BIRZEIT UNIVERSITY

Trees and Traversals

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COMP242

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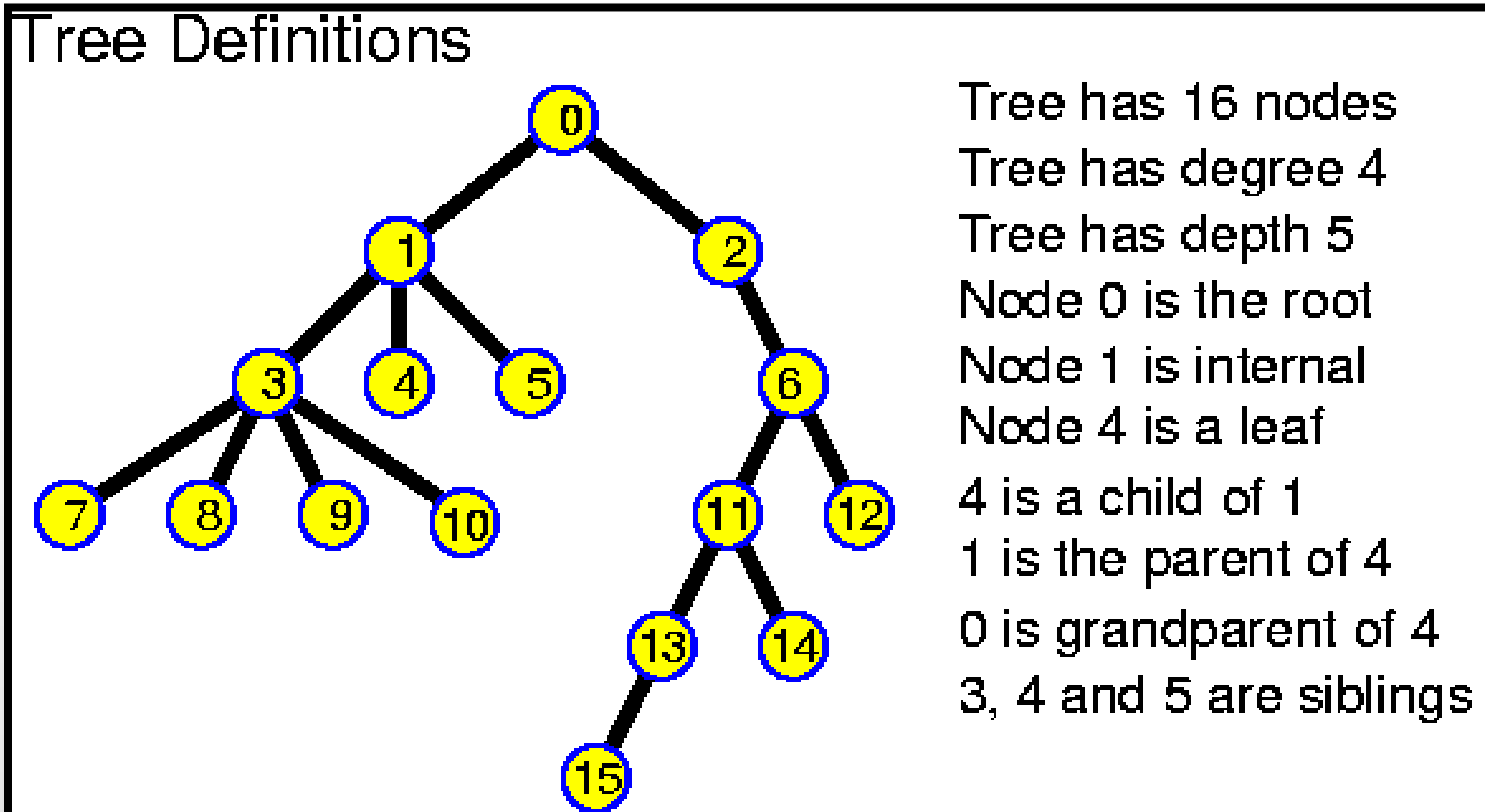
Tree

Motivation

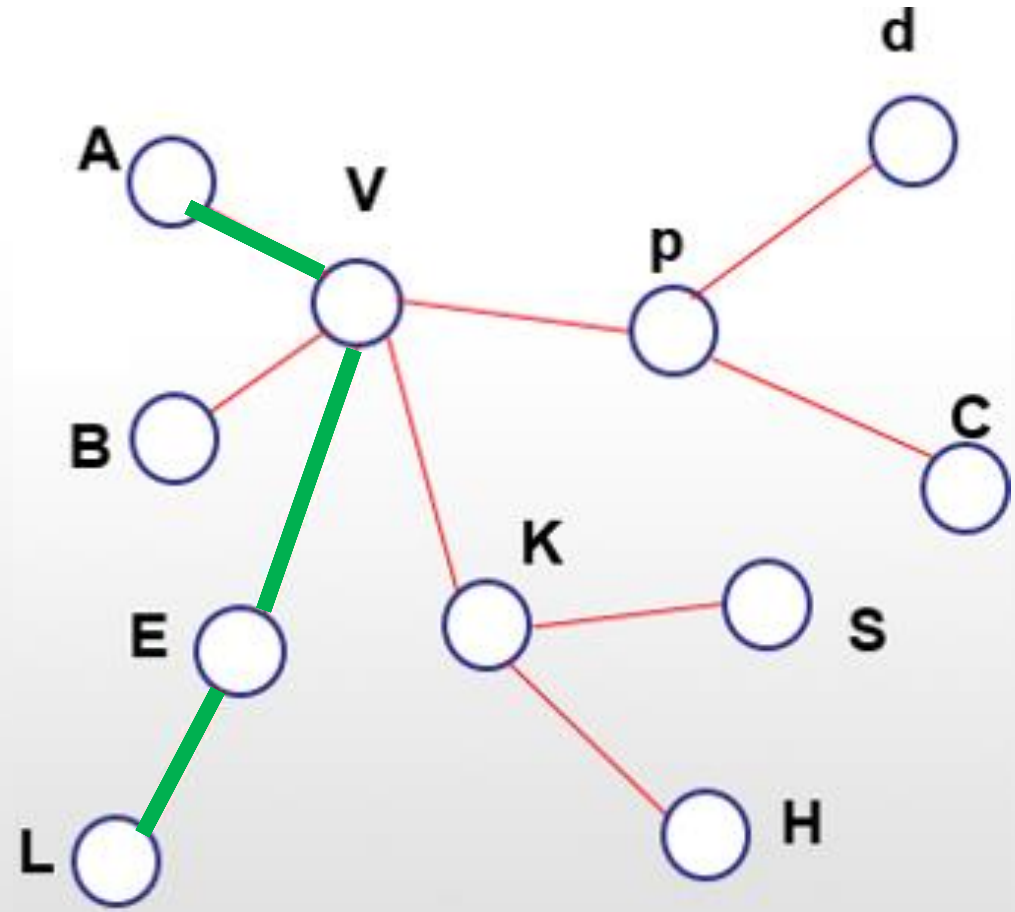

Tree Definitions
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
Tree has 16 nodes
Tree has degree 4
Tree has depth 5
Node 0 is the root
Node 1 is internal
Node 4 is a leaf
4 is a child of 1
1 is the parent of 4
0 is grandparent of 4
3, 4 and 5 are siblings

Tree

- **Tree:** **Set of nodes and edges that connect them.**

- **Exactly one path between any two nodes.**

- **Path: connected sequence of edges.**

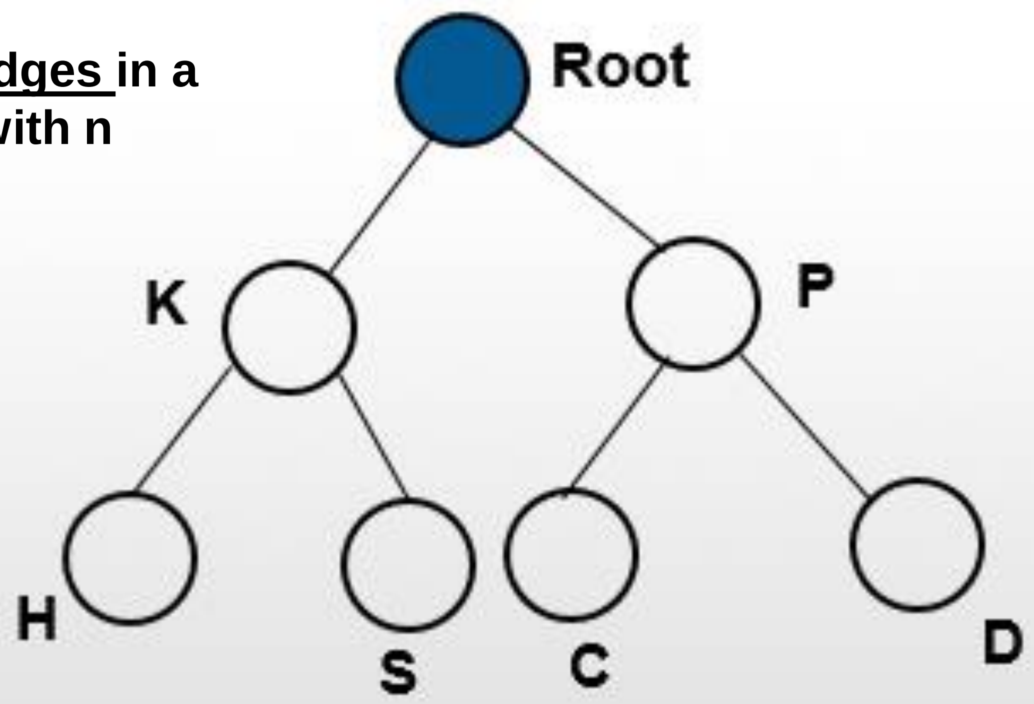


A **path** is a sequence of nodes such that the next node in the sequence is a child of the previous {A,V,E,L}

