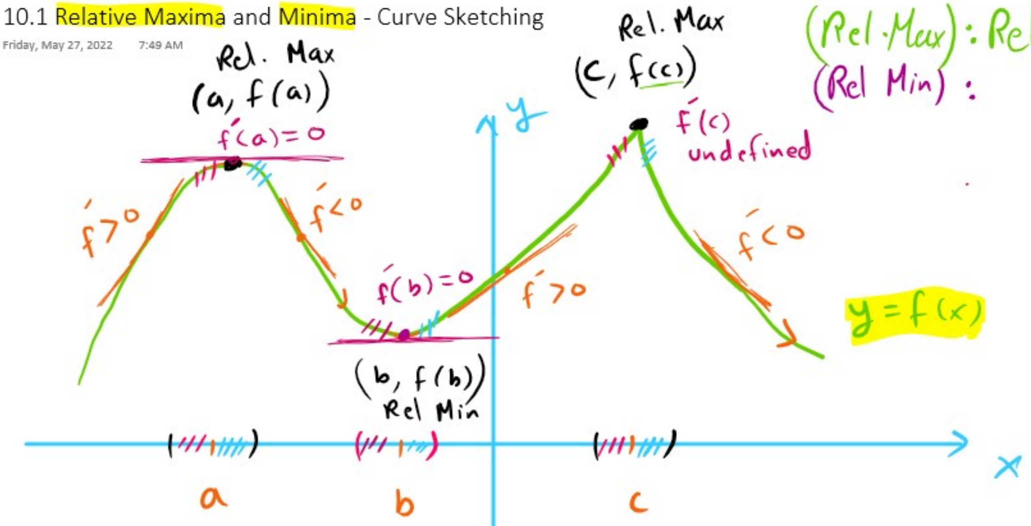
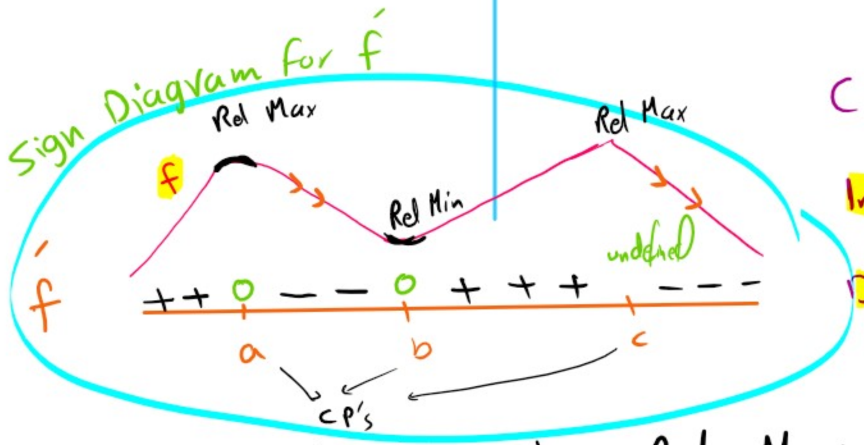
10.1 Relative Maxima and Minima - Curve Sketching

Friday, May 27, 2022 7:49 AM

(Rel.Max) : Relative Maxima (Maximum)

(Rel Min) : = Minima (Minimum)





CP : Critical Point نقطة حرجة

Increasing ↑

Decreasing ↓

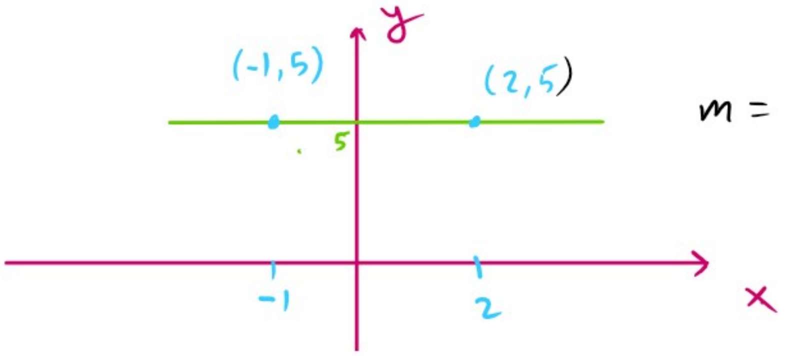
Def. The point $(c, f(c))$ is Rel Max if

