Chapter 5: Geometrical Optics

Monday, April 4, 2022 7:30 PM

λ is very smalls

Prizms

Monday, March 28, 2022 9:29 PM

Reflecting Prisms

We distinguish two major groups of prisms, *reflecting prisms* and *refracting prisms*. Reflecting prisms, unless they have a reflective coating, make use of total internal reflection. In Figure 1-10 we show several examples. In a *right-angle prism* the light is reflected at the hypotenuse. If the same prism is oriented so that

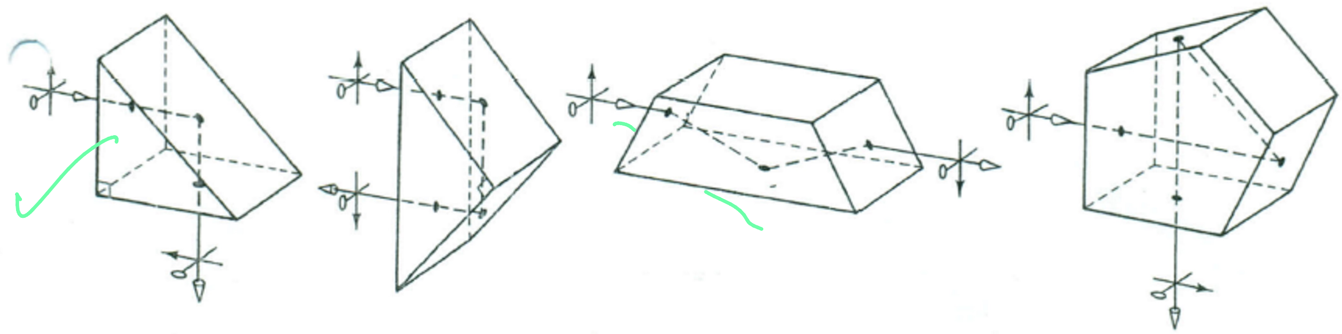

Figure 1-10 Some reflecting prisms, based on total internal reflection are (*from left to right*) right-angle prism, roof prism, Dove prism, and penta prism.

Refracting prisms

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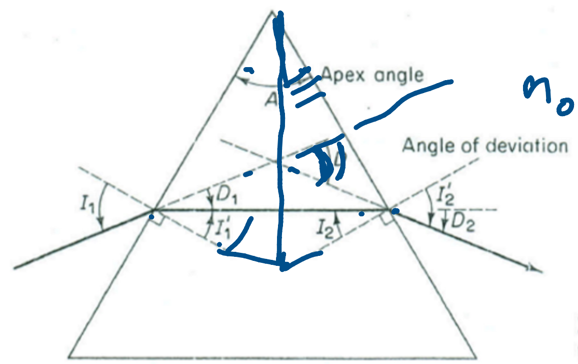
Refracting Prisms



A refracting prism has two plane surfaces that subtend a certain angle A, the *apex angle*. The face opposite the apex is called the *base*. The total angle by which the light changes direction is the *angle of deviation*, D.

When the light passes through the prism *symmetrically* (with equal angles of incidence and emergence), then from the construction in Figure 1-11 it follows that

$$I_1 = I_2'$$

$$I_1' = I_2 = \frac{A}{2}$$

$$D_1 = D_2 = \frac{D}{2} = I_2' - I_2 = I_2' - \frac{A}{2}$$

so that

$$I_2' = \frac{A}{2} + \frac{D}{2}$$

Now we apply Snell's law, $n \sin I = n' \sin I'$, to the second (right-hand) surface:

$$n_{\text{prism}} \sin I_2 = n_0 \sin I_2'$$

where n_0 is the index outside the prism. Solving for $\sin I_2'$ gives

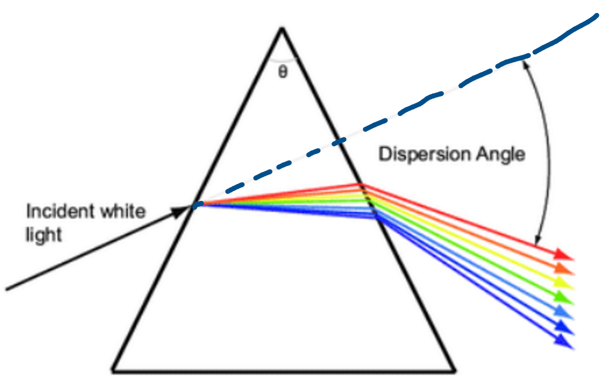
$$\sin I_2' = \left(\frac{n_{\text{prism}}}{n_0}\right) \sin I_2$$

and thus

$$\sin\left(\frac{A+D}{2}\right) = \left(\frac{n_{\text{prism}}}{n_0}\right) \sin\left(\frac{A}{2}\right)$$

$$\boxed{\frac{n_{\text{prism}}}{n_0} = \frac{\sin \frac{1}{2}(A + D)}{\sin \frac{1}{2}A}} \qquad [1\text{-}7]$$

which is the *prism equation*. It determines the *angle of minimum deviation*. For light not passing through symmetrically, the deviation is larger.



