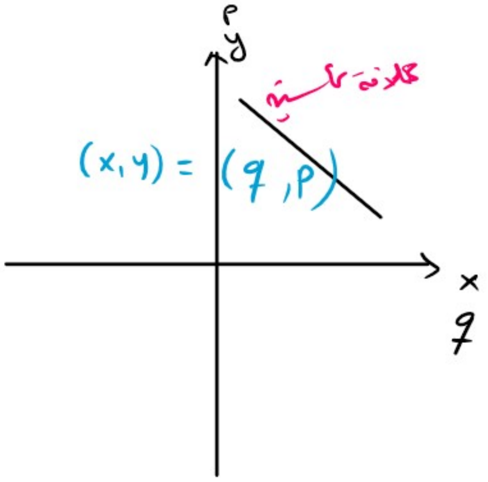
11.5 Applications in Business and Economics

Sunday, June 19, 2022 11:30 AM

Elasticity : measure for the responsiveness of consumers quantity demanded when prices changes

مرونة



$$E = \overset{\text{eta}}{\eta} = - \frac{\text{Relative change in quantity}}{\text{Relative change in Prices}}$$

$$= - \frac{\dfrac{\Delta q}{q_1}}{\dfrac{\Delta p}{p_1}} \div$$

$\Delta q \rightarrow q_2 - q_1$

$\Delta p \rightarrow p_2 - p_1$

$$= - \frac{\Delta q}{q} \cdot \frac{p}{\Delta p}$$

$$\boxed{E = \eta = - \frac{p}{q} \frac{\Delta q}{\Delta p}}$$

Point elasidity of demand

$$\lim_{\Delta p \to 0} - \frac{p}{q} \left(\frac{\Delta q}{\Delta p} \right)$$

$$\boxed{q = f(p)}$$

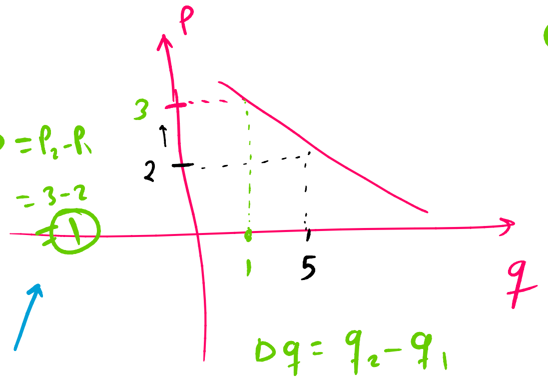
$$q' = \frac{dq}{dp}$$

$$\boxed{\eta = - \frac{p}{q} \frac{dq}{dp}}$$

النقطة ↳ (q, p)

Three cases for $E = \eta$

1. If $E = \eta > 1$ then the demand is elastic. $\Rightarrow$ % of ↓ in demand > % of ↑ in price



$\Delta p = P_2 - P_1$
$= 3 - 2$
$= 1$

زيادة 1

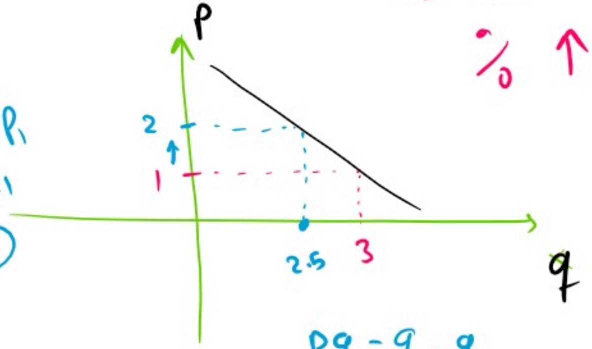
$\Delta q = q_2 - q_1$
$= 1 - 5$
$= -4$ ← نقص

نقص طلب

(2) If $\eta < 1$ then the demand is inelastic.