$f(c) \geq f(x)$ for all x in small interval around c

فترة

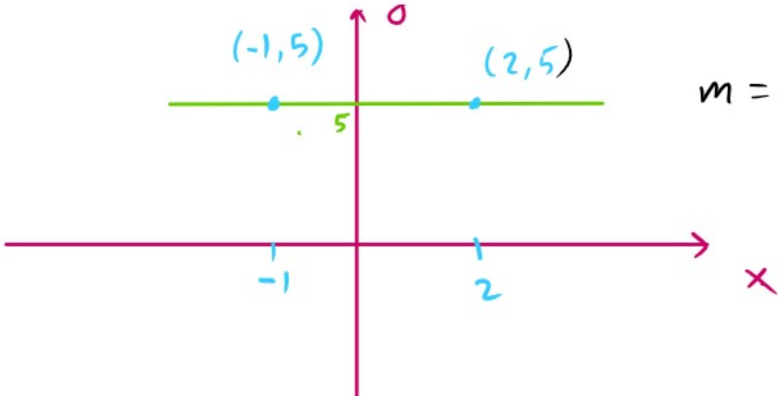
. The point $(b, f(b))$ is Rel Min if

$f(b) \leq f(x)$ for all x in small interval around c



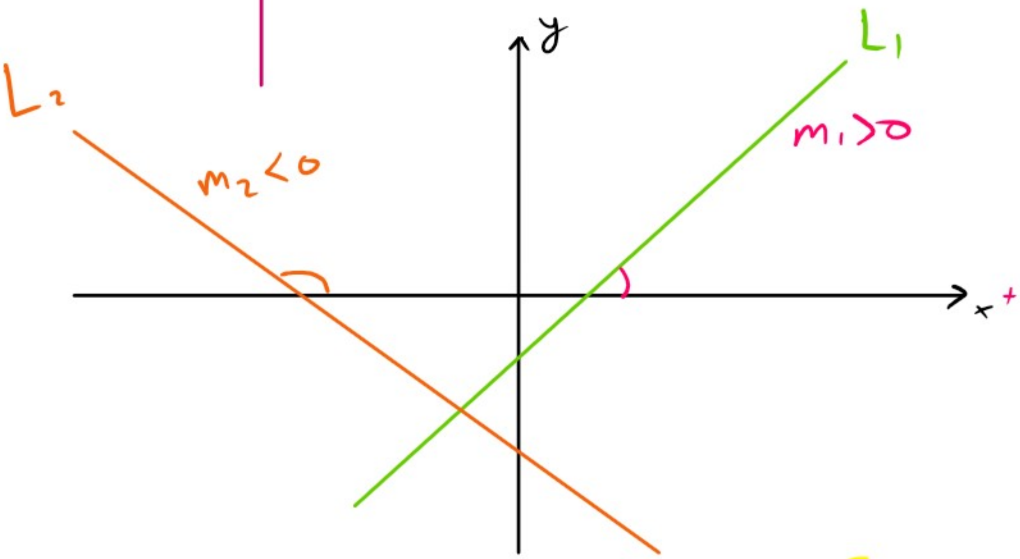
$$m = \text{slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5-5}{2--1} = \frac{0}{3} = 0$$



$$m = \text{slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5 - 5}{2 - -1} = \frac{0}{3} = 0$$



CP's : occurs when → $f' = 0$ or → f' undefined

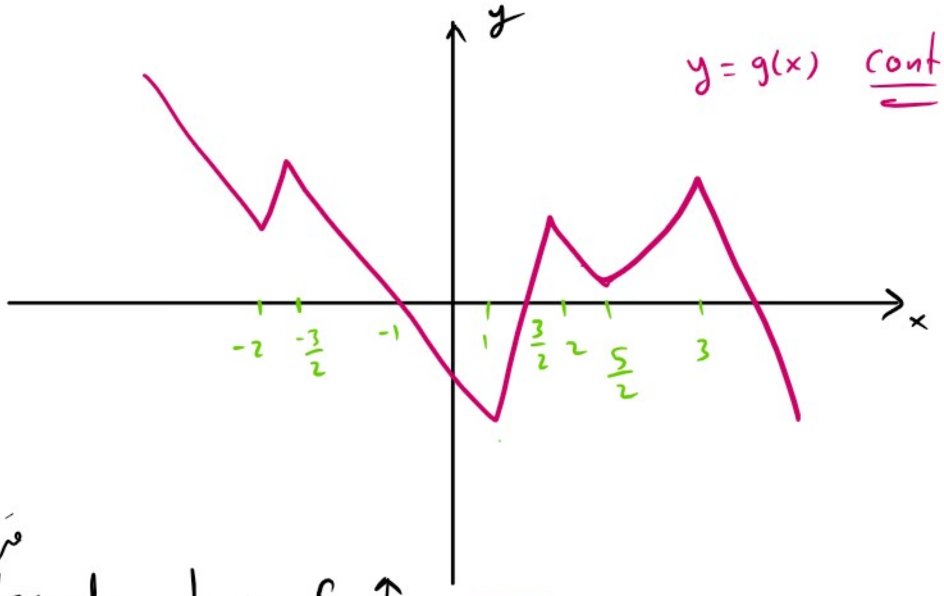
Critical Points

$(a, f(a))$ is CP since $f'(a) = 0$

$(b, f(b))$ is CP since $f'(b) = 0$

$(c, f(c))$ is CP since $f'(c)$ undefined

Critical Values occure at $x = a, b, c$



$f'(-2)$ undefined

$f'(-\frac{3}{2}) =$

$f'(\frac{5}{2}) =$

$f'(3) =$

$\vdots$

فترات

- Intervals where f ↑

f is increasing on $(-\infty, a) \cup (b, c)$

- intervals where f ↓

f is decreasing on $(a, b) \cup (c, \infty)$

Def (intervals where f ↑ or f ↓)

Assume f differentiable قابل للاشتقاق

- If $f'(x) > 0$ for all $x \in (c_1, c_2)$ then

  f is increasing on (c_1, c_2)