Rooted Tree

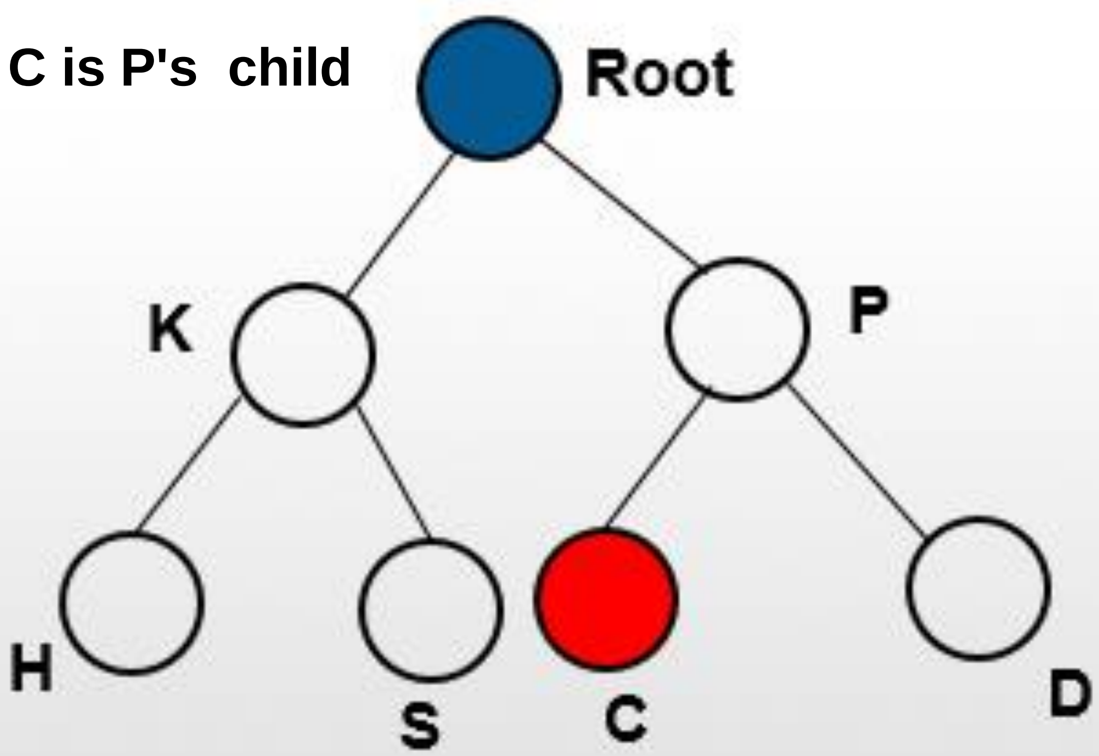
- **<u>Rooted tree</u>: One distinguished node is called the root.**

- **<u>Number of edges</u> in a rooted tree with n nodes = n-1**



Rooted Tree

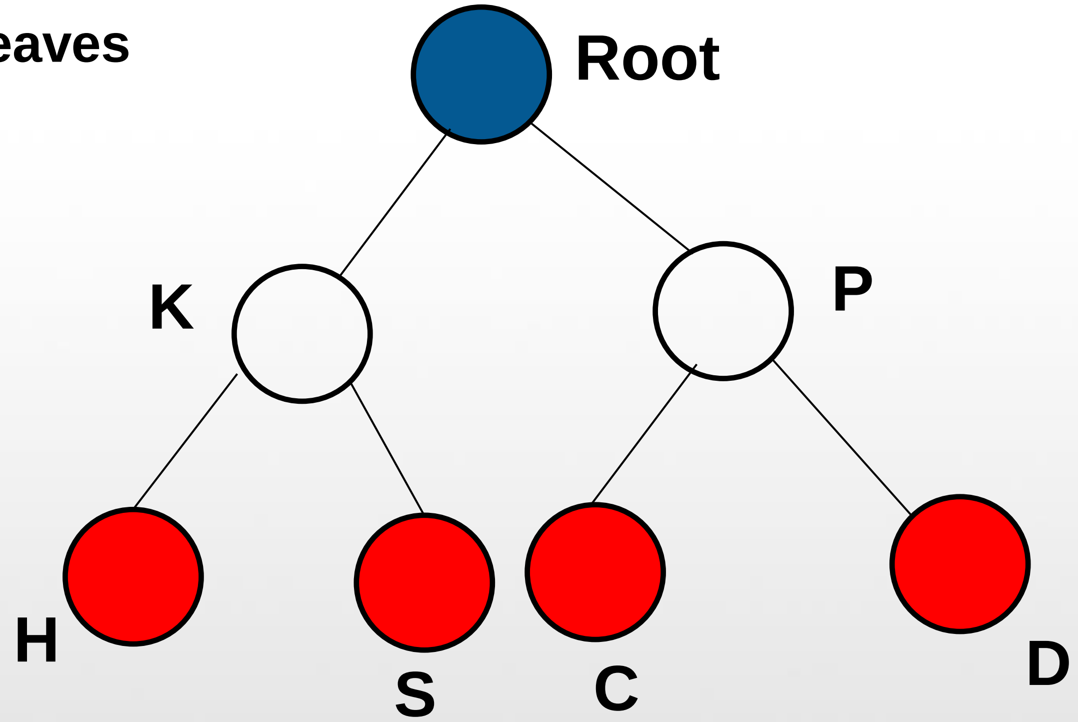
- Every node C, except root, has one parent P, the first node on path form c to the root.
- if p is parent of C,then C is P's child
- Root has no parents



Rooted Tree

<u>Leaf</u> : **Node with no children.**

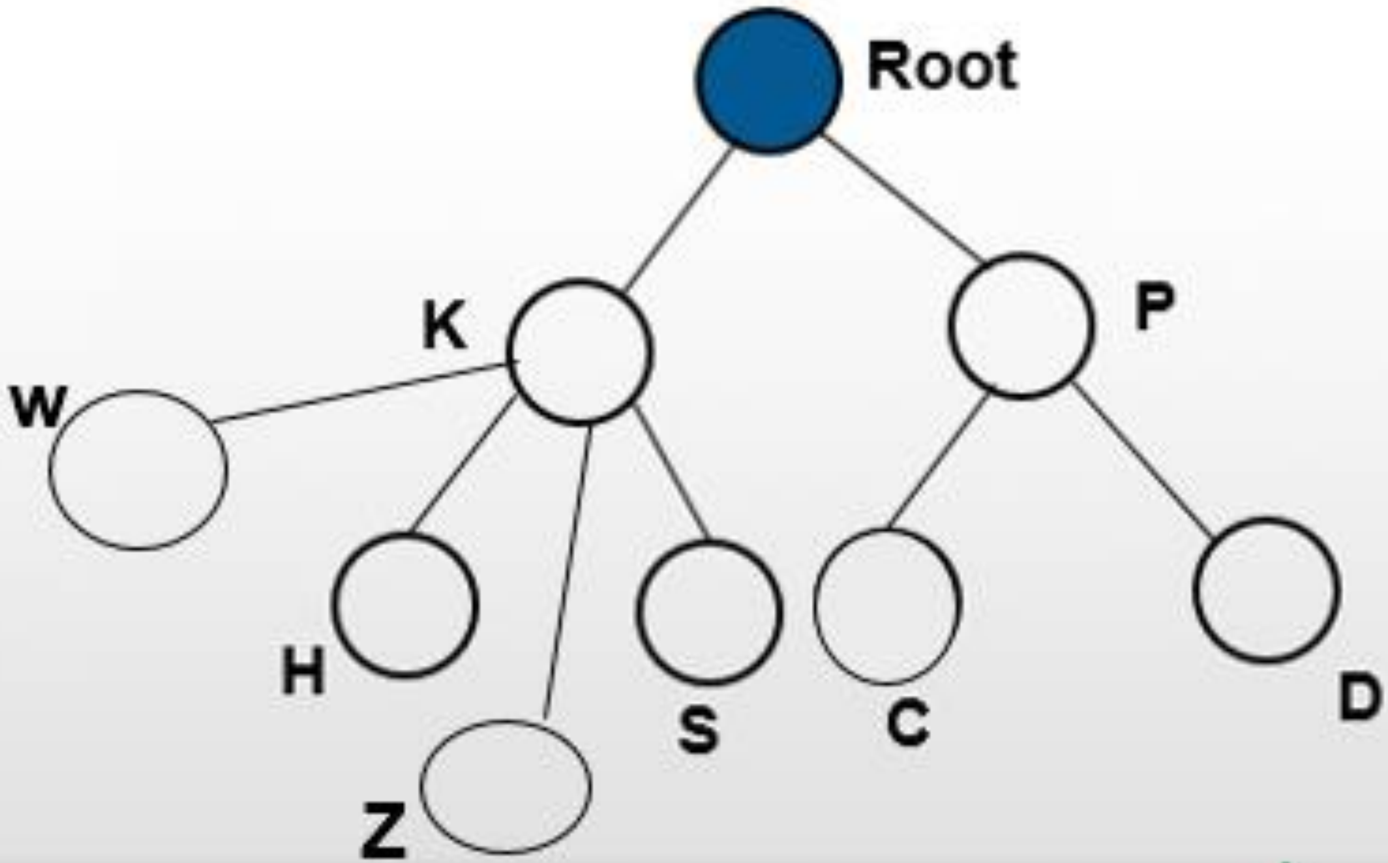
Example. H,S,C, and D are leaves in our tree