$\Rightarrow$ % ↓ in demand <

% ↑ in price

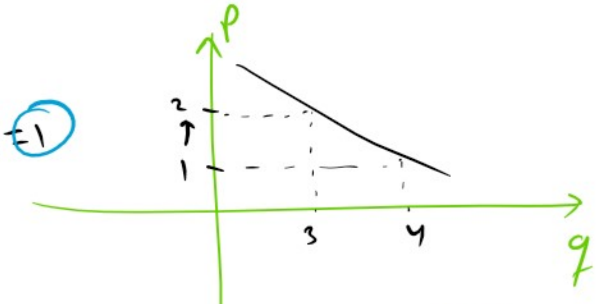


$\Delta p = p_2 - p_1$
$= 2 - 1$
$= \textcircled{1}$

$\Delta q = q_2 - q_1$
$= 2.5 - 3$
$= -\textcircled{0.5}$

(3) If $\eta = E = 1$ then the demand is unitary elastic. $\Rightarrow$ % ↓ in demand $\simeq$

% ↑ in prices



$\Delta p = \textcircled{1}$

$\Delta q = -\textcircled{1}$

$\boxed{q = f(p)}$

Exp Given this demand $p + 5q = 100$

① Find Elasticity

$1 + 5q' = 0$

$5q' = -1$

$E = \eta = -\frac{p}{q}\frac{dq}{dp}$

$$E = \eta = -\frac{p}{q}\frac{dq}{dp}$$

$$5q' = -1$$

$$q' = \frac{-1}{5}$$

$$\boxed{\frac{dq}{dp} = -\frac{1}{5}}$$

$$= -\frac{p}{q}\,\frac{-1}{5}$$

$$\boxed{E = \frac{p}{5q}}$$

② What is the type of elasticity for the prices ⓐ \$40 ⓑ \$60 ⓒ \$50

ⓐ $p = 40$

$$p + 5q = 100$$

$$40 + 5q = 100$$

$$5q = 60$$

$$q = \frac{60}{5}$$

$$\boxed{q = 12}$$

$$E = \frac{p}{5q}\Bigg|_{(q,p)=(12,40)}$$

$$= \frac{40}{5(12)} = \frac{40}{60} = \frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{6}}} = \frac{2}{3}$$

$E = \frac{2}{3} < 1 \Rightarrow$ demand is inelastic

ⓑ $p = 60$

$$p + 5q = 100$$

$$60 + 5q = 100$$

$$5q = 40$$

$$q = 8$$

$$E = \frac{p}{5q}\Bigg|_{(q,p)=(8,60)}$$

$(q, p) = (8, 60)$

$q = 8$

$$= \frac{60}{5(8)} = \frac{60}{40} = \frac{6}{4} = \frac{3}{2}$$

$E = \frac{3}{2} > 1 \Rightarrow$ demand is elastic

ⓒ $p = 50$

$p + 5q = 100$

$50 + 5q = 100$

$5q = 50$

$\boxed{q = 10}$

$$E = \frac{p}{5q} \Big|_{(q,p) = (10, 50)}$$

$$= \frac{50}{5(10)} = \frac{50}{50} = 1$$

demand is unitary elastic

Exp Given this demand $\boxed{p = \frac{1000}{(q+1)^2}}$

q: demand for product whose price is p

① Find elasticity

$$E = -\frac{p}{q} \boxed{\frac{dq}{dp}}$$

$$p = 1000 (q+1)^{-2}$$

$$1 = 1000(-2)(q+1)^{-3} q'$$

$$1 = -2000 \frac{1}{\quad^3} q'$$

$p \quad (q+1)^3$

$$= -\frac{p}{q}\,\frac{(q+1)^3}{-2000}$$

$$\boxed{\eta = \frac{p}{q}\,\frac{(q+1)^3}{2000}}$$

$$1 = -2000\,\frac{1}{(q+1)^3}\,q'$$

$$1 = \frac{-2000}{(q+1)^3}\,q'$$

$$(q+1)^3 = -2000\,q'$$

$$\frac{(q+1)^3}{-2000} = \frac{dq}{dp}$$

② Find E when $q = 19$ and classify

$$p = \frac{1000}{(q+1)^2} = \frac{1000}{(\boxed{19}+1)^2} = \frac{1000}{(20)^2}$$

$$= \frac{\overset{5}{\cancel{1000}}}{\underset{2}{\cancel{400}}} = 2.5 = \frac{5}{2}$$

$$E\Big|_{(q,p)=(19,\frac{5}{2})} = \frac{p}{q}\,\frac{(q+1)^3}{2000}\Big|_{(19,\frac{5}{2})} = \frac{2.5}{19}\,\frac{(19+1)^3}{2000}$$

$$= \frac{\frac{5}{\cancel{2}}}{19}\,\frac{(2\cancel{0})(\cancel{20})(\cancel{20})}{\cancel{2000}}$$

$$= \frac{10}{19} < 1$$

demand is inelastic

demand is

Q: How $\underset{\text{elasticity}}{\eta}$ is related to $\underset{\text{Revenue}}{R}$?