- If $f'(x) < 0$ for all $x \in (d_1, d_2)$ then

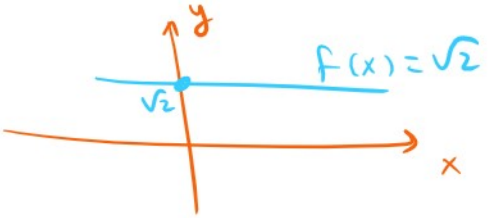
  f is decreasing on (d_1, d_2)

f is decreasing on (d_1, d_2)

- If $f'(x) = 0$ then f is constant

Exp $f(x) = \sqrt{2} \Rightarrow \boxed{f'(x) = 0}$

$\sqrt{2}$ → constant

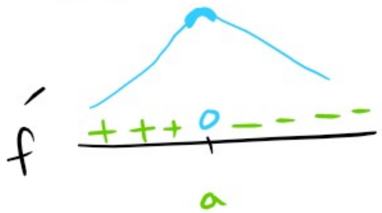


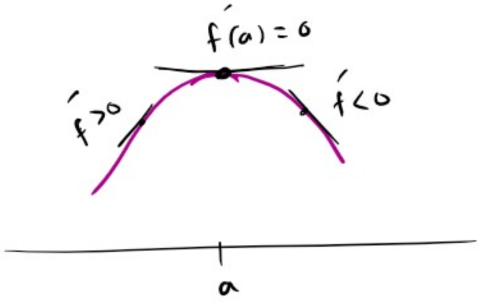
Classification of CP's

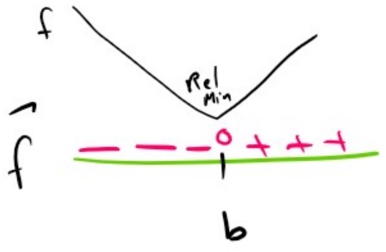
give → Rel Max

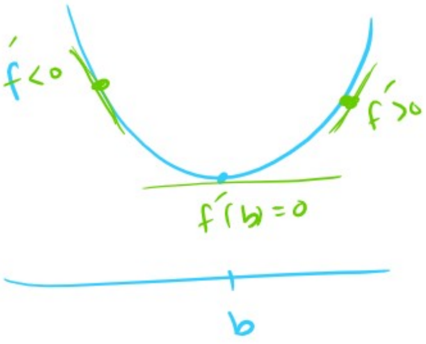
or → Rel Min

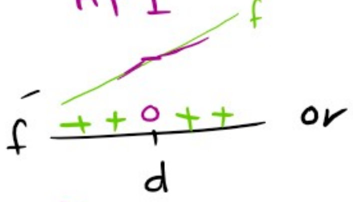
or → Horizontal Point of Inflection HPI

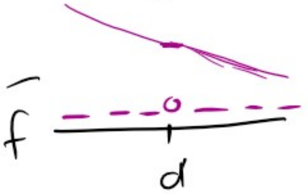


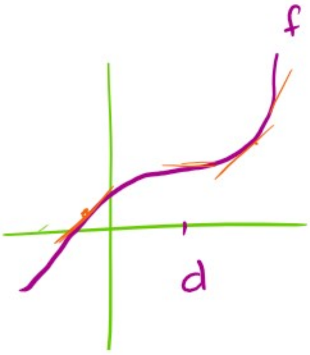


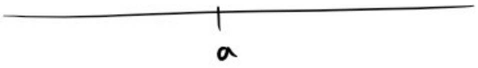


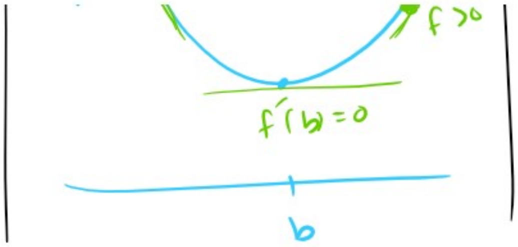


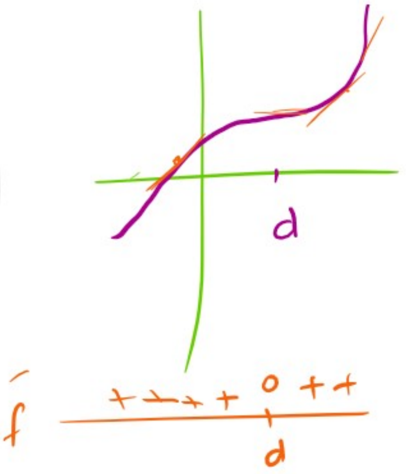












Exp $f(x) = x^3 - 3x^2 - 1$ Find

① critical values

$f'(x) = 3x^2 - 6x$

critical values $x = 0, 2$

CP

or

$f' = 0$ ✓

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$3x = 0, \quad x - 2 = 0$

$\boxed{x=0}, \quad \boxed{x=2}$