(a)

Refraction at Spherical surfaces

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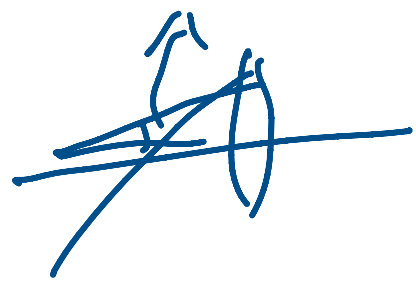
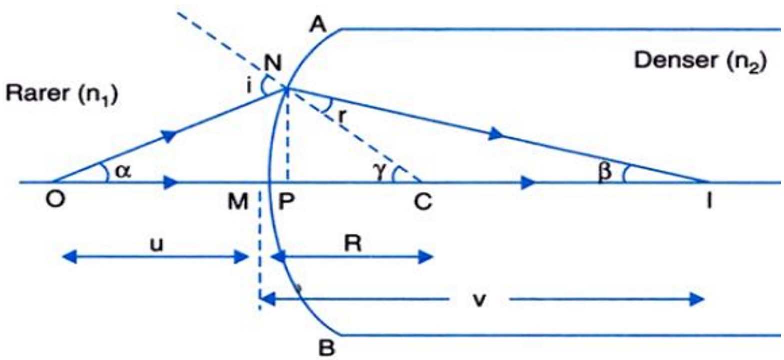
$\sin\theta = \tan\theta$
$= \theta$

Paraxial appeoximation

Here, while considering the refraction at spherical surfaces, we assume:

1. The object taken here is point sized, and is lying on the principal axis of the spherical refracting surface.
2. The aperture of the spherical refracting surface is small.
3. The incident and the refracted rays make small angles with the principal axis of the spherical surface so that $\sin i \approx i$ and $\sin r \approx r$.





Sign conventions

Friday, April 1, 2022 12:57 PM

Sign Conventions for Refraction at Spherical Surfaces

While studying refraction at spherical surfaces, we follow the below mentioned sign convention:

1. All distances are measured from the pole of the spherical refracting surface.
2. The distances measured in the direction of incidence of light are taken as positive, and the distances measured in a direction opposite to the direction of incidence of light are taken as negative.
3. The distances measured in the perpendicular direction above the principal axis are taken as positive. And the distances measured in the perpendicular direction below the principal axis are taken as negative.