A: Total Revenue $R(x) = p\,x$

$q = f(p)$

$R(q) = \underset{\text{الأول}}{p}\;\underset{\text{الثاني}}{q}$

$$\frac{dR}{dp} = \underbrace{p\,\frac{dq}{dp}}_{\text{قريبة من تعريف } E} + q \quad (1)$$

$$= (-q)\underbrace{\left(-\frac{p}{q}\,\frac{dq}{dp}\right)}_{E=\eta} + q$$

$q = f(p)$

$(p)' = 1$

$(q)' = \frac{dq}{dp}$

$$= -q\eta + q$$

$$\boxed{\underbrace{\frac{dR}{dp}}_{\text{علاقة بين السعر والعائدات}} = q(1-\eta)}$$

Q. How $\underset{\text{Revenue}}{R}$ depends on $\underset{\text{price}}{p}$?

A. ① If $\eta > 1$ "Demand elastic" then there is negative relationship between P and R

⇒ If P ↑ ⇒ R ↓

If P ↓ ⇒ R ↑

② If $\eta < 1$ "Demand inelastic" then there is postive relationship between P and R

⇒ If P ↑ ⇒ R ↑

If P ↓ ⇒ R ↓

③ If $\eta = 1$ "Demand is unitary elastic", then if p ↑ or ↓ ⇒ no change on R

⇒ Revenue is optimized at this point

Exp Given this demand

$$q = \sqrt{900 - P} \quad , \quad 0 \le P \le 900$$

① Find elasticity of demand as function of P

$q = (900 - P)^{\frac{1}{2}}$

① Find elasticity of

$$\eta = -\frac{p}{q}\frac{dq}{dp}$$

$$= \ominus \frac{p}{q}\,\frac{\ominus 1}{2\sqrt{900-p}}$$

$$= \frac{p}{\sqrt{900-p}\;2\sqrt{900-p}}$$

$$\boxed{E = \frac{p}{2(900-p)}}$$ ✓

$$q = (900-p)^{\frac{1}{2}}$$

$$q' = \frac{1}{2}(900-p)^{-\frac{1}{2}}(0-1)$$

$$= -\frac{1}{2}\,\frac{1}{(900-p)^{\frac{1}{2}}}$$

$$\frac{dq}{dp} = \frac{-1}{2\sqrt{900-p}}$$

② Find the point at which the demand of unitary elasticity

$$\eta = 1 = \frac{p}{2(900-p)} \Rightarrow$$

$$2(900-p) = p$$

$$1800 - 2p = p$$

$$1800 = 3p$$

$$\boxed{600 = p}$$

$$q = \sqrt{900 - p}$$

$q = \sqrt{900 - P}$

$\boxed{600 = P}$

$= \sqrt{900 - 600}$

$= \sqrt{300} = \sqrt{(100)(3)} = \sqrt{100}\sqrt{3} = 10\sqrt{3}$

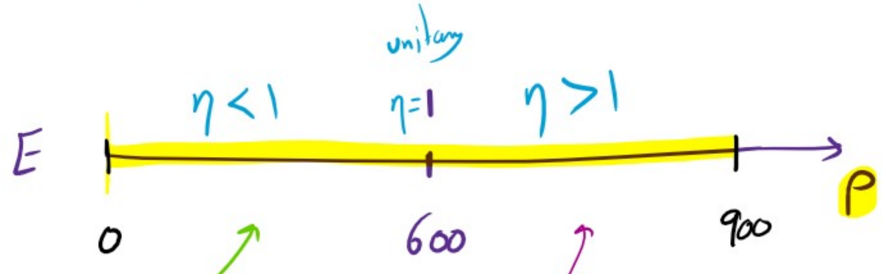
$\boxed{q = 10\sqrt{3}}$

$\text{Point} = (q, P) = (10\sqrt{3}, 600)$

③ Find intervals in which the demand is

ⓐ elastic ⓑ inelastic

$E = \dfrac{P}{2(900-P)}$



$E(P=300) = \dfrac{300}{2(900-300)} = \dfrac{\overset{1}{\cancel{300}}}{2(\underset{2}{\cancel{600}})} = \dfrac{1}{4} < 1 \Rightarrow$ demand inelastic