f' undefined

صفر في المقام

$\emptyset$

② Critical Points

$(0, f(0)) = (0, -1)$

$(2, f(2)) = (2, -5)$

$f(x) = x^3 - 3x^2 - 1$

$f(0) = 0 - 0 - 1 = -1$

$f(2) = \boxed{2}^3 - 3\boxed{2}^2 - 1$

$= 8 - 3(4) - 1$

$= 8 - 12 - 1$

$= -5$

③ Intervals where f is increasing

$f'(x) = 3x^2 - 6x$

+

f

$f'(-1) = 3\boxed{-1}^2 - 6\boxed{-1}$

$= 3 + 6$

$= 9 > 0$

$f'(1) = 3\boxed{1}^2 - 6\boxed{1}$

$= 3 - 6$

$= -3 < 0$

$f'(3) = 3\boxed{3}^2 - 6\boxed{3}$

$= 3(9) - 18$

$= 27 - 18 > 0$

f is increasing on $(-\infty, 0) \cup (2, \infty)$

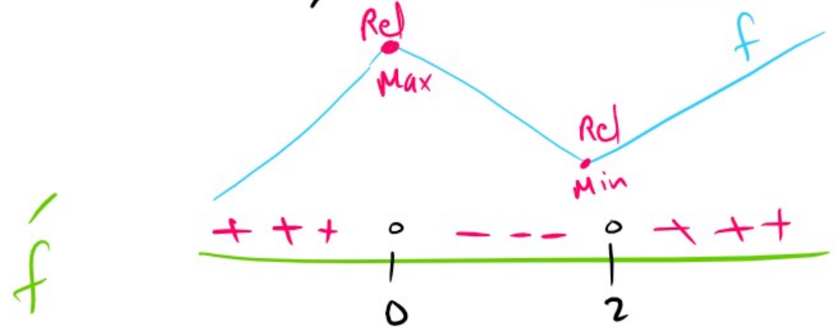
④ intervals where f decreasing

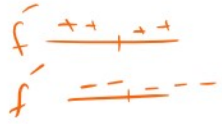
f is decreasing on $(0, 2)$

⑤ Classify the critical Points into

Rel Max / Rel Min / HPI

none





$(0, f(0)) = (0,-1)$ is Rel Max

$(2, f(2)) = (2,-5)$ is Rel Min

→ or f has Rel Max of $f(0) = -1$ occurs at $x = 0$

→ or f 〃 〃 Min of $f(2) = -5$ 〃 〃 $x = 2$

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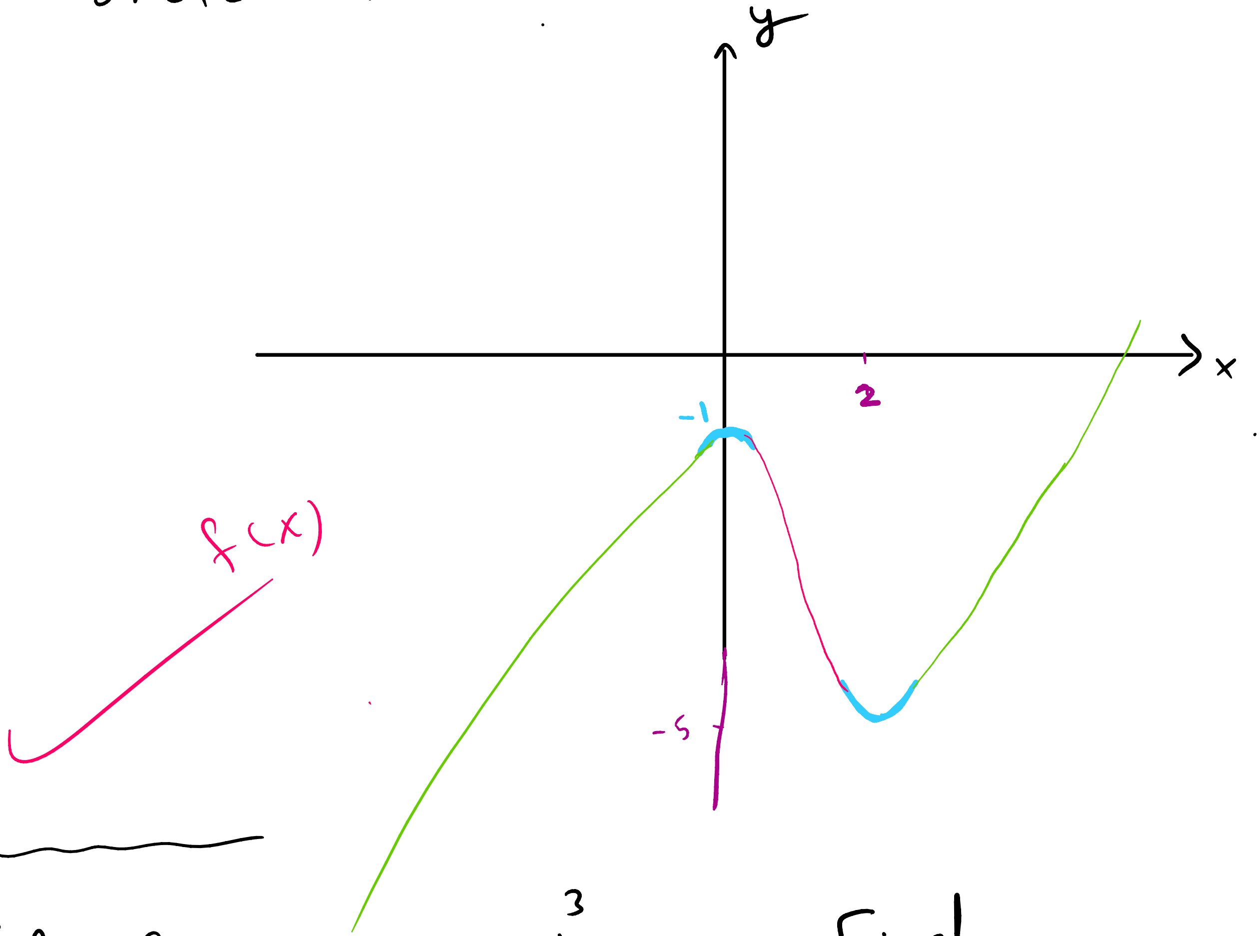
⑥ Sketch $f(x)$



