - newly generated nodes are added to the fringe
- search strategy
 - determines the selection of the next node to be expanded
 - can be achieved by ordering the nodes in the fringe
 - e.g. queue (FIFO), stack (LIFO), “best” node w.r.t. some measure (cost)

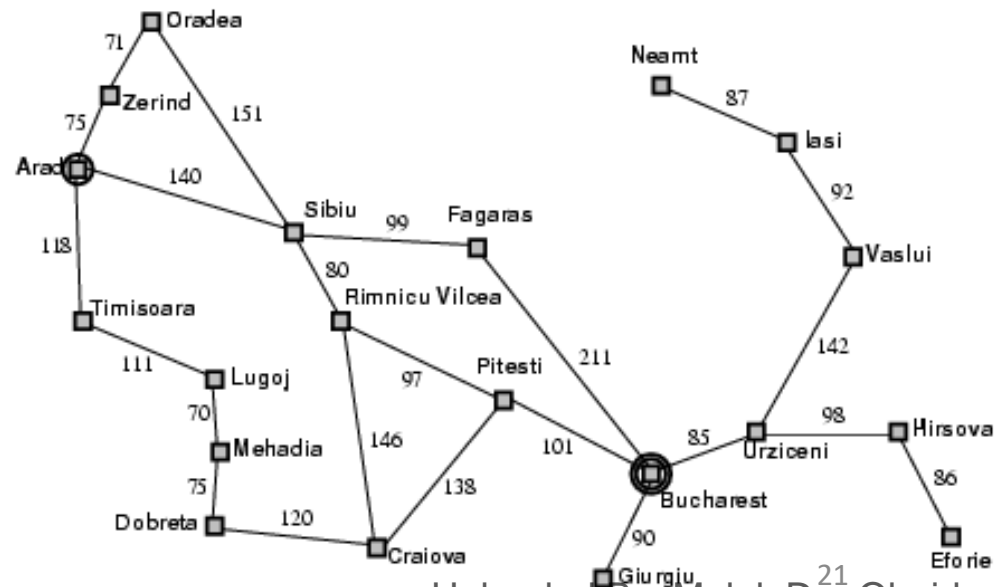
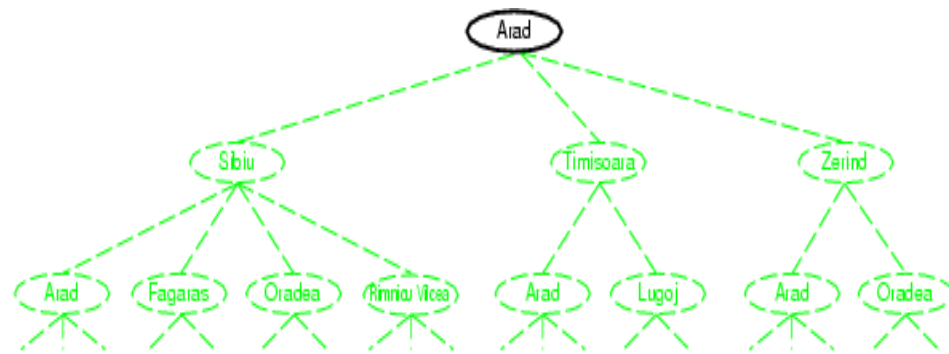
Example: Graph Search

- describes the search (state) space
 - each node represents one state in the search space
 - e.g. a city to be visited in a routing or touring problem
- additional information
 - names and properties for the states
 - links between nodes, specified by the successor function
 - properties for links (distance, cost, name, ...)



