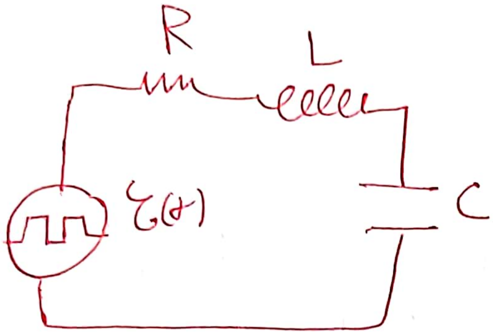
Exp 7 : Damped Oscillations

Theory : RLC circuit



- Voltage across the capacitor in this circuit are described using

$$Q(t) = A_1 e^{\lambda_+ t} + A_2 e^{\lambda_- t}$$

A_1, A_2 constants

$$\lambda_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

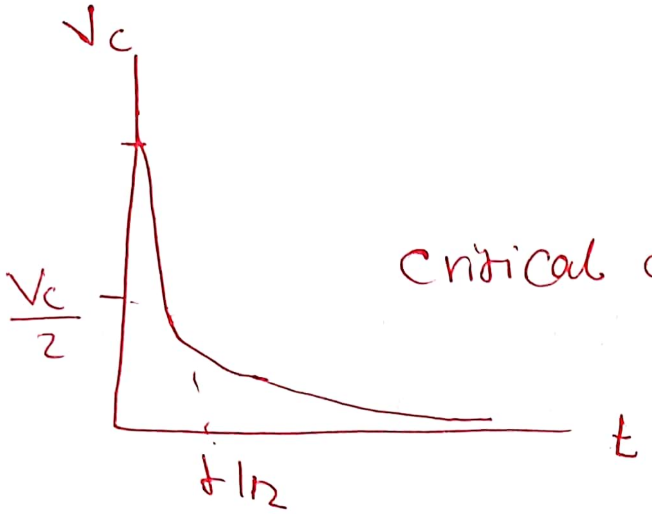
$$\lambda_2 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

as R change the value of λ change $\therefore$ Q(t) on the capacitor will change too.

Case 1 : Critical damping

$$\text{if } \left(\frac{R}{2L}\right) = \frac{1}{LC}$$

$\therefore \lambda_1 = \lambda_2 = -\dfrac{R}{2L}$



critical damping.

$$\therefore Q(t) = A e^{(-R/2L)t} + B e^{(-R/2L)t}$$

Constant

∴ then the charge (which lead to the voltage to settle same) will decay exponentially

→ This case it serves as a boundary between overdamping and under damping

if $R > R_c$ ∴ overdamping

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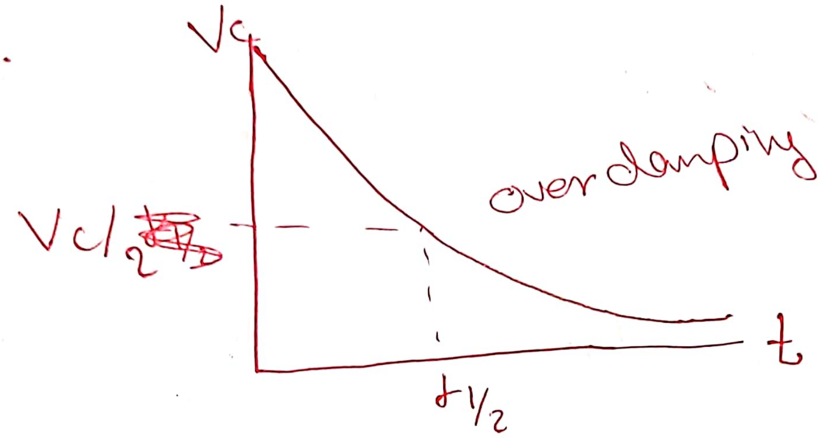
Case 2 : over-damping

if $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

→ also exponentially decay



Case 3 : under damping.