Exp $f(x) = \frac{1}{3}(x+1)^3$ Find

① Critical Values

CP

$f'(x) = \cancel{\frac{1}{3}}\,(\cancel{3})\,(\underbrace{x+1}_{})^{2} \rightarrow (1)$

$f'(x) = (x+1)^{2}$

Critical values $x = -1$

CP

$f' = 0$ | f' undefined

$(x+1)^{2} = 0$ | صفر في المقام

$x + 1 = 0$ | $\emptyset$

$\boxed{x = -1}$

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② Critical Points

$\left(-1, f(-1)\right) = (-1, 0)$

$f(x) = \frac{1}{3}(x+1)^{3}$

$f(-1) = \frac{1}{3}(-1+1)^{3} = 0$

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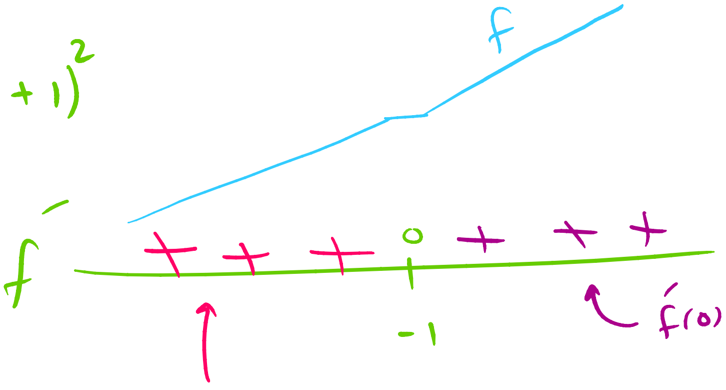
③ Intervals where f is increasing / Decreasing

$f'(x) = (x+1)^{2}$



$f'(0) = (0+1)^{2} = 1 > 0$

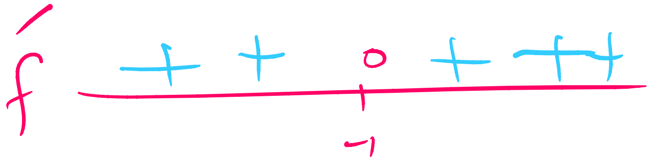
$f'(-2) = (-2+1)^{2} = (-1)^{2} = 1 > 0$

f is increasing on $(-\infty, \infty) = \mathbb{R}$

$\cancel{\exists}$ intervals where f is decreasing

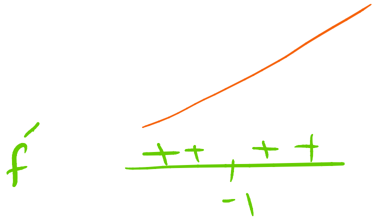
④ Classify the Critical Points

④ Classify the Critical Points

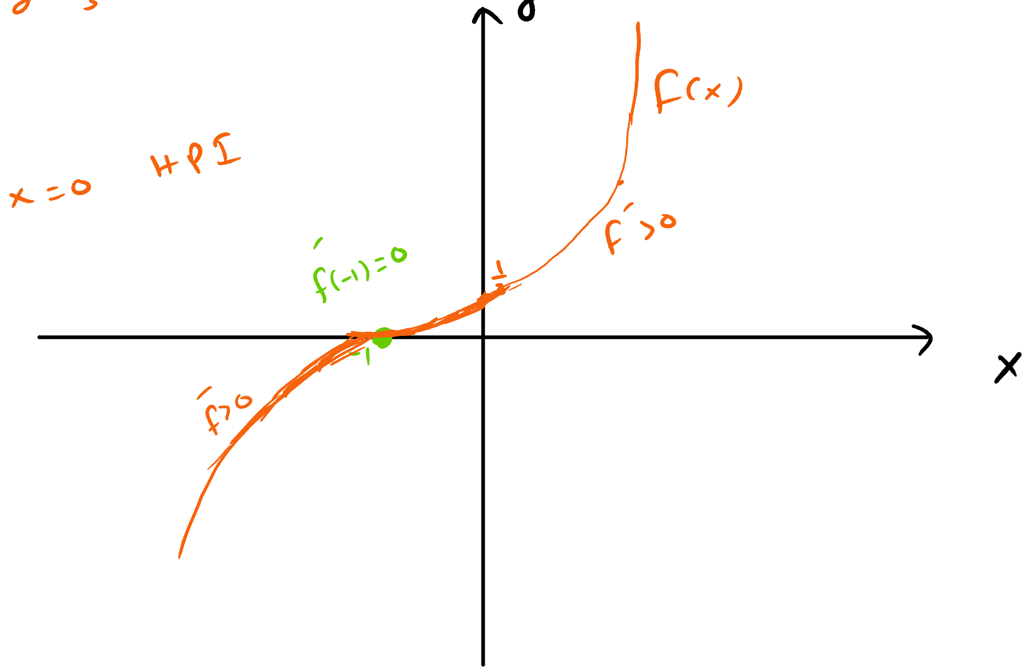


$(-1, f(-1)) = (-1, 0)$ is HPI

⑤ Sketch $f(x) = \frac{1}{3}(x+1)^3$



$x=0 \Rightarrow y = \frac{1}{3}(0+1)^3 = \frac{1}{3}$



Exp $f(x) = \sqrt[3]{(x+2)^2}$ Find

① Critical Values

$\frac{2}{3}$ $\Rightarrow f'(x) = \frac{2}{3}(x+2)^{\frac{2}{3}-1}$, (1) ① Critical values