Leaf node also called external node, all other nodes are internal

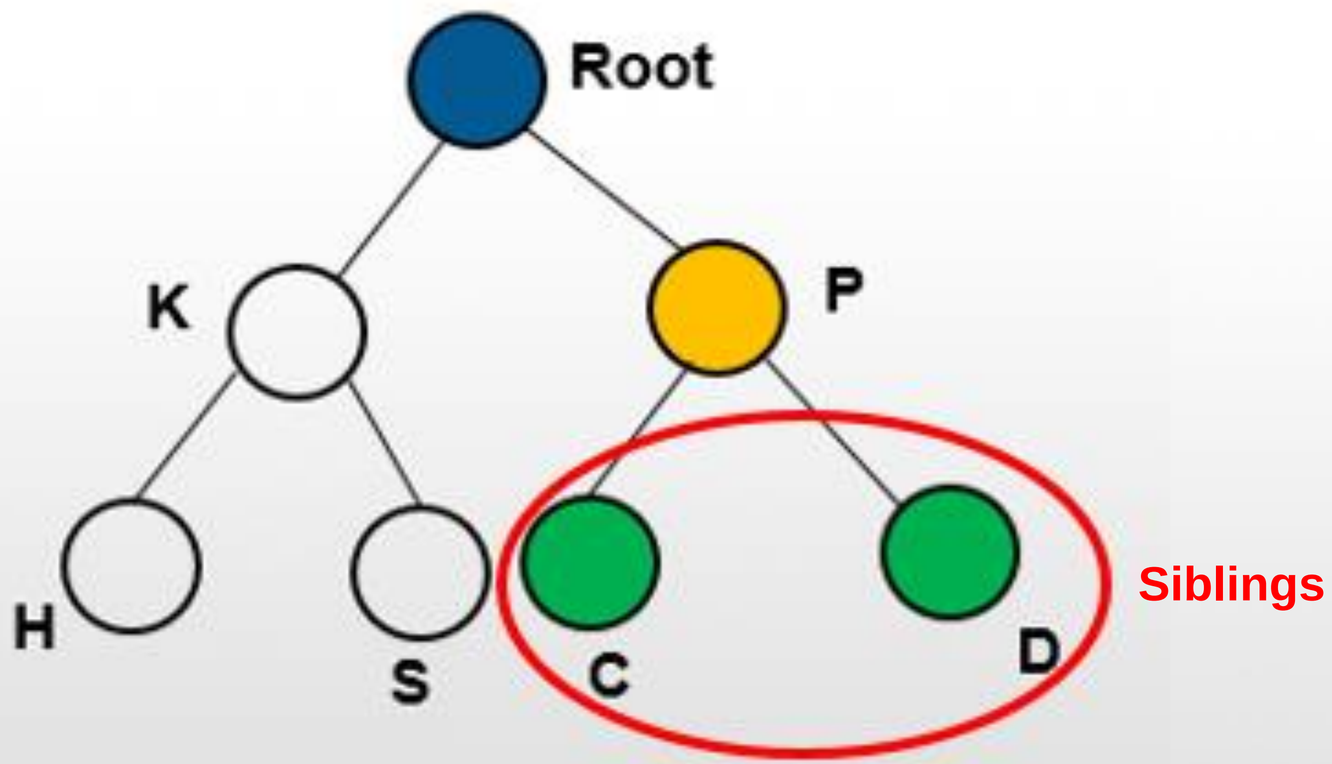
Rooted Tree

- A node can have any number of children



Rooted Tree

- **<u>Siblings</u> : Nodes with same parent.**

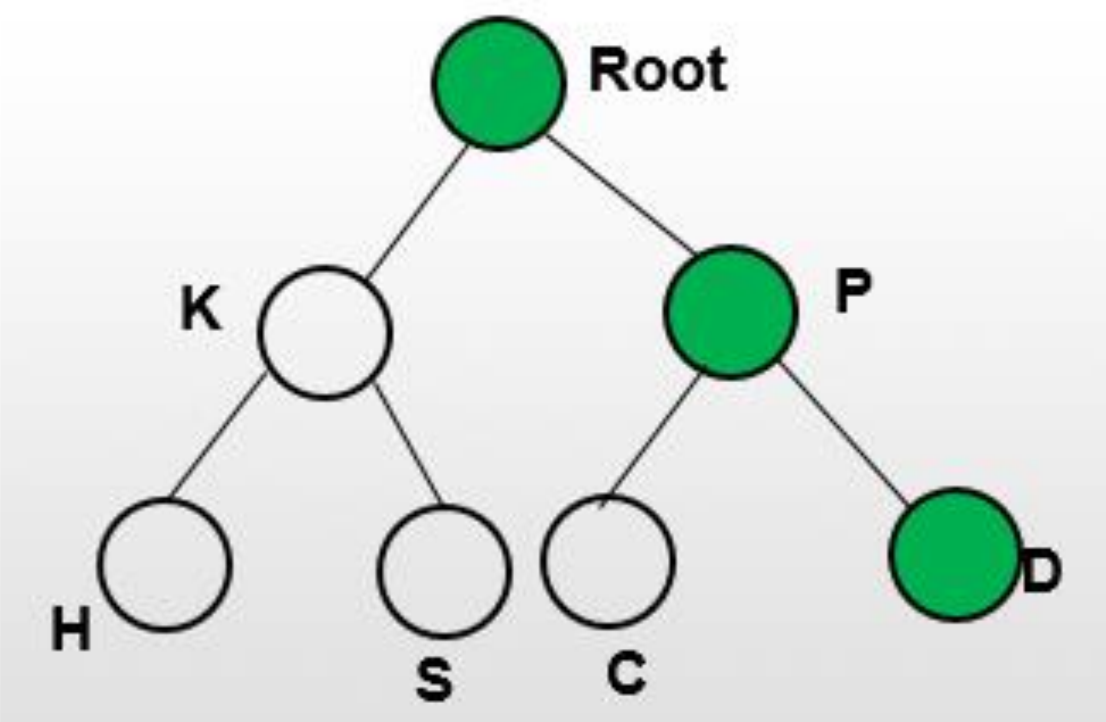


P is parent of C, and P is parent of D

Rooted Tree

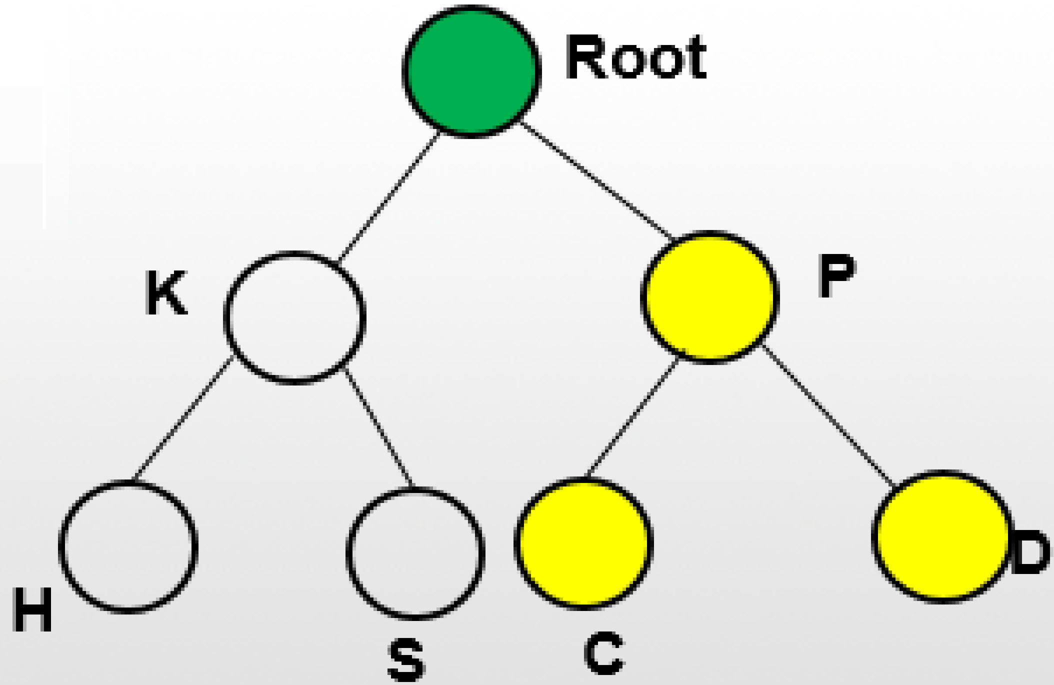
- Ancestors of a node D : nodes on path from D to root, including D, D's parent, D's grandparent,...root (included).