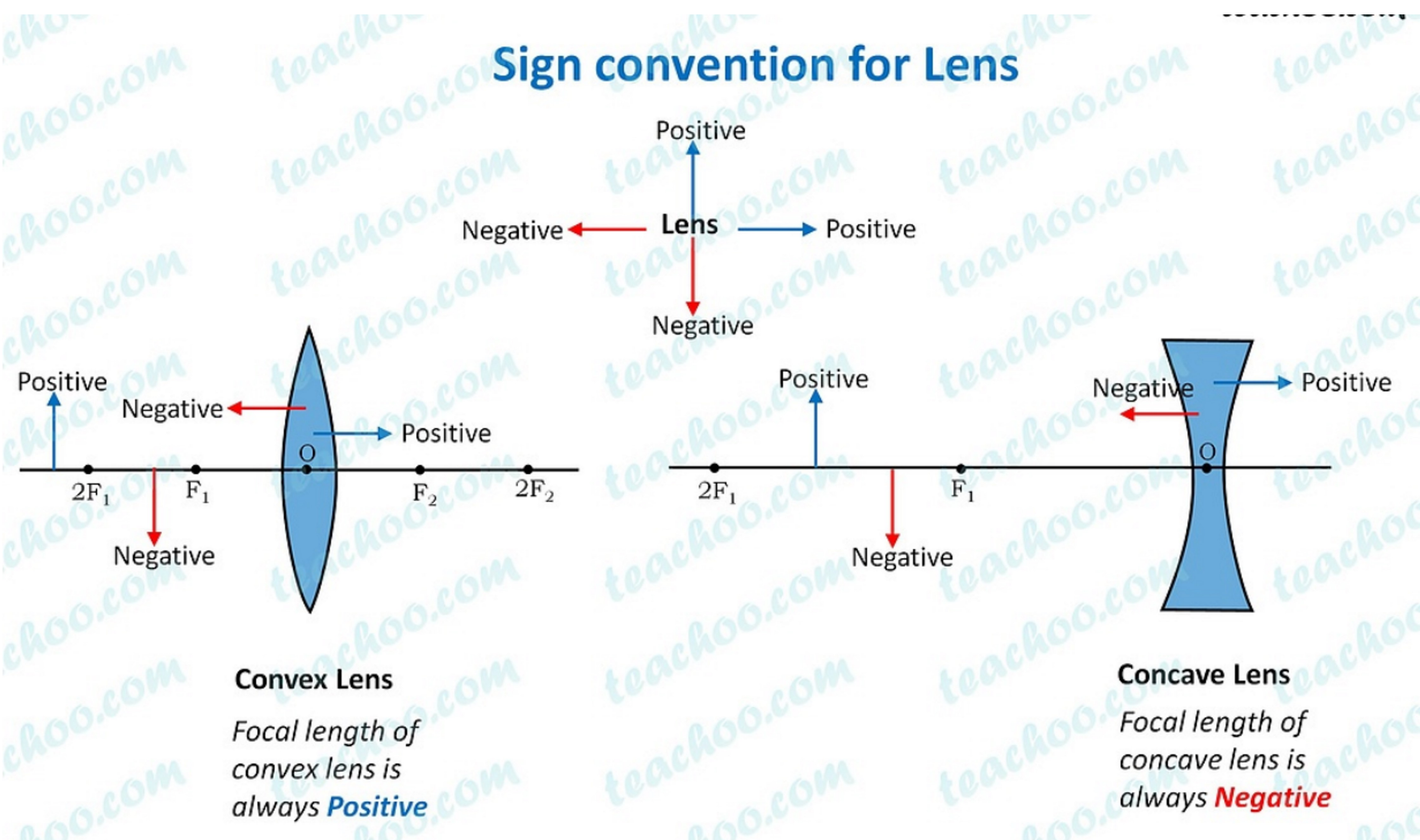


Image formation at a spherical interface

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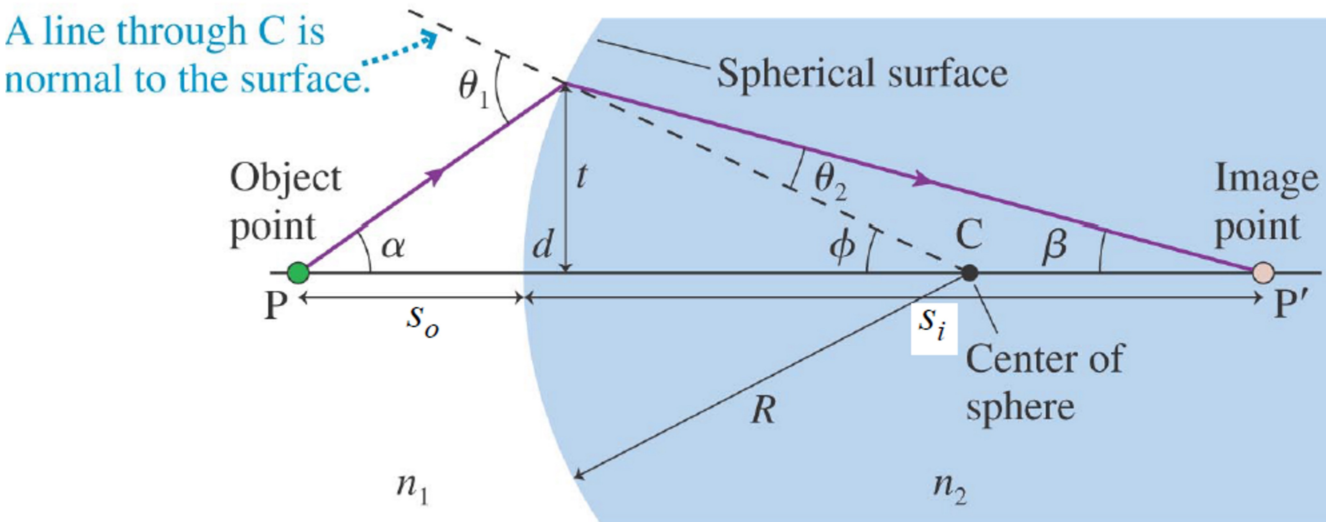
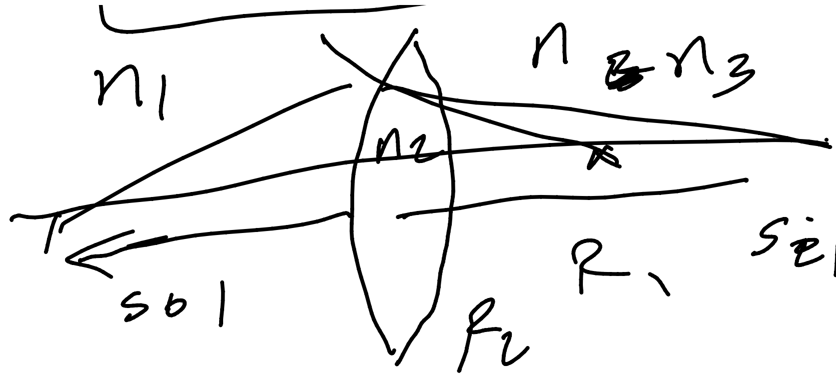
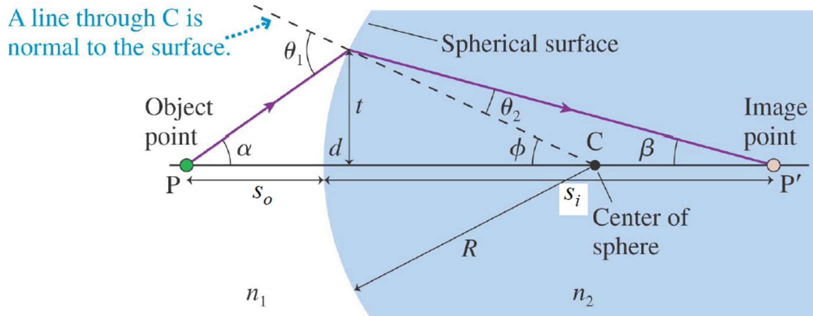