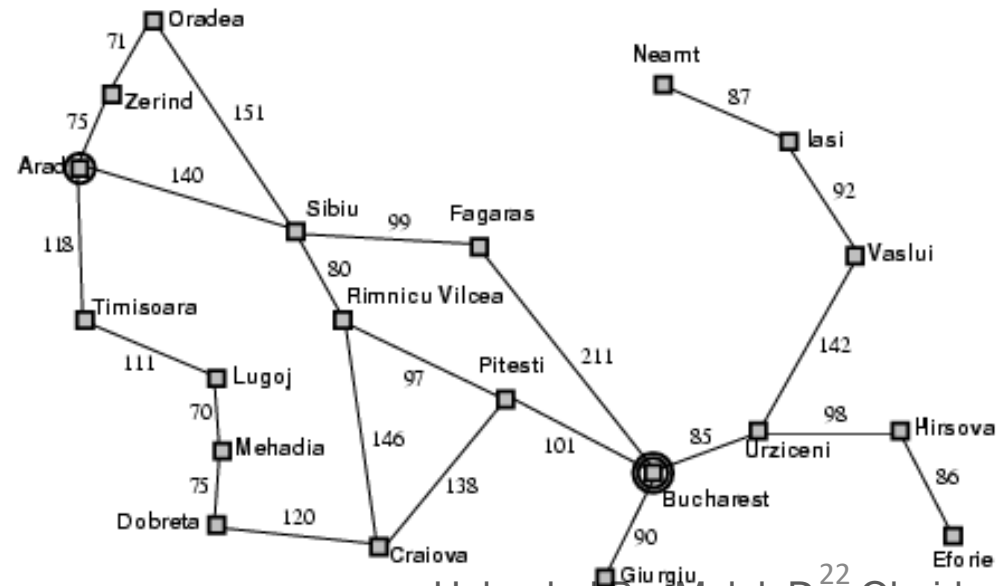
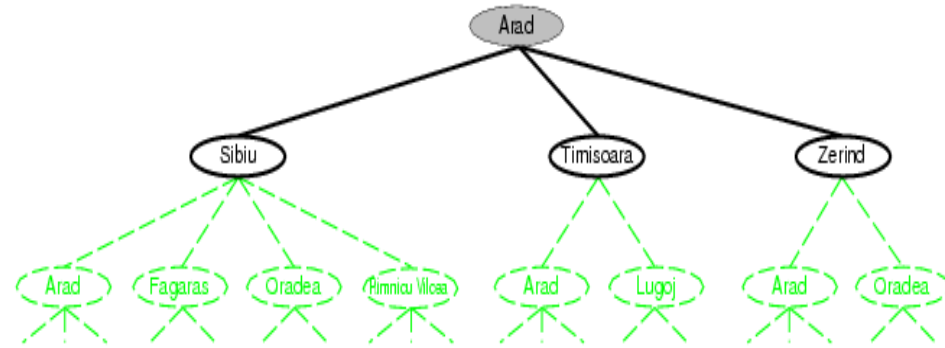
Graph and Tree

- the tree is generated by traversing the graph
- the same node in the graph may appear repeatedly in the tree
- the arrangement of the tree depends on the traversal strategy (search method)
- the initial state becomes the root node of the tree
- in the fully expanded tree, the goal states are the leaf nodes
- cycles in graphs may result in infinite branches



