Discussion $\boxed{8.4}$

17) $\displaystyle\int_0^1 \frac{x^3\,dx}{x^2+2x+1}$ $\Rightarrow$ we use long Division $\Rightarrow$

$$\begin{array}{r|l} & x-2 \\ \hline x^2+2x+1 & x^3 \\ & \cancel{-x^3} \mp 2x^2 \mp x \\ \hline & \cancel{-2x^2} - x \\ & \cancel{+2x^2} + 4x + 2 \\ \hline & 3x+2 \end{array}$$

$$= \int_0^1 \left(x-2 + \frac{3x+2}{x^2+2x+1} \right) dx$$

$$= \frac{x^2}{2} - 2x \Big|_0^1 + \int_0^1 \frac{3x+2}{(x+1)^2}\,dx$$

$$= \left(\frac{1}{2}-2\right) - (0-0) + \int_0^1 \left(\frac{A}{x+1} + \frac{B}{(x+1)^2} \right) dx$$

since $\displaystyle\frac{3x+2}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2} = \frac{A(x+1)}{(x+1)^2} + \frac{B}{(x+1)^2} = \frac{A(x+1)+B}{(x+1)^2}$

$3x+2 = A(x+1) + B$

$3x+2 = Ax + A + B$

$\boxed{A=3}$ and $A+B=2$

$\boxed{B=-1}$

$$= -\frac{3}{2} + \int_0^1 \frac{3}{x+1}\,dx - \int_0^1 \frac{dx}{(x+1)^2}$$

$$= -\frac{3}{2} + 3\ln|x+1| \Big|_0^1 + \frac{1}{x+1}\Big|_0^1$$

$u = x+1$

$du = dx$

$$\int \frac{dx}{u^2} = \int u^{-2}\,du = \frac{u^{-1}}{-1} = -\frac{1}{u} = -\frac{1}{x+1}$$

$$= -\frac{3}{2} + (3\ln 2 - 3\ln 1) + \left(\frac{1}{2} - \frac{1}{1}\right)$$

$$= -\frac{3}{2} + 3\ln 2 - 0 + \frac{1}{2} - 1$$

$$= 3\ln 2 - 2$$

(30) $$\int \frac{x^2+x}{x^4-3x^2-4}\,dx = \int \frac{x^2+x}{(x^2-4)(x^2+1)}\,dx = \int \frac{(x^2+x)\,dx}{(x-2)(x+2)(x^2+1)}$$

$$\frac{x^2+x}{(x-2)(x+2)(x^2+1)} = \frac{A}{x-2} + \frac{B}{x+2} + \frac{Cx+D}{x^2+1}$$

$$= \frac{A(x+2)}{x^2-4} + \frac{B(x-2)}{x^2-4} + \frac{Cx+D}{x^2+1}$$

$$= \frac{A(x+2)+B(x-2)}{x^2-4} + \frac{Cx+D}{x^2+1}$$

$$= \frac{A(x+2)(x^2+1)+B(x-2)(x^2+1)+(Cx+D)(x^2-4)}{(x^2-4)(x^2+1)}$$

$$x^2+x = A(x+2)(x^2+1) + B(x-2)(x^2+1) + (Cx+D)(x^2-4)$$

when $x=2 \Rightarrow 4+2 = A(4)(5)+0+0 \Rightarrow A = \frac{6}{20} = \frac{3}{10}$

$x=-2 \Rightarrow 4-2 = 0 + B(-4)(5)+0 \Rightarrow B = \frac{2}{-20} = \frac{-1}{10}$

$x=0 \Rightarrow 0+0 = A(2)(1) + B(-2)(1) + D(-4)$

$0 = \frac{6}{10} + \frac{2}{10} - 4D \Rightarrow 4D = \frac{8}{10} \Rightarrow D = \frac{2}{10} = \frac{1}{5}$

$x=1 \Rightarrow 1+1 = A(3)(2) + B(-1)(2) + (C+\frac{1}{5})(-3)$

$2 = \frac{18}{10} + \frac{2}{10} - 3C - \frac{6}{10} \Rightarrow 3C = \frac{14}{10} - \frac{20}{10} = \frac{-6}{10}$

$C = \frac{-2}{10} = \frac{-1}{5}$

$$\int \frac{x^2+x}{x^4-3x^2-4}\,dx = \int \frac{\frac{3}{10}}{x-2}\,dx - \int \frac{\frac{1}{10}}{x+2}\,dx + \int \frac{-\frac{1}{5}x+\frac{1}{5}}{x^2+1}\,dx$$

$$= \frac{3}{10}\ln|x-2| - \frac{1}{10}\ln|x+2| - \frac{1}{5\times 2}\int \frac{2x}{x^2+1}\,dx + \frac{1}{5}\int \frac{dx}{x^2+1}$$

$$= \frac{3}{10}\ln|x-2| - \frac{1}{10}\ln|x-2| - \frac{1}{10}\ln(x^2+1) + \frac{1}{5}\tan^{-1}x + C$$

(34) $\int \frac{x^4}{x^2-1}\,dx \Rightarrow$ we use long Division

$$\begin{array}{r} x^2+1 \\ x^2-1 \overline{\big) \; x^4 \qquad\quad} \\ \underline{\cancel{-x^4}+x^2} \\ x^2 \quad \\ \underline{-x^2+1} \\ 1 \quad \end{array}$$

$$\int \left(x^2+1+\frac{1}{x^2-1}\right)dx$$

$$= \frac{x^3}{3}+x+\int\frac{dx}{(x-1)(x+1)}$$

$$= \frac{x^3}{3}+x+\int\left(\frac{A}{x-1}+\frac{B}{x+1}\right)dx \qquad A=\frac{1}{\boxed{1}+1}=\frac{1}{2}$$

$$= \frac{x^3}{3}+x+\int\frac{\frac{1}{2}}{x-1}\,dx-\int\frac{\frac{1}{2}}{x+1}\,dx \qquad B=\frac{1}{\boxed{-1}-1}=-\frac{1}{2}$$

$$= \frac{x^3}{3}+x+\frac{1}{2}\ln|x-1|-\frac{1}{2}\ln|x+1|+C$$

$$= \frac{x^3}{3}+x+\frac{1}{2}\Big[\ln|x-1|-\ln|x+1|\Big]+C$$

$$= \frac{x^3}{3}+x+\frac{1}{2}\ln\frac{|x-1|}{|x+1|}+C$$

$$= \frac{x^3}{3}+x+\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|+C$$