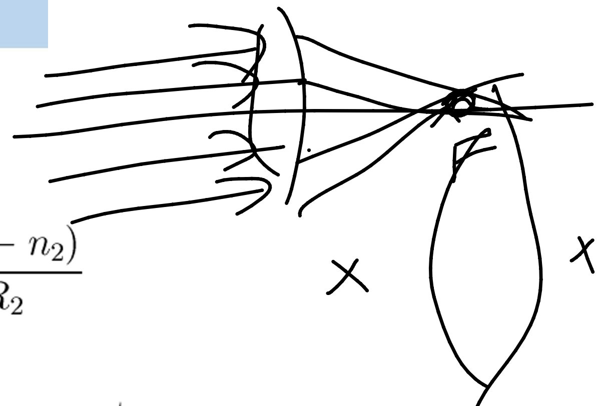
$$
\begin{aligned}
n_1 \sin(\theta_1) &= n_2 \sin(\theta_2) \\
n_1\theta_1 &\approx n_2\theta_2 \\
\theta_1 &= \alpha + \phi \\
\theta_2 &= \phi - \beta \\
n_1(\alpha + \phi) &\approx n_2(\phi - \beta) \\
n_1\alpha + n_2\beta &\approx (n_2 - n_1)\phi \\
n_1\frac{t}{s_{o1}} + n_2\frac{t}{s_{i1}} &\approx (n_2 - n_1)\frac{t}{R_1} \\
\frac{n_1}{s_{o1}} + \frac{n_2}{s_{i1}} &\approx \frac{(n_2 - n_1)}{R_1}
\end{aligned}
$$

n_1 n n_3



n_1
n_3
n_2
s_{o1}
R_1
R_2
s_{i1}





For the second surface

$$\frac{n_2}{s_{o2}} + \frac{n_3}{s_{i2}} \approx \frac{(n_3 - n_2)}{R_2}$$

But $s_{o2} = -s_{i1}$, By adding the two equations, we get

$$\frac{n_1}{s_{o1}} + \frac{n_3}{s_{i2}} \approx n_2\left(\frac{1}{R_1} - \frac{1}{R_2}\right) - \left(\frac{n_1}{R_1} - \frac{n_3}{R_2}\right)$$

Putting $s_{o1} = s_o$, $s_{i2} = s_i$, $n_2 = n_l$, $n_1 = n_3 = n_m$ The lensmaker's formula becomes

$$\boxed{\frac{1}{s_o} + \frac{1}{s_i} \approx \left(\frac{n_l}{n_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