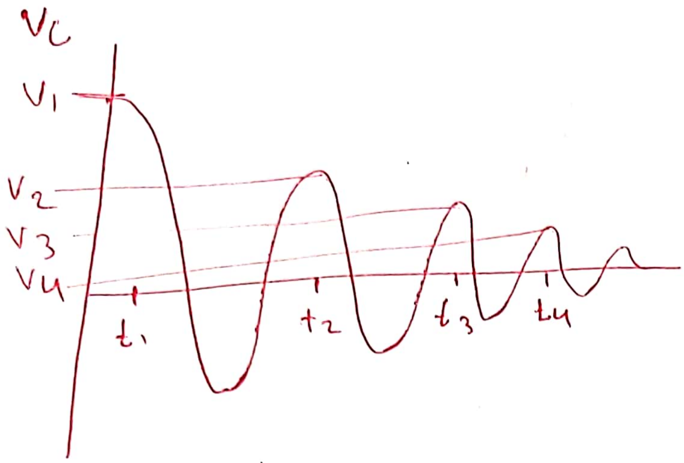
if $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$ ∴ the term under the square root (negative)

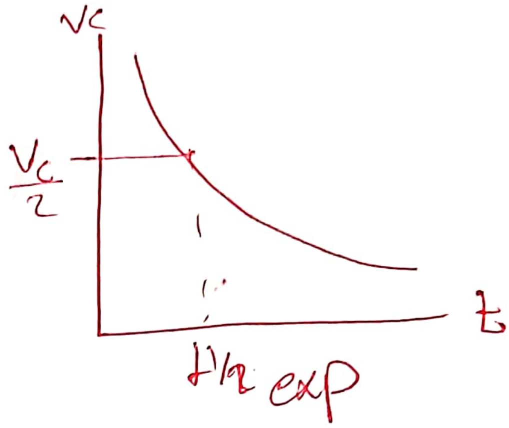
The solution become → ∴ $Q(t) = Q_0 e^{-\delta t} \cos(\omega' t + \theta_0)$

$$\delta = \frac{R}{2L}$$

$$\omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

→ this eq represent a sinusoidal function with a amplitude that decaying exponentially





$$\frac{Q_0}{2} = Q_0 e^{-\delta t_{1/2}} \longleftrightarrow \delta = \frac{R}{2L}$$

Substituting

$$t_{1/2} = \frac{2L(\ln(2))}{R} \quad \text{then} \quad t_{1/2} = \frac{\ln(2)}{\delta}$$

$$V_C = \frac{Q(t)}{C}$$

$\therefore V_C$ behave exactly the same as $Q(t)$ in all three cases.

$\uparrow R = \text{zero}$

* $R_{under} = R_o = \frac{(2L)\ln(2)}{t_{1/2\,exp}} \approx 138.6\,\Omega$

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* $t_{1/2} \approx 100\,\mu s$

$f_{input} \approx 700$

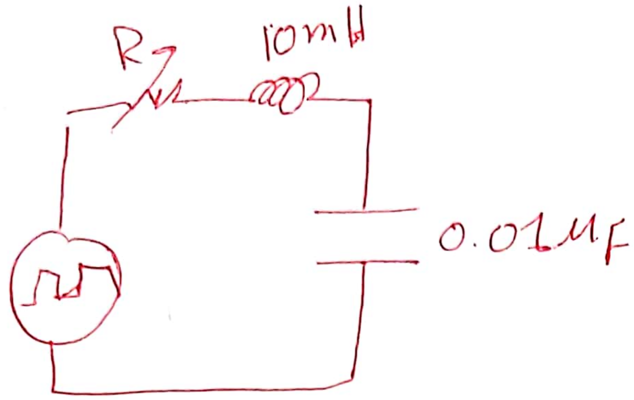
* $\delta_{exp} = \frac{\ln(2)}{t_{1/2}}$

$L = 10\,mH$

$C \approx 25\,nF$

* f_{exp}

Procedure :



1. underdamping

find $t_{1/2}$, f_{exp}

2. Critical and overdamping

find decay constant (δ)

3. define the range of values of R in each case

$t_{1/2}$ → Graph (in 3 cases)

δ → decay constant $\delta = \frac{\ln(2)}{t_{1/2}}$

R → calculate R in each case

find f in underdamping.