- If P is <u>ancestor</u> of D, then D is <u>descendant</u> of P



Rooted Tree

Example: **Descendants of P are** ***P,C, and D***
Ancestors of H are H,K, and Root

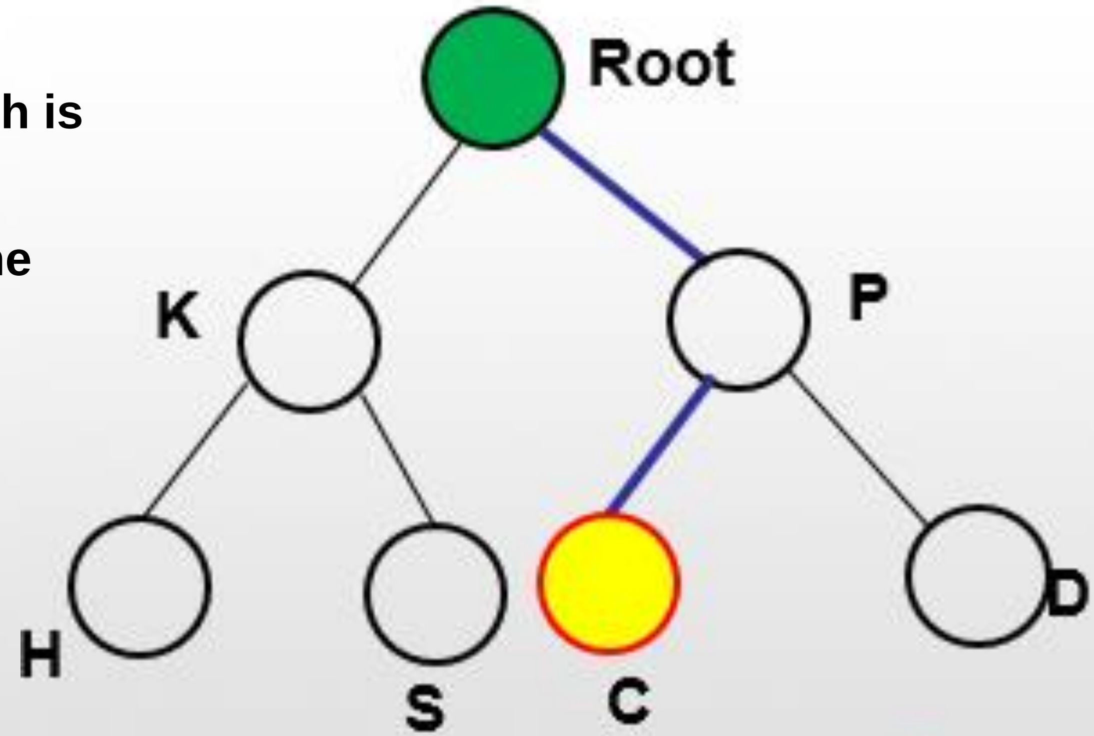


Rooted Tree

- ❑ **Length of path: number of edges in path.**

Example:

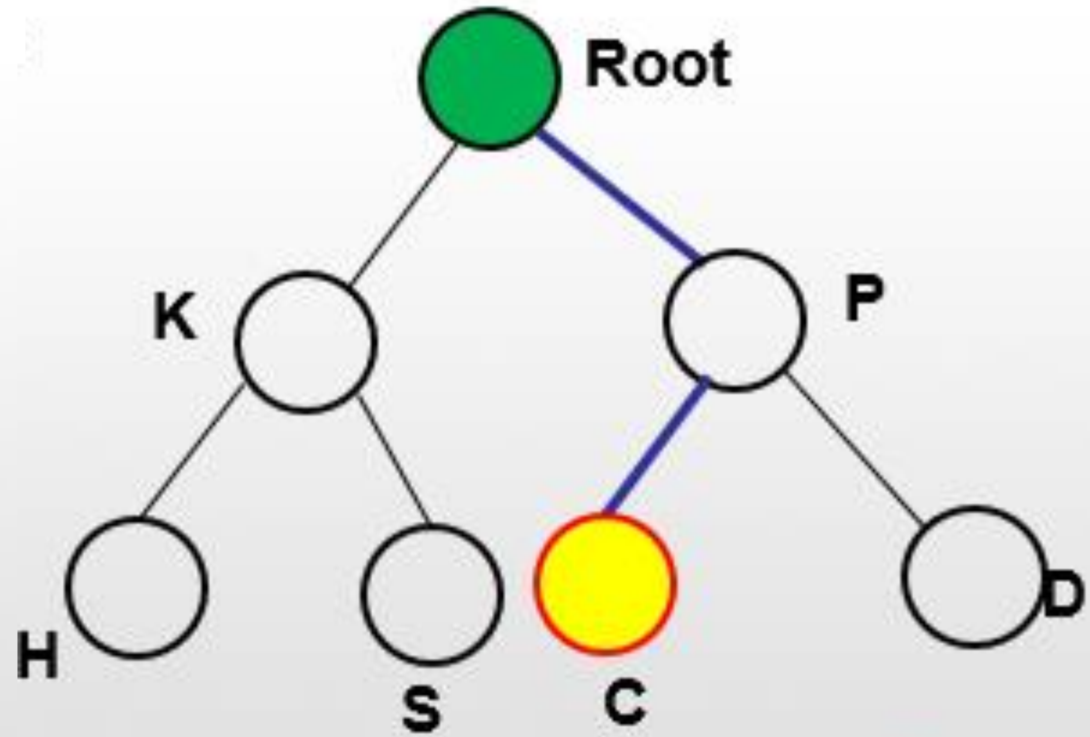
- ❑ **path from c to itself , the length is zero (empty path or no path)**
- ❑ **path from c to p, length is 1 (one edge in path)**
- ❑ **path from c to root, length is 2**



Rooted Tree

- Depth of node n is length of path from n to root.

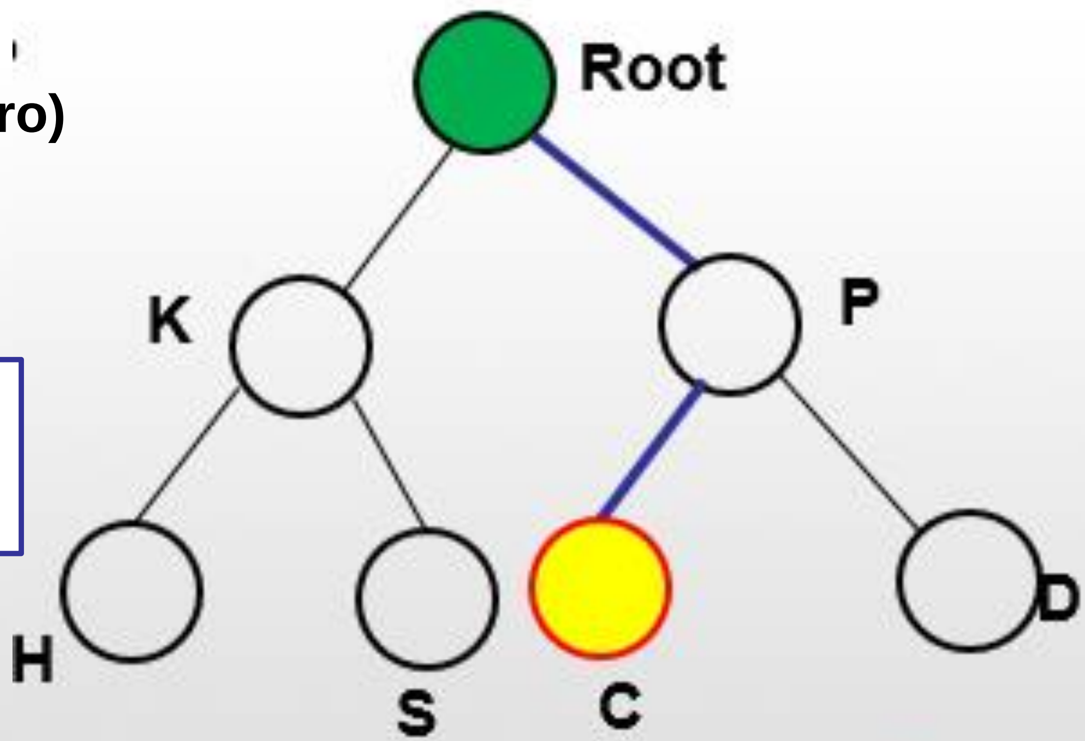
EX: (Depth of root is zero)
EX: (Depth of C is 2)