$R_1 > 0$

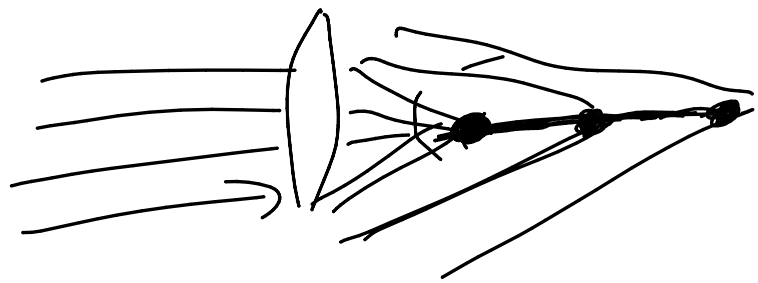
$R_2 < 0$

$S_o = \infty$, $\frac{1}{S_i} = \frac{1}{f}$

$$\frac{1}{f} = \left(\frac{n_l}{n_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$n_m = 1$

$$\frac{1}{f} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$ air



Focal Length

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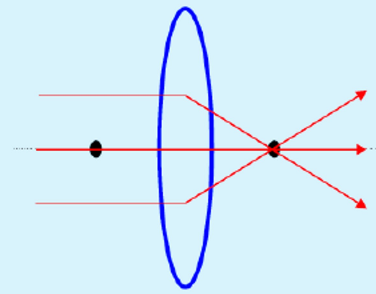
Focal length

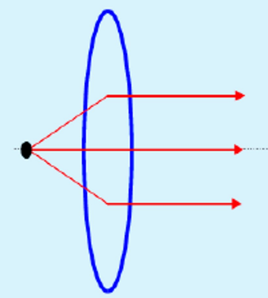
If the object is infinitely far from the lens $s_o = \infty$ then the image will be at a distance $s_i = f$ defined as:



$$\frac{1}{f} = \frac{n_l - n_m}{n_m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

If the object is at a distance f ($s_o = f$) from the lens,

the image will move infinitely far from the lens $s_i = \infty$



This special distance f is called the **focal length**.

We can rewrite the Lensmaker's formula in a form of

the Thin Lens Equation: $$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Example

Saturday, April 2, 2022 12:25 AM

The lensmaker's equation - example

A 4.0 mm object is placed 22 cm to the left of a planoconvex lens. The left surface of the lens is flat, the right surface has a radius of curvature (magnitude) of 12 cm. The refractive index of the lens is 1.6. Calculate the location, size and type of image formed by the lens.

$R_1 \to \infty$, R_2 = - 12 cm

$$\frac{1}{f} = (0.6)\left(\frac{1}{\infty} - \frac{1}{-12}\right) \Rightarrow f = 20\,\text{cm}$$

$$s' = \frac{sf}{s-f} = \frac{22 \times 20}{22-20} = 220\,\text{cm}$$

$$m = -\frac{s'}{s} = -\frac{220}{22} = -10$$

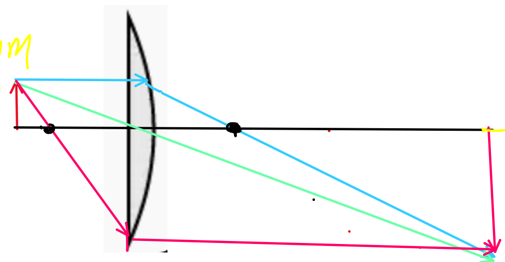


Image 40mm height, real, inverted

$\left(\frac{1}{12} - \frac{1}{\infty}\right)$

$$\frac{1}{f} = \frac{0.6}{12}$$

$$f = \frac{12}{0.6} = 20\,\text{cm}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{s-f}{fs} \Rightarrow s' = \frac{fs}{s-f}$$

Thin Lenses

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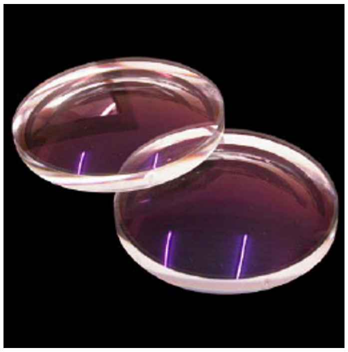
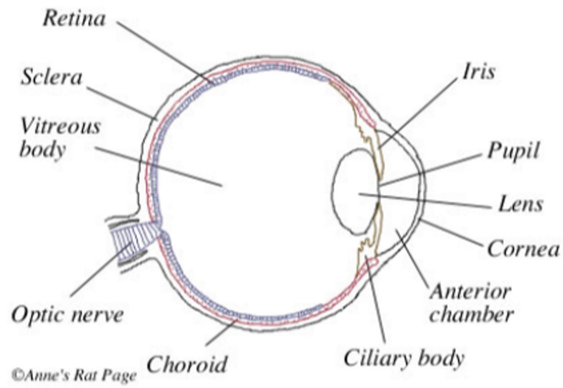




