Mathematics Department

Math B21 _ Worksheet #4

"10.4 _ 10.5"

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Name: ----

Q1. Which of the following series converge, and which diverge

1. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{2+\ln n}$

2. $\sum_{n=1}^{\infty} \frac{10n+1}{n(n+1)(n+2)}$

3. $\sum_{n=1}^{\infty} \left(1-\frac{1}{n}\right)^{n^2}$

4. $\sum_{n=1}^{\infty} \left(1-\frac{1}{n}\right)^{2n}$

5. $\sum_{n=1}^{\infty} \frac{1}{1+\ln n}$

6. $\sum_{n=1}^{\infty} n! \left(\frac{3}{n}\right)^n$

7. $\sum_{n=1}^{\infty} 1-\cos\left(\frac{1}{n}\right)$

8. $\sum_{n=1}^{\infty} \frac{n!\ln n}{n(n+2)!}$

Hint: $\frac{n!\ln n}{n(n+2)!} = \frac{\ln n}{n(n+2)(n+1)}$

Short Answers.

1 $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{2+\ln n}$ Div. by D.C.T

Since $\frac{\sqrt{n}}{2+\ln n} > \frac{\sqrt{n}}{2+n} \geq \frac{\sqrt{n}}{3n}$ And $\sum_{n=1}^{\infty} \frac{1}{3\sqrt{n}}$ Div.

2 $\sum_{n=1}^{\infty} \frac{10n+1}{n(n+1)(n+2)}$ Con. by D.C.T

$\frac{10n+1}{n(n+1)(n+2)} \leq \frac{11n}{n(n+1)(n+2)} \leq \boxed{\frac{11}{n^2}}$

Conv. by P-series

3 $\sum_{n=1}^{\infty} \left(1-\frac{1}{n}\right)^{n^2}$ Conv. by nth Root Test $\rho = \frac{1}{e} < 1$

4 $\sum_{n=1}^{\infty} \left(1-\frac{1}{n}\right)^{2n}$ Div by nth term test

$\lim_{n\to\infty} \left(1-\frac{2}{2n}\right)^{2n} = e^{-2} \neq 0$

5 $\sum_{n=1}^{\infty} \frac{1}{1+\ln n}$ Div. by D.C.T

$\frac{1}{1+\ln n} > \frac{1}{1+n} \geq \boxed{\frac{1}{2n}}$

Div. Harmonic series

6 $\sum_{n=1}^{\infty} n! \left(\frac{3}{n}\right)^n$ Div. by Ratio Test $\rho = \frac{3}{e} > 1$

7 $\sum_{n=1}^{\infty} 1-\cos\left(\frac{1}{n}\right)$ Con. by L.C.T with $\sum_{n=1}^{\infty} \frac{1}{n^2}$

[8] $\sum_{n=1}^{\infty} \frac{n!\ \ln n}{n(n+2)!}$ Conv. by D.C.T.

→ If you try use Ratio Test $\rho = 1$ Inconclusive

→ $\frac{n!\ \ln n}{n(n+2)!} \leq \frac{\ln n}{n(n+2)(n+1)} \leq \frac{n}{n(n+2)(n+1)} \leq \frac{1}{n^2}$