Rooted Tree

- Height of node n is the length of path from n to its deepest descendant

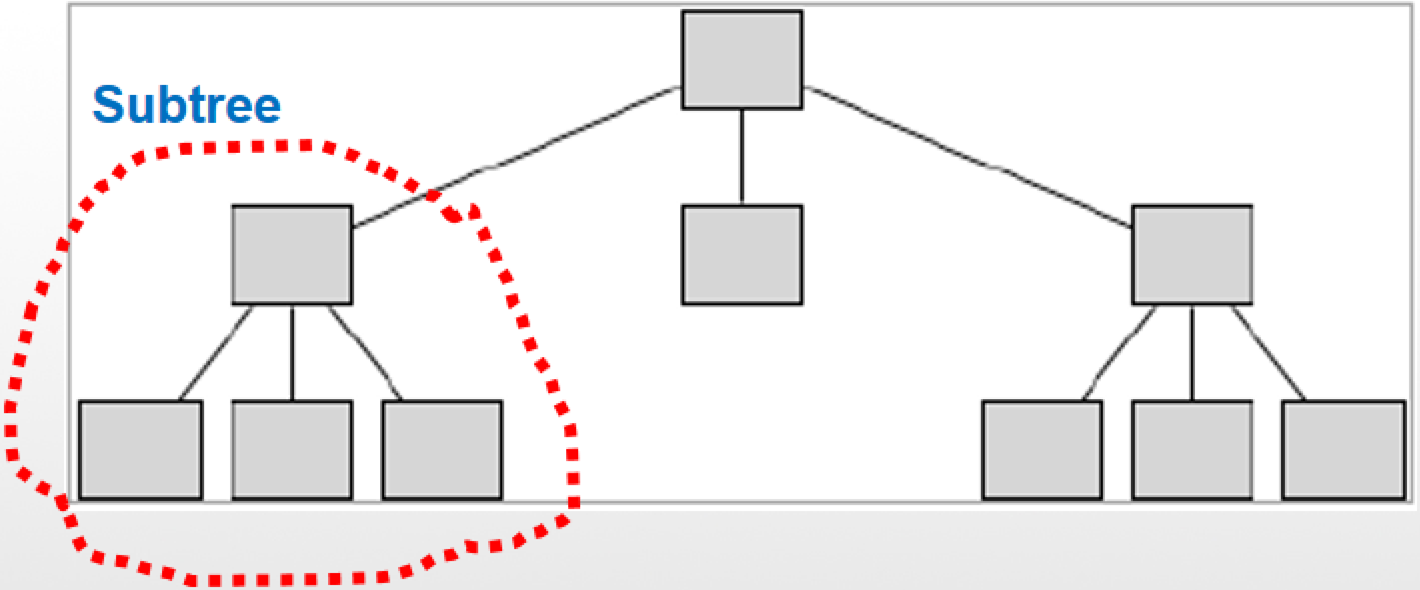
Examples:
(Height of any leaf node is zero)
(Height of Root node is 2)
(Height of P node is 1)



Height of a tree= height of the root
(The longest path length from
 the root to a leaf)

Rooted Tree

- **<u>Degree</u> : the maximum number of possible children for every node in the tree.**



The height of tree is 2 and the degree 3

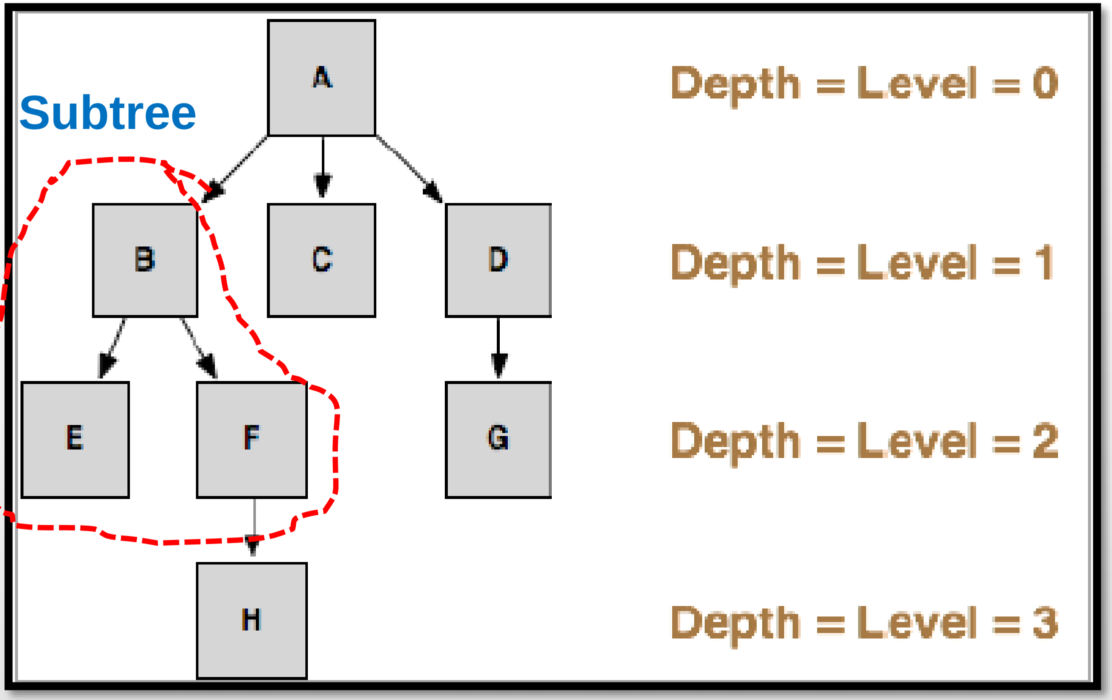
Rooted Tree

Node level & node depth : is the path length from the root

- **The root is level 0 and depth 0**
- **Other nodes depth is 1 + depth of parent**

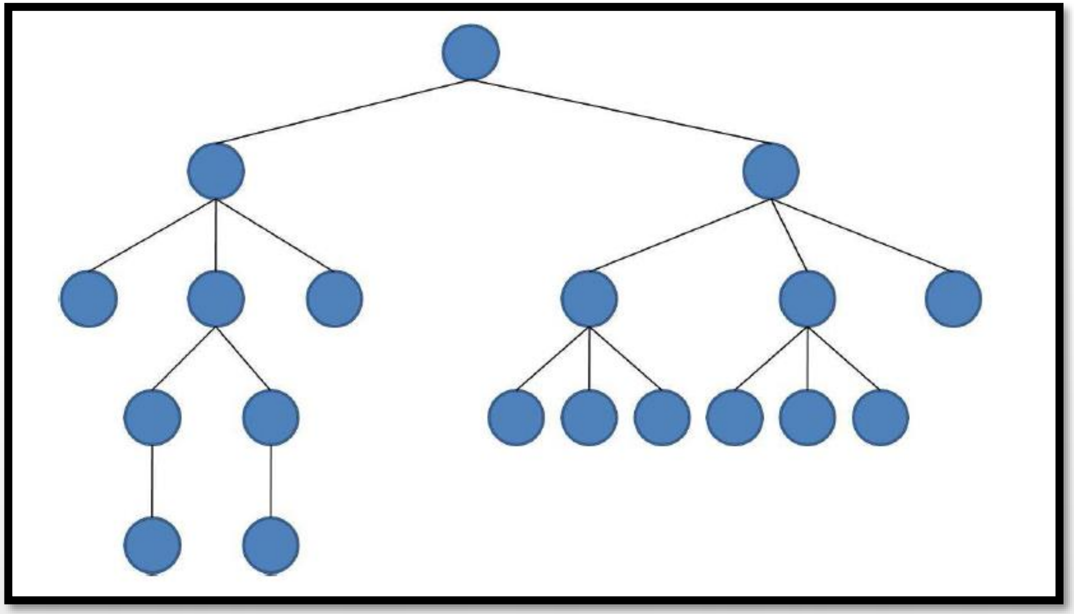
depth(A)=0
depth(B)=1
depth(E)=2
depth(H)=3

height(A)=3
height(F)=1
height(C)=0
height(E)=0
height(D)=1
height(B)=2



Rooted Tree: H.W

You have one week to do the following



- ❑ **Explain the values of the main characteristics of the tree shown in the figure.**

NOTE: These characteristics are grade(degree) of the tree, height, number of nodes, external and internal nodes.

Rooted Tree: H.W

You have one week to do the following

Given the following properties of a tree, draw a tree that satisfies them:

