| 7.8 | 7 | 6, 8 |

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \begin{cases} 0 & \Rightarrow f \text{ grows slower than } g \text{ as } x\to\infty \\ & \Rightarrow g \text{ " faster " } f \text{ " " } \\ \infty & \Rightarrow f \text{ grows faster than } g \text{ as } x\to\infty \\ & \Rightarrow g \text{ " slower " } f \text{ " " } \\ C\neq 0 & \Rightarrow \boxed{f \text{ and } g} \text{ grow at same rate as } \boxed{x\to\infty} \end{cases}$$

(6) Compare with $\underline{\underline{\ln x}}$

(which function grows faster/slower/same rate as $x\to\infty$)

(a) $\log_2 x^2 \Rightarrow \lim_{x\to\infty} \dfrac{\log_2 x^2}{\ln x} = \lim_{x\to\infty} \dfrac{\frac{\ln x^2}{\ln 2}}{\ln x} \div$

$$= \lim_{x\to\infty} \frac{2\cancel{\ln x}}{\ln 2} \cdot \frac{1}{\cancel{\ln x}} = \frac{2}{\ln 2}$$

عدد

$\log_2 x^2$ and $\ln x$ grow at same rate as $x\to\infty$

(c) $\dfrac{1}{\sqrt{x}} \Rightarrow \lim_{x\to\infty} \dfrac{\frac{1}{\sqrt{x}}}{\ln x} = \lim_{x\to\infty} \dfrac{1}{\sqrt{x}\,\ln x} = \dfrac{1}{\infty} = \boxed{0}$

$\frac{1}{\sqrt{x}}$ grows slower than $\ln x$ as $x \to \infty$

or $\ln x$ " faster " $\frac{1}{\sqrt{x}}$ " "

e) $x - 2\ln x \Rightarrow \lim_{x \to \infty} \frac{x - 2\ln x}{\ln x} \quad \underline{\underline{\infty - \infty}}$

$= \lim_{x \to \infty} \left(\frac{x}{\ln x} - 2 \right) = \lim_{x \to \infty} \left(\frac{1}{\frac{1}{x}} - 2 \right)$

$= \lim_{x \to \infty} (x - 2) = \infty - 2 = \boxed{\infty}$

$x - 2\ln x$ grows faster than $\ln x$ as $x \to \infty$

or $\ln x$ " slower " $x - 2\ln x$ " "

f) $e^{-x} \Rightarrow \lim_{x \to \infty} \frac{e^{-x}}{\ln x} = \lim_{x \to \infty} \frac{1}{e^{x} \ln x} = \frac{1}{\infty} = \boxed{0}$

e^{-x} grows slower than $\ln x$ as $x \to \infty$

or $\ln x$ " faster " e^{-x} " "