$f(x) = (x+2)^{\frac{2}{3}} \Rightarrow f'(x) = \frac{2}{3}(x+2), (1)$

$= \frac{2}{3}(x+2)^{-\frac{1}{3}}$

$= \frac{2}{3\sqrt[3]{x+2}}$

cp

or

$f' = 0$ $\qquad$ f' undefined

$\frac{2}{3\sqrt[3]{x+2}} = 0$

$\cancel{2 = 0}$

$\boxed{x = -2}$

$x+2=0$

$x=-2$

critical value

$x = -2$

---

② Critical Point

$f(x) = \sqrt[3]{(x+2)^2}$

$f(-2) = \sqrt[3]{(-2+2)^2} = 0$

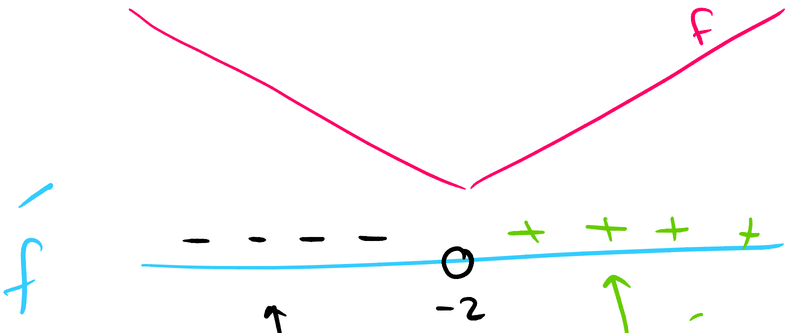
$(-2, f(-2)) = (-2, 0)$

③ Intervals where f increasing/decreasing

$f'(x) = \frac{2}{3\sqrt[3]{x+2}}$

f is increasing on $(-2, \infty)$

f is decreasing on $(-\infty, -2)$



$f'(0) = \frac{2}{3\sqrt[3]{(0)+2}} = \frac{2}{3\sqrt[3]{2}} > 0$

1

$f(0) = \frac{}{3\sqrt[3]{(0)+2}} = \frac{}{3\sqrt[3]{2}}$

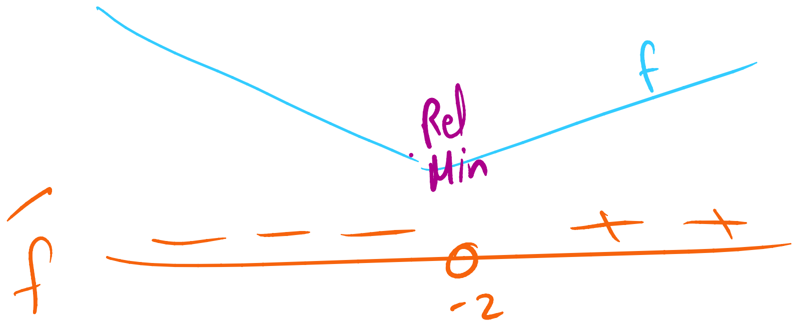
$f'(-3) = \frac{2}{3\sqrt[3]{(-3)+2}} = \frac{2}{3\sqrt[3]{-1}} = \frac{2}{3(-1)} = \frac{-2}{3} < 0$

④ Classify the critical Points

Rel Max

Rel Min

H.P.I



$(-2, f(-2)) = (-2, 0)$ is Rel Min

or f has Min value of 0 occurs at $x = -2$

Exp $f(x) = \frac{x^4}{4} - \frac{x^3}{3} - x^2$ Find

① Critical Values

$f'(x) = x^3 - x^2 - 2x$

critical values $x = -1, 0, 2$

CP

$f' = 0$

$x^3 - x^2 - 2x = 0$

$x(x^2 - x - 2) = 0$

f' undefined

صفر في المقام

$\emptyset$

$x(x^2-x-2)=0$

$x(x-2)(x+1)=0$

$x=0,\quad x=2,\quad x=-1$

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② Critical Points

$(0, f(0)) = (0,0)$

$(2, f(2)) = (2, -\frac{8}{3})$

$(-1, f(-1)) = (-1, -\frac{5}{12})$

$f(2) = \frac{16}{4} - \frac{8}{3} - 4$

$= \cancel{4} - \frac{8}{3} - \cancel{4}$

$= -\frac{8}{3}$

$f(x) = \frac{x^4}{4} - \frac{x^3}{3} - x^2$

$f(-1) = \frac{1}{4} - \frac{-1}{3} - 1$

$= \frac{1}{4} + \frac{1}{3} - 1$

$= \frac{1}{4} - \frac{2}{3}$

$= \frac{-5}{12}$

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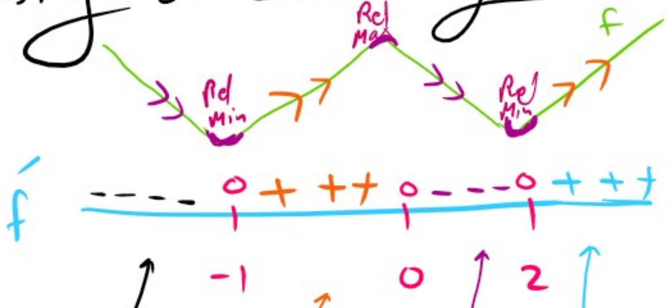
③ Intervals where f is increasing or decreasing