1. Degree of the tree: 3

2. Number of nodes: 14

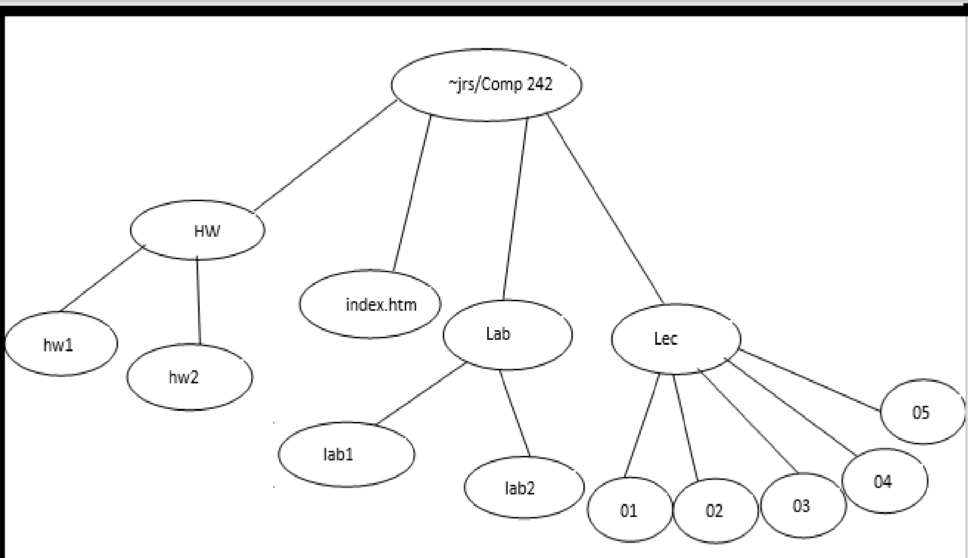
3. Height of the tree: 3

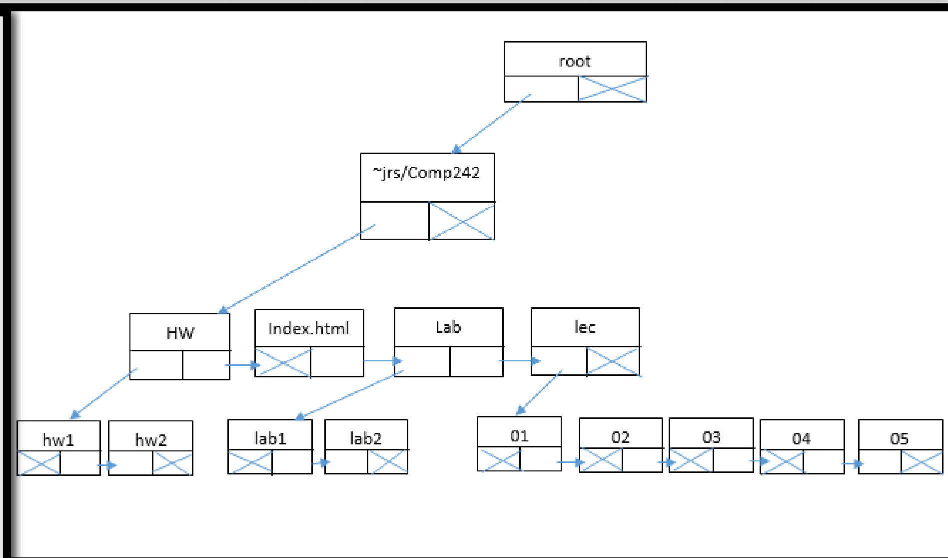
4. Number of nodes with depth=2: 6

Implementation: Rooted Tree

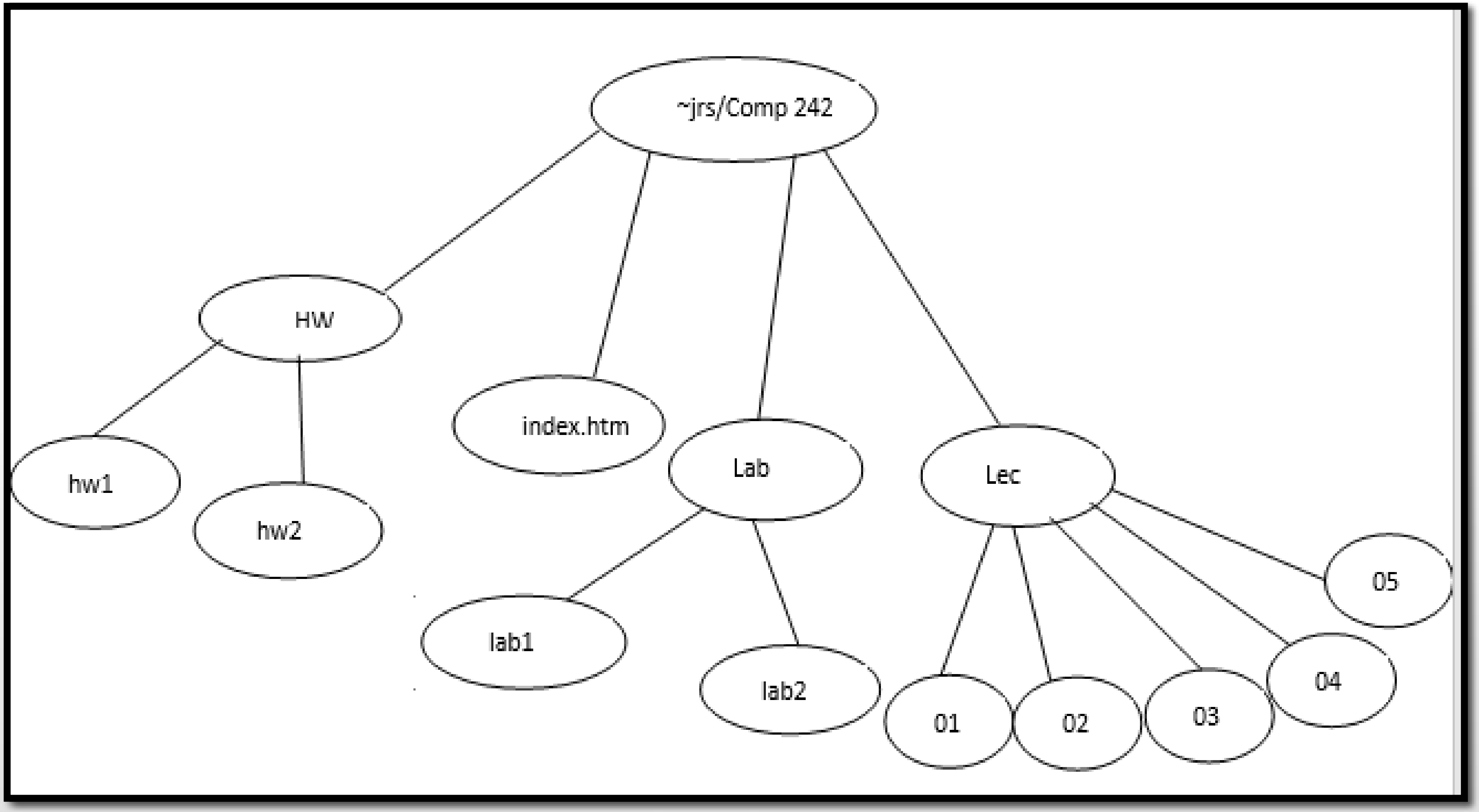
```java
//Class Node for the rooted Tree
public class RootedTreeNode {
  Object element;          //store data
  RootedTreeNode firstChild;    // reference for left tree
  RootedTreeNode nextSibling; //reference for right tree

}
```