$E(P=800) = \dfrac{800}{2(900-800)} = \dfrac{\overset{4}{\cancel{800}}}{\cancel{2}(\cancel{100})} = 4 > 1 \Rightarrow$ demand elastic

- $0 < P < 600 \Rightarrow \eta < 1 \Rightarrow$ demand inelastic

- $0 \le p < 600 \Rightarrow \eta < 1 \Rightarrow$ demand inelastic
- $600 < p \le 900 \Rightarrow \eta > 1 \Rightarrow$ demand elastic
- $p = 600 \Rightarrow \eta = 1 \Rightarrow$ demand unitary

④ A $p = \$500 \Rightarrow$ should price ↑ or ↓ in order to increase Revenue?

at price \$500 ⇒ demand inelastic ⇒ $\eta < 1$

$\Rightarrow \frac{dR}{dP} > 0 \Rightarrow$ so to increase R just ↑ the price

⑤ At $p = 800$ dollar ⇒ should we ↑ or ↓ the price in order to ↑ the revenue

at $p = \$800 \Rightarrow$ demand elastic $\Rightarrow \eta > 1$

$\Rightarrow \frac{dR}{dp} < 0 \Rightarrow$ to increase R we must ↓ the price

⑥ What is the Max Revenue

Revenue is optimized when $\eta = 1$

$\Rightarrow p = \$600 \quad \Rightarrow \quad q = \sqrt{300} = 10\sqrt{3}$

$(q, p) = (10\sqrt{3}, \overset{\checkmark}{600})$ Max Revenue

(S1)

$R(q) = p\,q = 600(10\sqrt{3})$

$= 6000\sqrt{3}$ dollars

$$\frac{dR}{dp} = q(1-\underset{\underset{1}{\uparrow}}{\eta})$$

$\eta = 1 \Rightarrow$

$$\boxed{\frac{dR}{dp} = 0}$$

(S2)

$R = q\,p \qquad q = \sqrt{900-p} = (900-p)^{\frac{1}{2}}$

$= \sqrt{900-p}\;p$

$R' = \sqrt{900-p}\,(1) + p\;\frac{1}{2}(900-p)^{-\frac{1}{2}}(-1)$

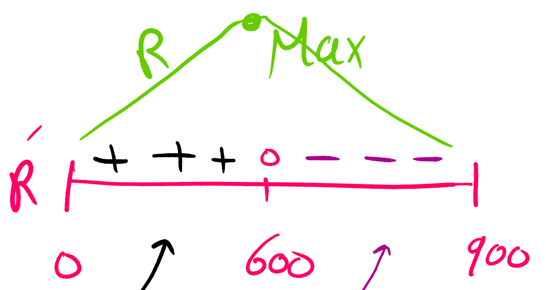
$$= \sqrt{900-p} - \frac{1}{2} p \frac{1}{\sqrt{900-p}}$$

$$= \frac{\sqrt{900-p}\ \overset{2\sqrt{900-p}}{\swarrow}}{2\sqrt{900-p}} - \frac{p}{2\sqrt{900-p}}$$

$$= \frac{2(900-p) - p}{2\sqrt{900-p}}$$

$$= \frac{1800-2p-p}{2\sqrt{900-p}}$$

$$R' = \frac{1800-3p}{2\sqrt{900-p}} = 0$$



$$1800 - 3p = 0$$

$$1800 = 3p$$

$$\boxed{600 = p}$$

$$R'(100) = \frac{1800-3(100)}{2\sqrt{900-100}} = \frac{1500}{2\sqrt{800}} > 0$$

$$R'(700) = \frac{1800-3(700)}{2\sqrt{900-700}} = \frac{1800-2100}{2\sqrt{200}}$$

$$= \frac{-300}{2\sqrt{900}} < 0$$

$= \frac{-300}{2\sqrt{900}} < 0$

Max Revenue at $p = 600$

$q = \sqrt{900 - p} = \sqrt{900 - 600} = \sqrt{300} = 10\sqrt{3}$

Max Rev. $R = pq$

$= 600\,(10\sqrt{3})$

$= \$\,6000\sqrt{3}$