$f'(x) = x^3 - x^2 - 2x$

$= x(x^2 - x - 2)$

$= x(x-2)(x+1)$



$f'(-2) = (-2)(-4)(-1) < 0$

$f'(-\frac{1}{2}) = (-\frac{1}{2})(-2.5)(\frac{1}{2}) > 0$

$f'(1) = (1)(-1)(2) < 0$

$f'(3) = (3)(1)(4) > 0$

f is increasing on

$(-1,0) \cup (2,\infty)$.

f is decreasing on

$(-\infty,-1) \cup (0,2)$

④ classify the critical points into Rel Max
Rel Min
HPI

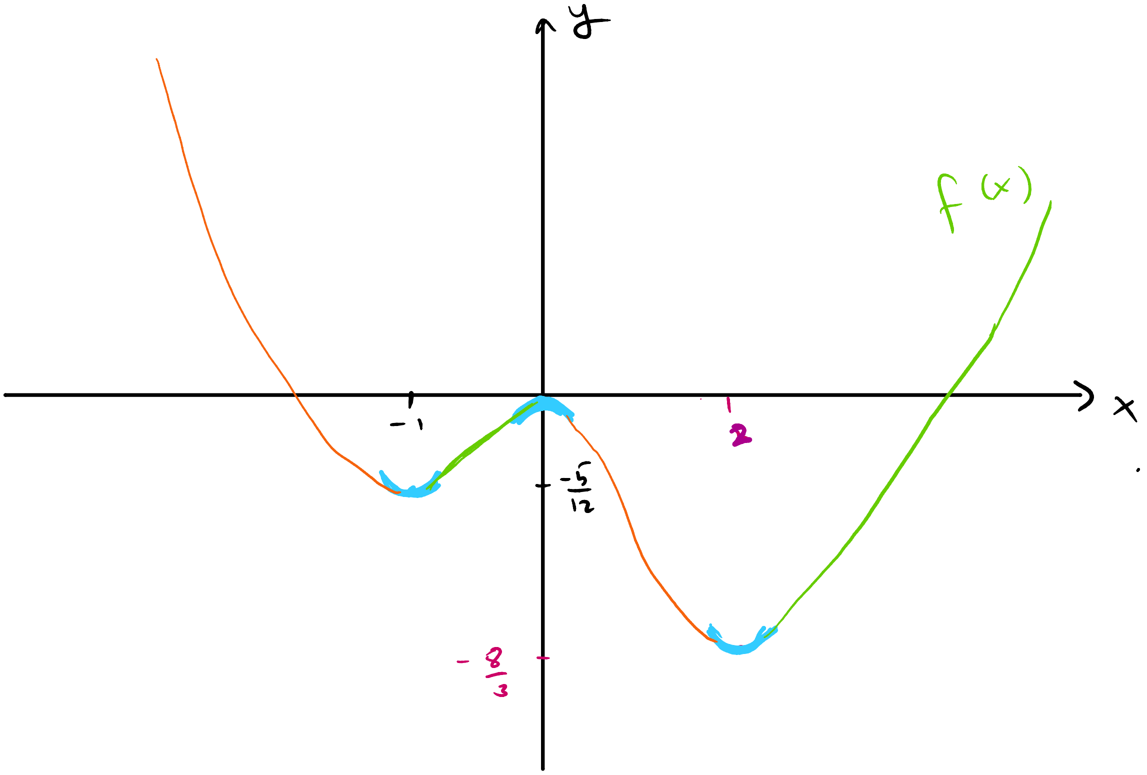
$(-1, f(-1)) = (-1, \frac{-5}{12})$ is Rel Min

$(0, f(0)) = (0,0)$ is Rel Max

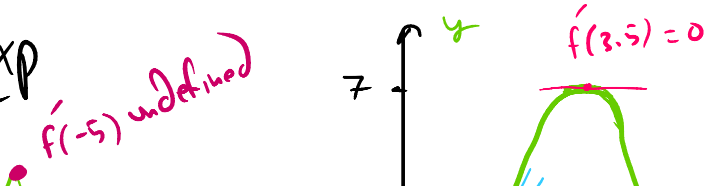
$(2, f(2)) = (2, -\frac{8}{3})$ is Rel Min