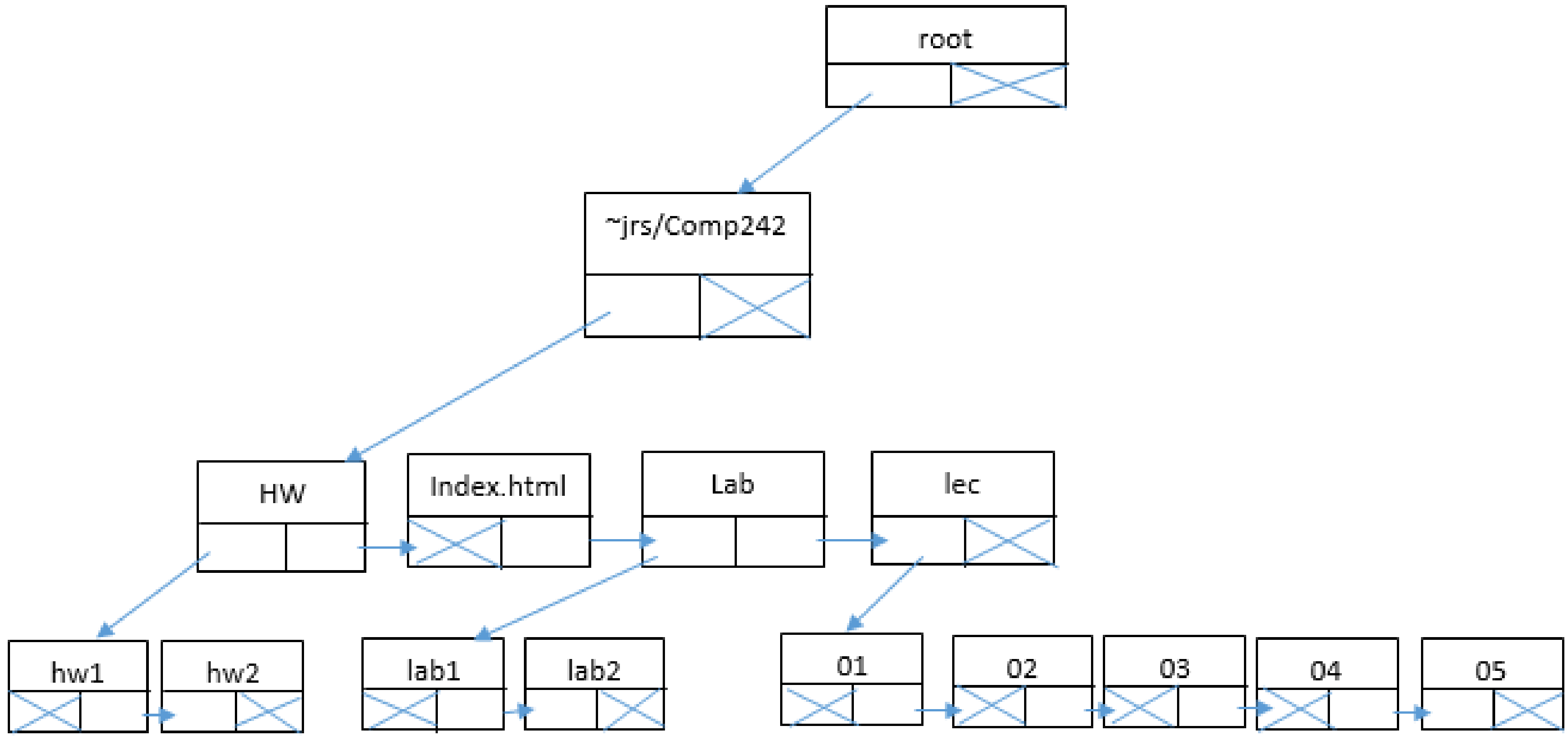




Implementation: Rooted Tree



Implementation: Rooted Tree



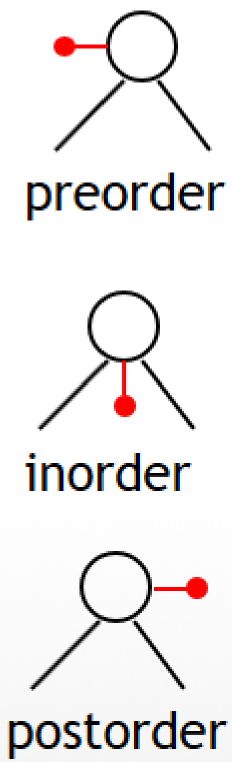
Tree Traversals

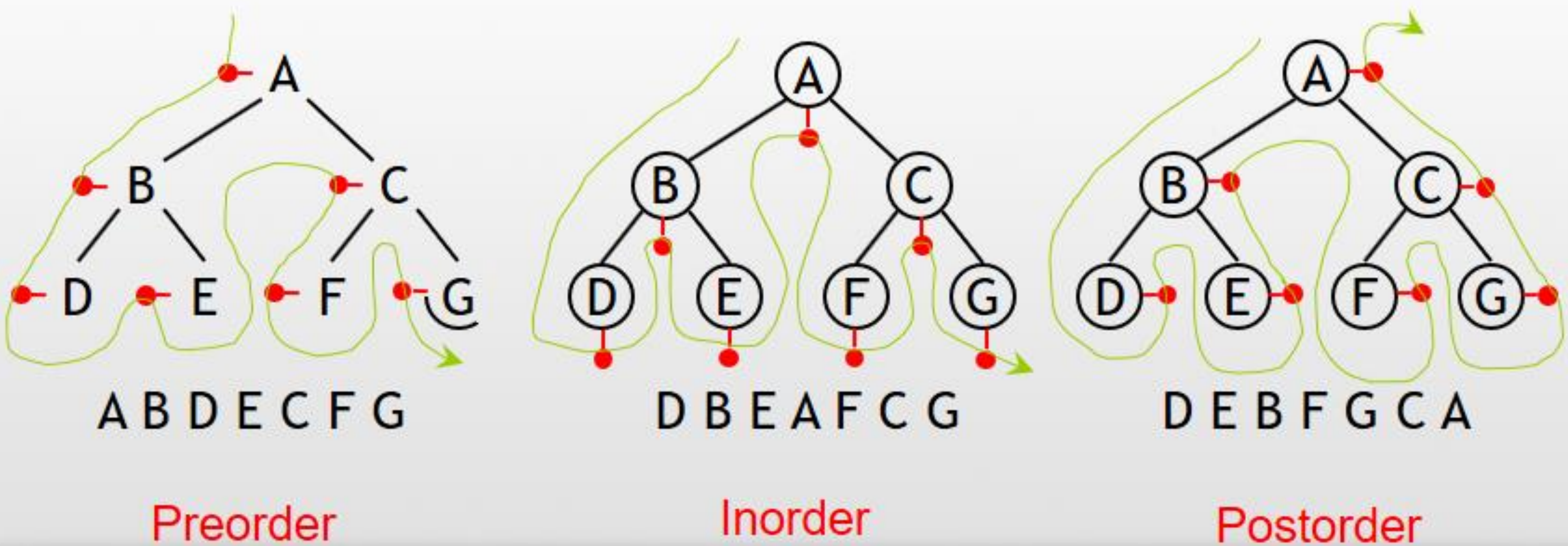
- A traversal is a manner of visiting each node in a tree once.

- What you do when visiting any particular node depends on the application; for instance, you might print a node’s value, or perform some calculation upon it. There are several different traversals, each of which orders the nodes differently

Tree Traversal

- Preorder: visits nodes as root → left → right
- Inorder: visits nodes as left → root → right
- Postorder: visits nodes as left → right → root





Tree Traversal: More Details

Preorder traversal

Let T be a tree with root r and subtrees $T_1, T_2, \ldots, T_n$. In Preorder traversal, we visit the root r first, then traverse the subtree T_1 in preorder, then traverse the subtree T_2 in preorder, and so on up to the traversal of the subtree T_n in preorder.

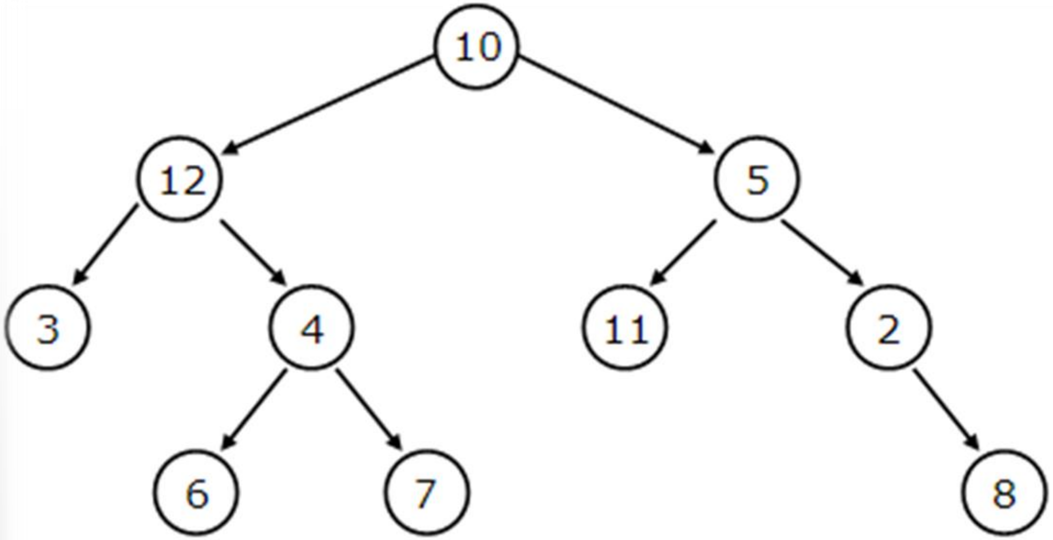
Inorder traversal

Let T be a tree with root r and subtrees $T_1, T_2, \ldots, T_n$. In an Inorder traversal, we traverse the subtree T_1 in inorder, then we visit the root r, then traverse the subtree T_2 in inorder, and so on up to the traversal of the subtree T_n in inorder.

Postorder traversal

Let T be a tree with root r and subtrees $T_1, T_2, \ldots, T_n$. In a Postorder traversal, we traverse the subtree T_1 in postorder, then traverse the subtree T_2 in postorder, and so on up to the traversal of the subtree T_n in postorder, and finally we visit the root r.

Tree Traversal



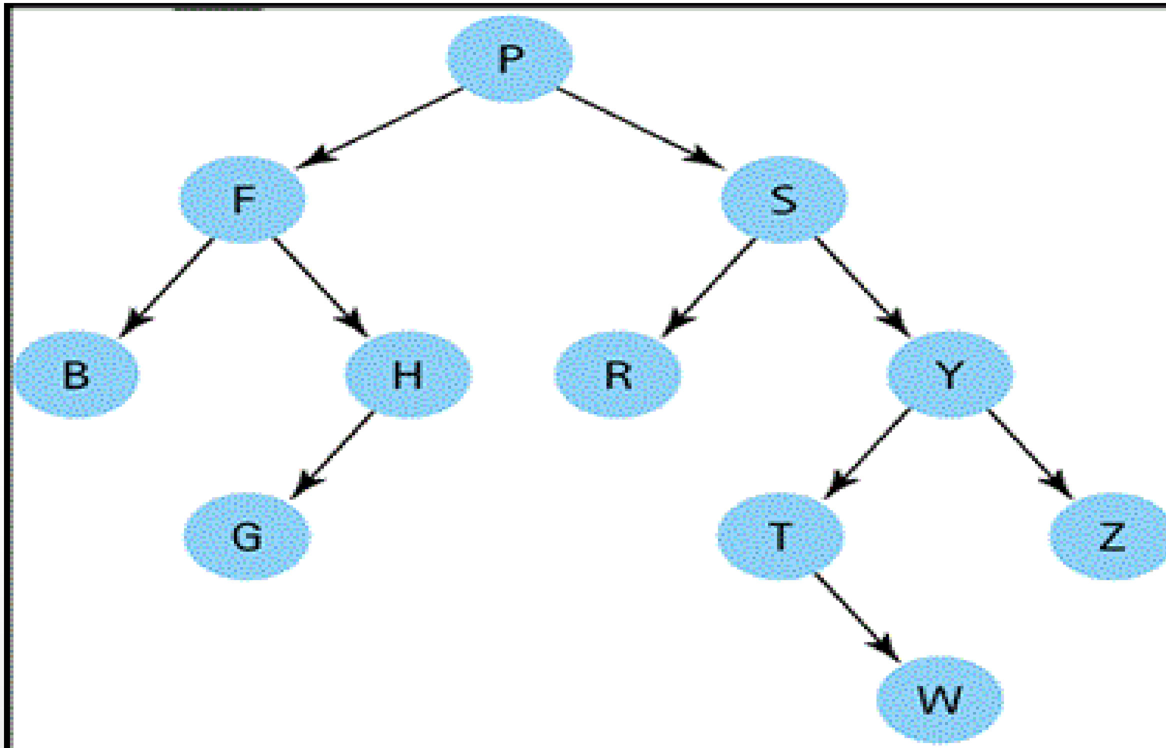
Levelorder tree traversal
10, 12, 5, 3, 4, 11, 2, 6, 7, 8