Convex and concave lenses

Friday, April 1, 2022 9:55 PM

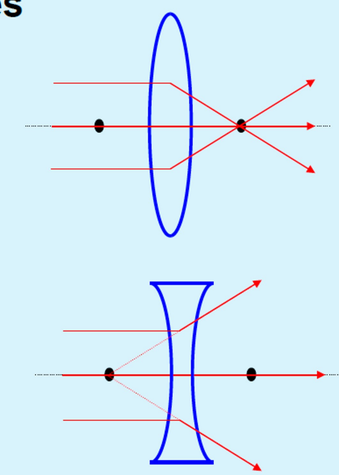
Convex and Concave Lenses

'A lens is a refracting device that reconfigures a transmitted energy distribution'

Converging, convex or positive lens, the central section is thicker than the rim

Diverging, concave or negative lens, central section is thinner than the rim

Materials used for lenses in visible and near infra-red (IR): various types of glass and plastic



Types of thin Lenses

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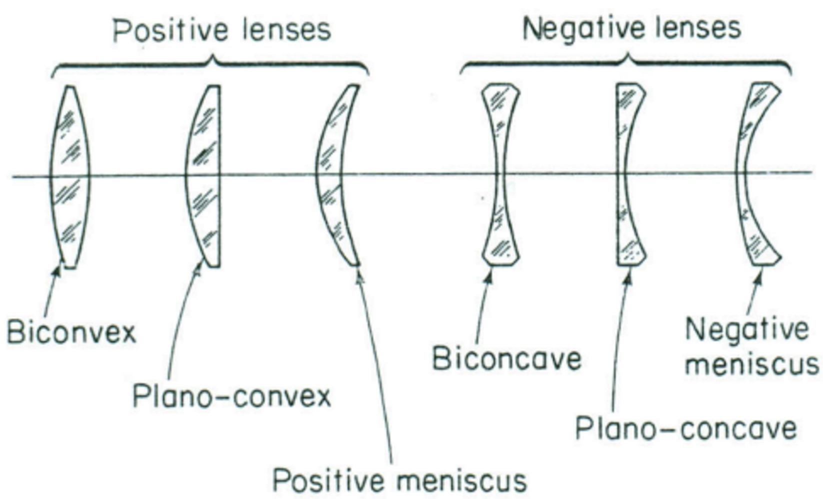


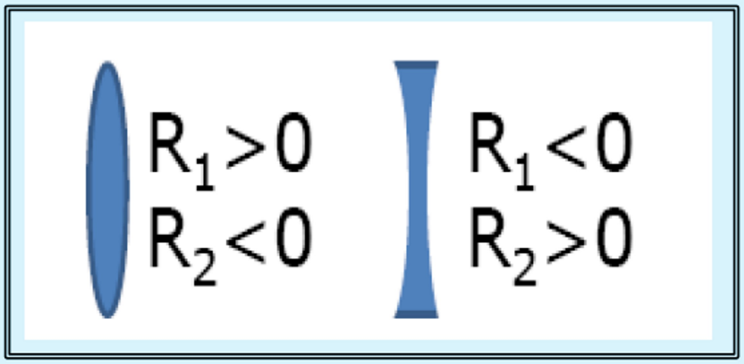
Figure 2-9 Types of lenses.

Sign conventions

Friday, April 1, 2022 10:02 PM

Sign conventions

If light is incident from the left (as will be considered in most of the questions and sketches) the signs of spherical surfaces are as follows:



A convex lens (left) has a positive focal length, a concave lens (right) has a negative focal length

Summary

Friday, April 1, 2022 10:04 PM

SUMMARY

We will use **paraxial approximation** in most cases: angles at which light propagates with respect to the optical axis are small.