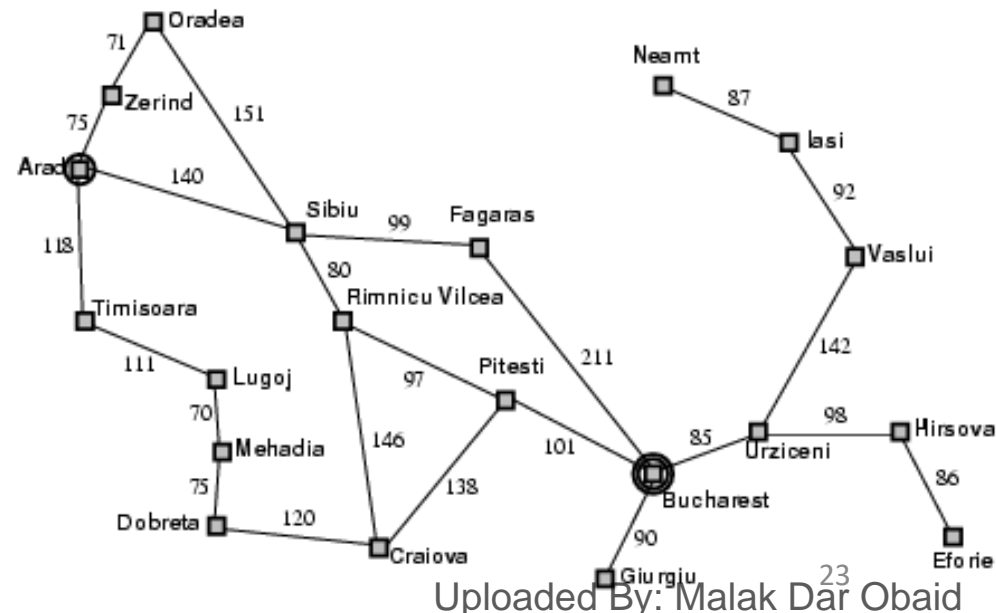
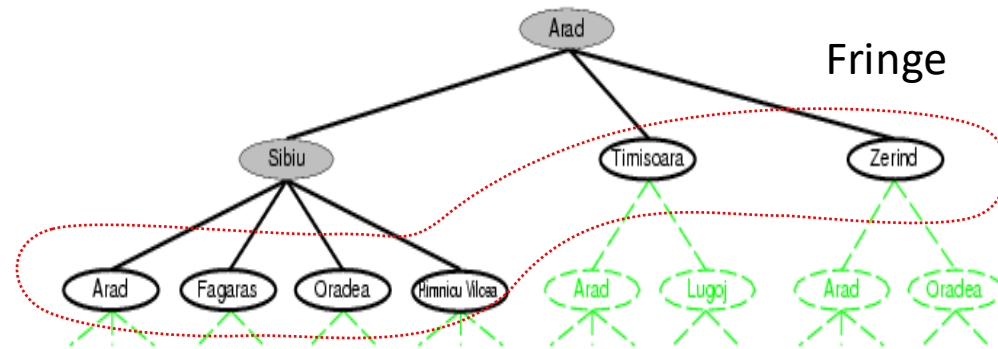
Graph and Tree

- the tree is generated by traversing the graph
- the same node in the graph may appear repeatedly in the tree
- the arrangement of the tree depends on the traversal strategy (search method)
- the initial state becomes the root node of the tree
- in the fully expanded tree, the goal states are the leaf nodes
- cycles in graphs may result in infinite branches



Graph and Tree

- the tree is generated by traversing the graph
- the same node in the graph may appear repeatedly in the tree
- the arrangement of the tree depends on the traversal strategy (search method)
- the initial state becomes the root node of the tree
- in the fully expanded tree, the goal states are the leaf nodes
- cycles in graphs may result in infinite branches



General Tree Search Algorithm

- generate the first node from the initial state of the problem
- Repeat
 - return failure if there are no more nodes in the fringe
 - examine the current node; if it's a goal, return the solution
 - expand the current node, and add the new nodes to the fringe

Terminology:

Fringe: Set of “visible” but unexplored nodes

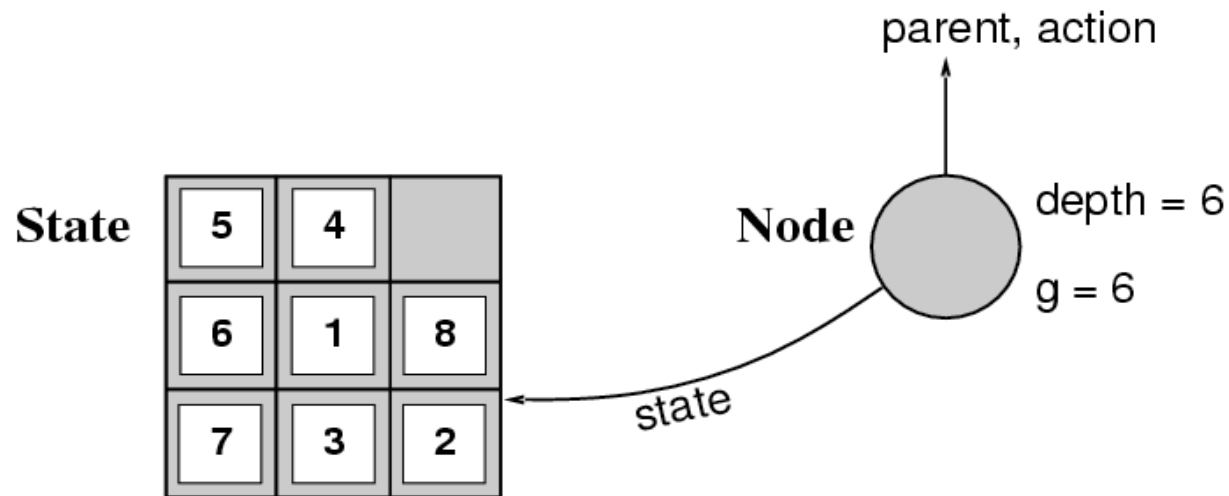
General Search Algorithm

```
function GENERAL-SEARCH(problem, QUEUING-FN) returns solution
    nodes := MAKE-QUEUE(MAKE-NODE(INITIAL-STATE[problem]))
    loop do
        if nodes is empty then return failure
        node := REMOVE-FRONT(nodes)
        if GOAL-TEST[problem] applied to STATE(node) succeeds
            then return node
        nodes := QUEUING-FN(nodes, EXPAND(node,
                                         ActionS[problem]))
    end
```