Inorder tree traversal
3, 12, 6, 4, 7, 10, 11, 5, 2, 8

Preorder tree traversal
10, 12, 3, 4, 6, 7, 5, 11, 2, 8

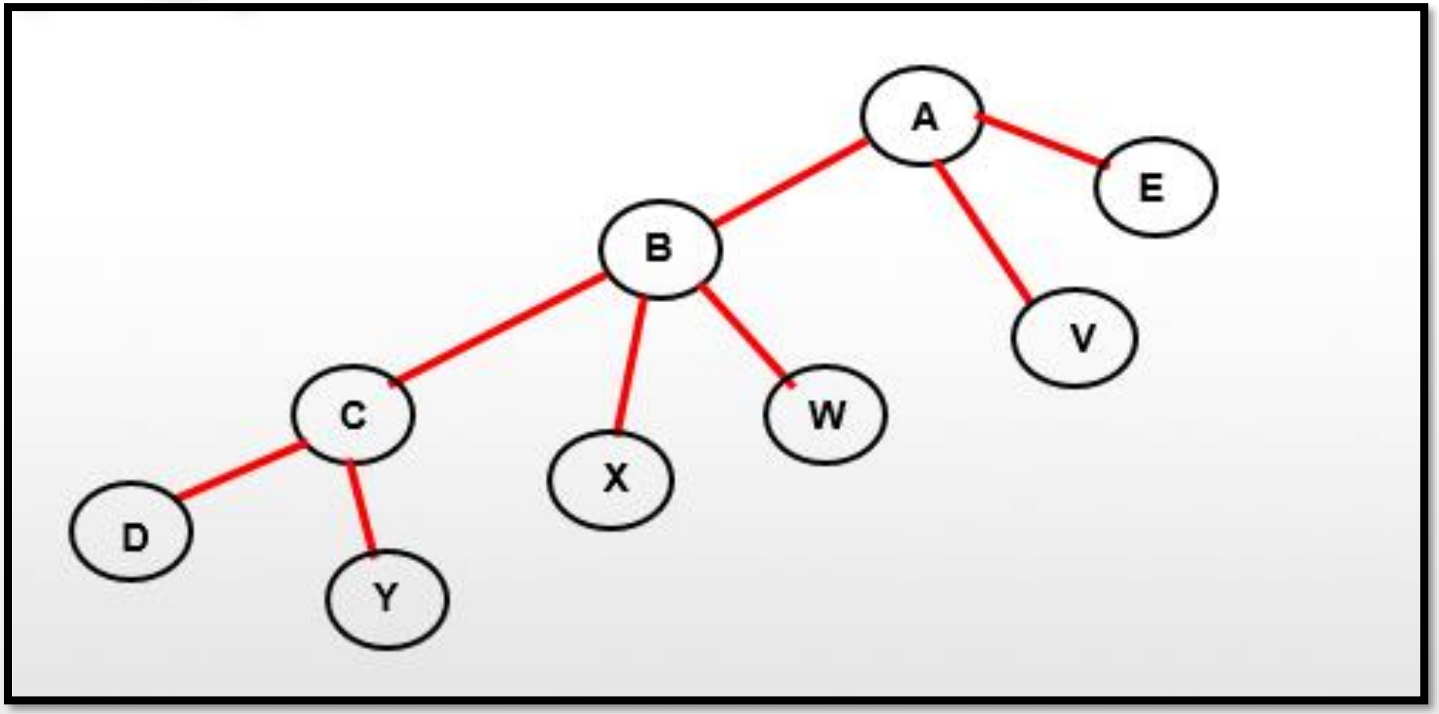
Postorder tree traversal
3, 6, 7, 4, 12, 11, 8, 2, 5, 10

Tree Traversal



Inorder: B F G H P R S T W Y Z
Preorder: P F B H G S R Y T W Z
Postorder: B G H F R W T Z Y S P

Tree Traversal

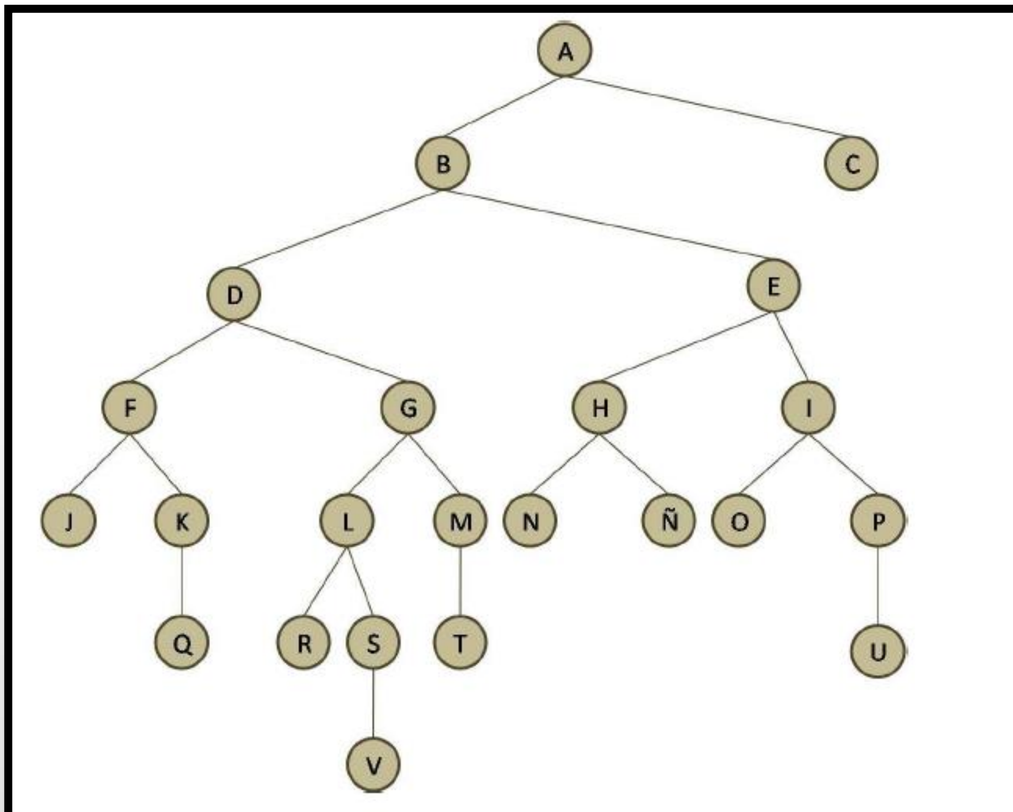


Pre-order: A,B,C,D,Y,X,W,V,E
In-order: D,C,Y,B,X,W,A,V,E
Post-order: D,Y,C,X,W,B,V,E,A

Rooted Tree: H.W

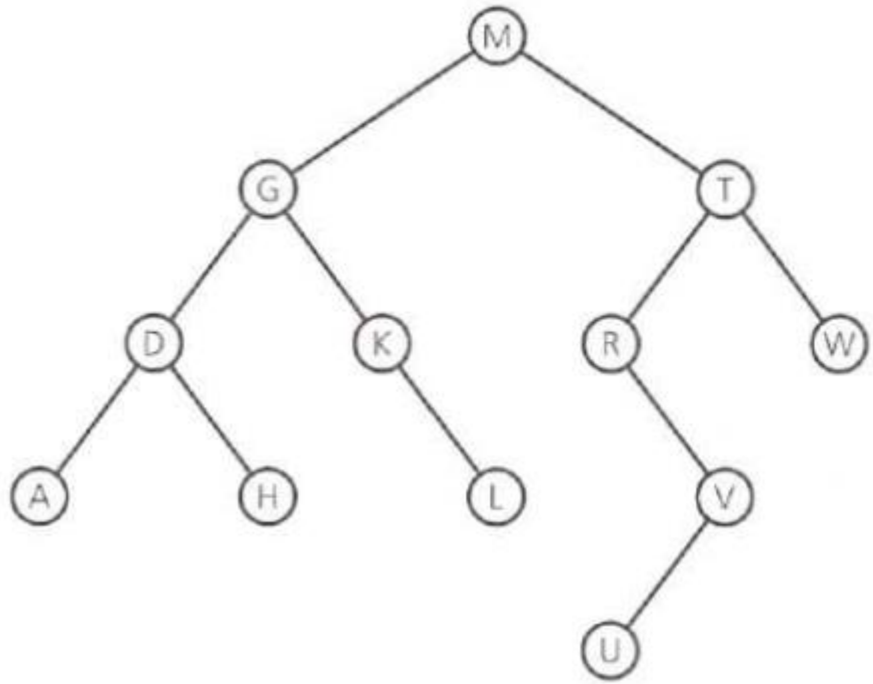
You have one week to do the following

Given the following tree, write the pre-order, in-order and post-order traversals.



Extra Exercises

Carrano, 4th edition, Chapter 10, Exercise 2: What are the preorder, inorder, and postorder traversals of the following binary trees:



Question?

“Success is the sum of small efforts, repeated day in and day out.”
Robert Collier

References:

1. Algorithms and Data Structures, Julian Moreno Schneider et al.
2. Fundamentals of Data Structures in C, Ellis Horowitz et al.
3. Data Structures and Problem Solving with C++: Walls and Mirrors
4. Analysis of algorithms robert Sedgewick
5. Prof. Sin-Min Lee Lecture Notes
6. Prof. Evan Korth Lecture Notes

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