The Lensmaker's equation:

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{n_l-n_m}{n_m}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

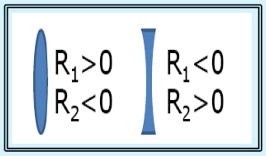
Can be rewritten in the form of **the Thin Lens Equation:**

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f}$$

using the expression for the **focal length:**

$$\frac{1}{f}=\frac{n_l-n_m}{n_m}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

The Lensmaker equation requires signs convention. Figure shows the case for light incident from the left:



$$\frac{1}{f}=\left(\frac{n_l}{n_m}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

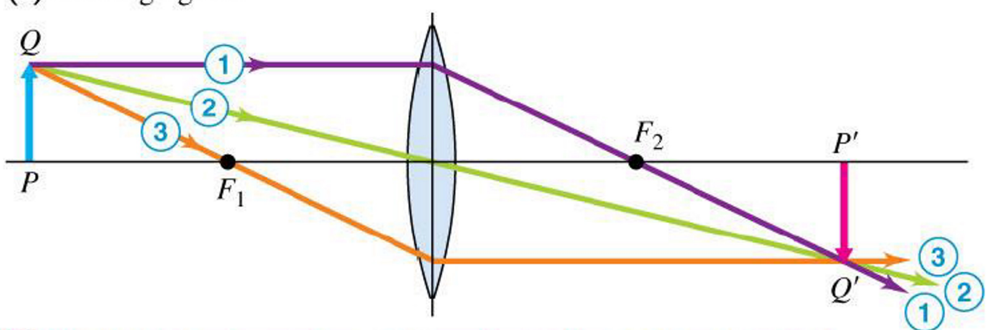
Graphical Methods

Friday, April 1, 2022 10:14 PM

Graphical Methods

- The primary rays for thin lenses are very similar to the ones we chose for mirrors:
  - A ray that is parallel to the lens axis and then passes through the second focal point.
  - A ray through the center of the lens.
  - A ray that passes through the first focal point and then emerges parallel to the axis.

(a) Converging lens



(1) Parallel incident ray refracts to pass through second focal point F_2.

(2) Ray through center of lens does not deviate appreciably.

(3) Ray through the first focal point F_1 emerges parallel to the axis.

Convex lens

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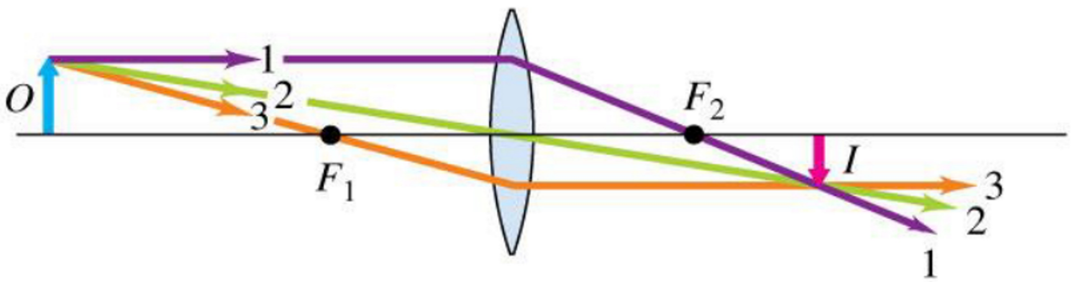
(a) Object O is outside focal point; image I is real.



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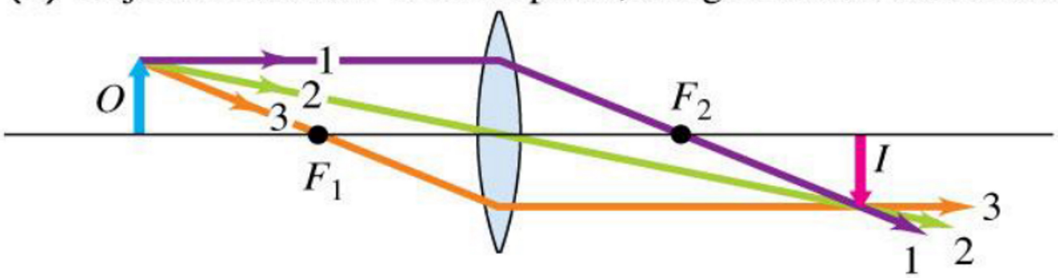
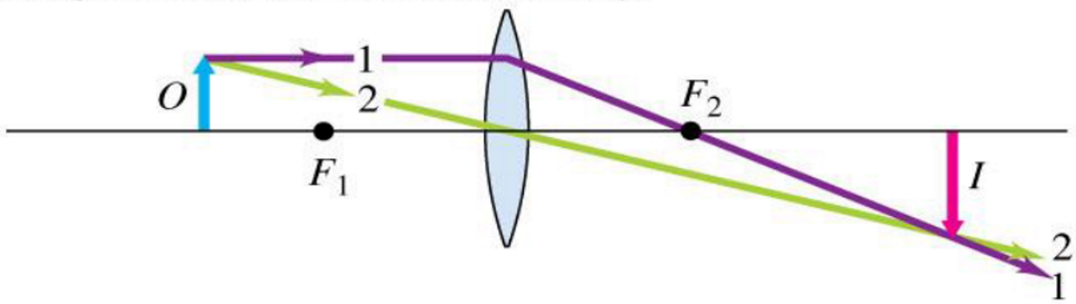
(b) Object O is closer to focal point; image I is real and farther away.