Note: QUEUING-FN is customizable which will be used to specify the search method

Implementation: states vs. nodes

- A **state** is a (representation of) a physical configuration
- A **node** is a data structure constituting part of a search tree includes **state**, **parent node**, **action**, **path cost $g(x)$** , **depth**



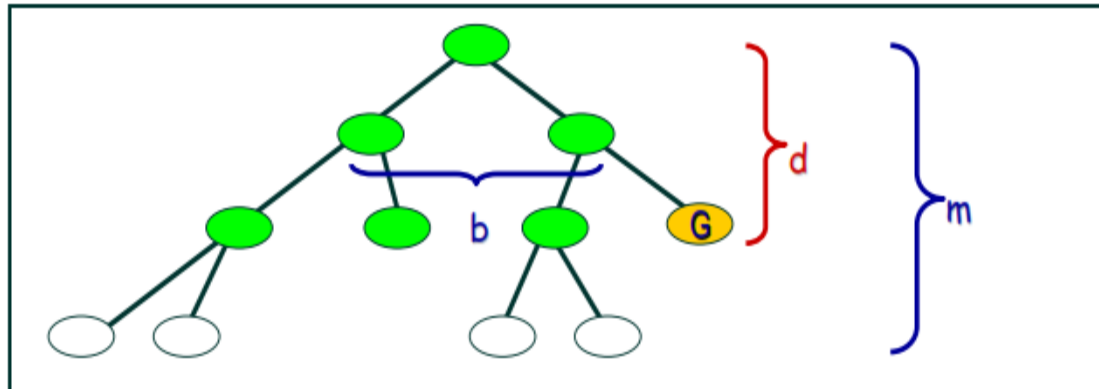
- The **Expand** function creates new nodes, filling in the various fields and using the **SuccessorFn** of the problem to create the corresponding states.

Search strategies

- A **search strategy** is defined by picking the order of node expansion
- Strategies are evaluated along the following dimensions:
 - **Completeness**: does it always find a solution if one exists?
 - **Optimality**: does it always find a least-cost solution?
 - **Time complexity**: time it takes to find the solution (number of nodes generated)
 - **Space complexity**: maximum number of nodes in memory

Search strategies

- Time and space complexity are measured in terms of
 - b : maximum branching factor of the search tree
 - d : depth of the least-cost solution
 - m : maximum length of any path in the state space (may be infinite)



Search Cost and Path Cost

- the search cost indicates how expensive it is to generate a solution
 - time complexity (e.g. number of nodes generated) is usually the main factor
 - sometimes space complexity (memory usage) is considered as well
- path cost indicates how expensive it is to execute the solution found in the search
 - distinct from the search cost, but often related
- total cost is the sum of search and path costs

Search strategies

- Uninformed Search

- breadth-first
- depth-first
- uniform-cost search
- depth-limited search
- iterative deepening
- bi-directional search

- Informed Search

- best-first search
- search with heuristics
- memory-bounded search
- iterative improvement search

- Local Search and Optimization

- hill-climbing
- simulated annealing
- local beam search
- genetic algorithms
- constraint satisfaction

- Others