HPI
None

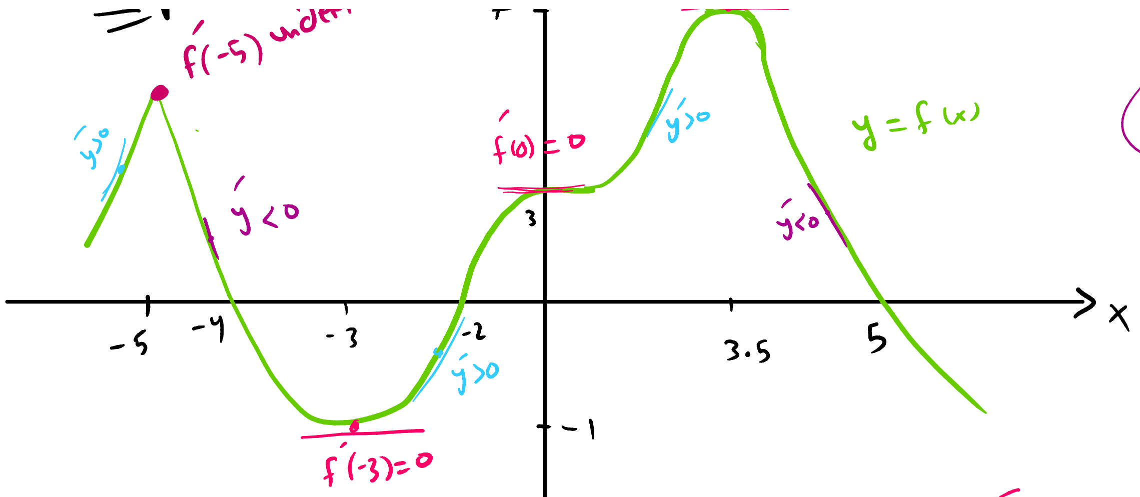
⑤ sketch f(x)



Exp



① Find values of x where $y' > 0$, $y' < 0$, $y' = 0$



x where $y'>0$, $y'<0$, $y'=0$, y' DNE

- $y'>0 \Rightarrow (-\infty,-5) \cup (-3,3.5)$
- $y'<0 \Rightarrow (-5,-3) \cup (3.5,\infty)$

- $y'=0 \Rightarrow x=-3,0,3.5$
- y' DNE $\Rightarrow x=-5$

Exp (Production Cost)

Suppose the average cost, in dollars, of producing a certain product is $\bar{C} = 5000x + \frac{125\,000}{x}$, $x>0$ where x is the number of machines used in the production process

① Find the CP's for this function

$\bar{c} = 5000x + 125000x^{-1}$

$\bar{c}' = 5000 - (-1)(125000)x^{-2}$

$$\bar{C}' = 5000 - \frac{125\,000}{x^2}$$

CP

$\bar{C}'=0$

$\bar{C}'$ undefined

$\Downarrow$

Critical Points

$(5, \bar{C}(5)) = (5, 30000)$

$5000 - \frac{125\,000}{}=0$

$(5, \bar{C}(5)) = (5, 30000)$

$$\bar{C}(x) = 5000x + \frac{125000}{x}$$

$$\bar{C}(5) = 5000(5) + \frac{125000}{(5)}$$

$$= 25000 + \frac{\overset{25}{\cancel{125000}}}{\cancel{5}}$$

$$= 25000 + 25000$$

$= \overset{5}{\cancel{3}}0{,}000$ dollars

$$5000 - \frac{125000}{x^2} = 0$$

$$\overset{1}{\cancel{5000}} = \frac{\overset{25}{\cancel{125000}}}{x^2}$$

$$1 = \frac{25}{x^2}$$

$$x^2 = 25$$

$$x = \pm 5$$

$\boxed{x = 5}$, $x = -5$ مرفوضة

$\Downarrow$

$x = 0$

مرفوضة

لأن

$x > 0$

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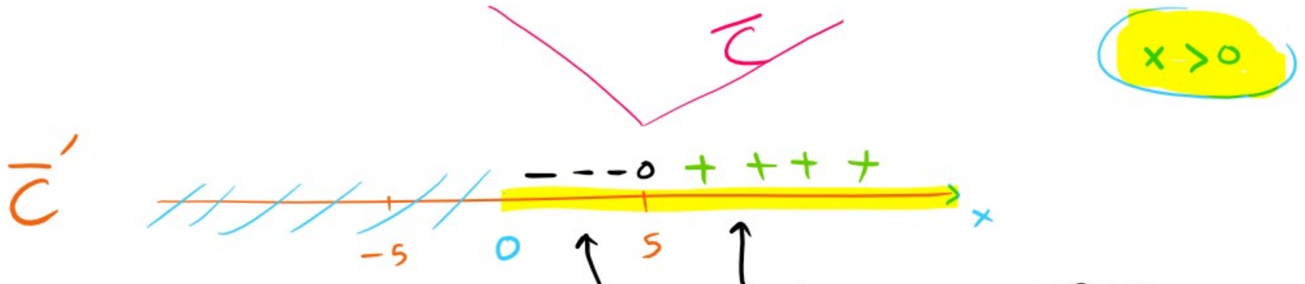
② Over what values of x does the average cost increase/decrease

$$\bar{C}'(x) = 5000 - \frac{125000}{x^2} = \frac{5000x^2 - 125000}{x^2}$$



$$\bar{C}'(10) = 5000 - \frac{12500\cancel{0}}{10\cancel{0}} > 0$$

$$\bar{C}'(1) = 5000 - 125000 < 0$$

$\bar{C}'(1) = 5000 - 125000 < 0$

The average cost is decreasing on $(0, 5)$

〃 〃 〃 〃 increasing on $(5, \infty)$