(c) Object O is even closer to focal point; image I is real and even farther away.



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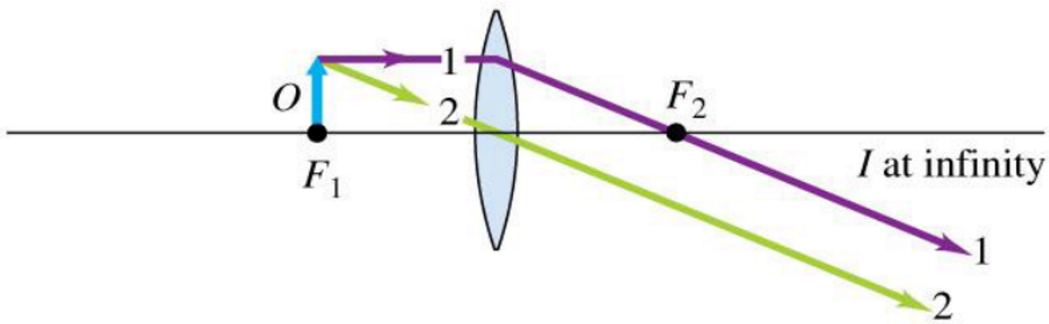
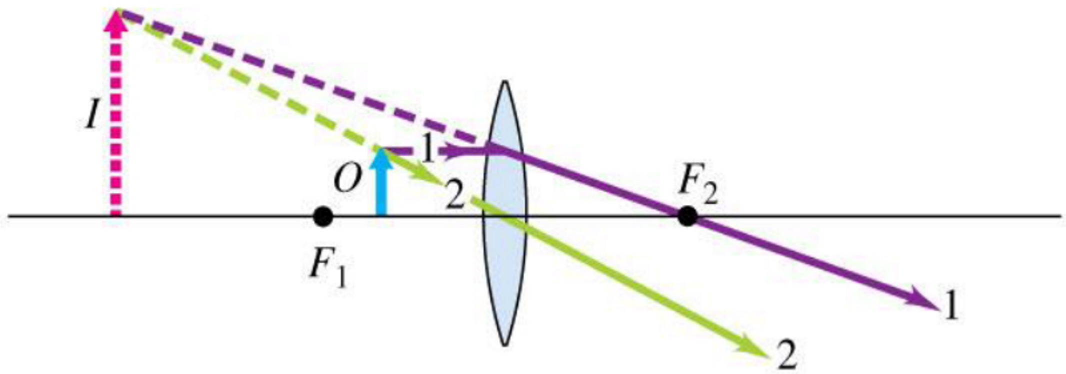
(d) Object O is at focal point; image I is at infinity.



(e) Object O is inside focal point;
image I is virtual and larger than object.

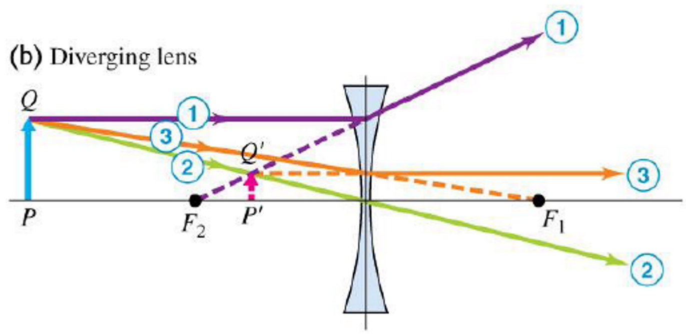


Concave lens

Friday, April 1, 2022 10:13 PM

Graphical Methods

- The primary rays for thin lenses are very similar to the ones we chose for mirrors:
  - A ray that is parallel to the lens axis and then passes through the second focal point.
  - A ray through the center of the lens.
  - A ray that passes through the first focal point and then emerges parallel to the axis.



(1) Parallel incident ray appears after refraction to have come from the second focal point F_2.
(2) Ray through center of lens does not deviate appreciably.
(3) Ray aimed at the first focal point F_1 emerges parallel to the axis.