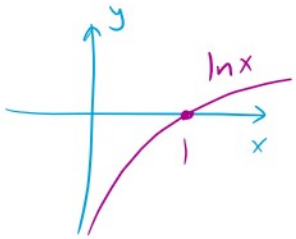
5.2 Part 2 Logarithmic Properties

Monday, January 17, 2022 6:53 PM

$$y = \log_a x = \frac{\ln x}{\ln a}, \quad x > 0, \ a > 0, \ a \neq 1$$

(1) $\log_a a = 1$

$\frac{\ln a}{\ln a} = 1$ or $a^1 = a$



Exp $\log_2 2 = 1$

Exp $\log_3 3 = 1$

Exp $\log_2 1 = 0$

$\frac{\ln 1}{\ln 2} = \frac{0}{\ln 2} = 0$

Exp $\log_a 1 = 0$

(2) $\log_a a^x = x \quad \forall x$

$a^x = a^x$ or

$\log_a a = x(1) = x$

If $a^y = a^x$ then

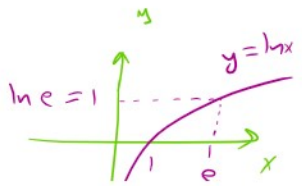
$y = x$

Exp Suppose $2^y = 2^3$ Find y

$y = 3$

Exp $\log_2 2^y = y$

Exp $\log_q 1 = \log_q q^0 = 0$



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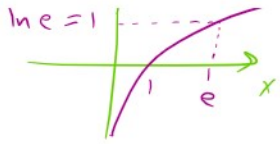
$\log_a 1$ $\log_a$

or $\log_a 1 = 0$



$\ln e = 1$

Exp $\ln e^4 = 4 \ln e = 4(1) = 4$

or $\ln e^4 = \log_e e^4 = 4$

Exp $\log_7 7 = 1$

or $\log_7 7^1 = 1$

(3) $a^{\log_a x} = x \quad \forall \quad x > 0$

Exp $3^{\log_3 7} = 7$

Exp $\left(\frac{1}{2}\right)^{\log_{\frac{1}{2}} 8} = 8$

Exp $e^{\ln x} = x$ (x > 0 عدد حقيقي)

$e^{\ln 7} = 7$, $e^{\ln \frac{1}{2}} = \frac{1}{2}$, $e^{\ln 10} = 10$

or $e^{\ln x} = e^{\log_e x} = x$

(4) $\log_a xy = \log_a x + \log_a y$

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(4) $\log_a xy = \log_a x + \log_a y$

(5) $\log_a \frac{x}{y} = \log_a x - \log_a y$

(6) $\log_a x^r = r \log_a x$ $\quad r \in \mathbb{R}$

Exp Find (1) $\log_3 27 = \log_3 3^3 = 3 \log_3 3 = 3$ (the $\log_3 3$ is raised to the power 1)

(2) $\log_4 \frac{1}{16} = \log_4 \frac{1}{4^2} = \log_4 4^{-2}$ (the -2 is the power)

$= -2 \log_4 4$ (the $\log_4 4$ is raised to the power 1)

$= -2$

Exp $\log_a x = 2$ and $\log_a y = 3$. Find

(1) $\log_a x^2 \cdot y^3 = \log_a x^2 + \log_a y^3$ (the 2 and 3 are the powers)

$= 2 \log_a x + 3 \log_a y$

$= 2(2) + 3(3)$

$= 4 + 9$

$= 13$

(2) $\log_a \sqrt{\frac{x}{y}} = \log_a \left(\frac{x}{y}\right)^{\frac{1}{2}}$

$= \frac{1}{2} \log_a \frac{x}{y}$

$= \frac{1}{2} \left[\log_a x^2 - \log_a y^3 \right]$

$= \frac{1}{2} [2 - 3]$

$= \frac{1}{2} [-1]$

$= -\frac{1}{2}$

Exp Find (1) $\log_3 \frac{9}{27} = \log_3 \frac{1}{3}$

$= \log_3 1 - \log_3 3$

$= 0 - 1$

$= -1$

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(2) $\log_2 \frac{1}{\sqrt{2}} = \log_2 1 - \log_2 \sqrt{2}$

$= 0 - \log_2 2^{\frac{1}{2}}$

$= -\frac{1}{2} \log_2 2 \rightarrow 1$

$= -\frac{1}{2}$

(3) $\log_2 32 = \log_2 2^5 = 5 \log_2 2 \rightarrow 1$

$= 5$

$32 = 2 \times 2 \times 2 \times 2 \times 2$

$= 2^5$

(4) $\ln x^3 e^2 = \ln x^3 + \ln e^2$

$= 3 \ln x + 2 \ln e \rightarrow 1$

$= 3 \ln x + 2$

or $\ln x^3 e^2 = \log_e x^3 e^2$

$= \log_e x^3 + \log_e e^2$

$= 3 \log_e x + 2 \log_e e \rightarrow 1$

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$= 3 \log_e x + 2 \log_e e$

$= 3 \dfrac{\ln x}{\ln e} + 2$

$\ln e = 1$

$= 3 \ln x + 2$

7) $\log_b x = \dfrac{\ln x}{\ln b} = \dfrac{\frac{\ln x}{\ln a}}{\frac{\ln b}{\ln a}} = \dfrac{\log_a x}{\log_a b}$

Convert the base b to the base a

$\log_b x = \dfrac{\ln x}{\ln b} = \dfrac{\frac{\ln x}{\ln e}}{\frac{\ln b}{\ln e}} = \dfrac{\log_e x}{\log_e b}$

convert the base b to the base e

$\log_b x = \dfrac{\ln x}{\ln b} = \dfrac{\frac{\ln x}{\ln 10}}{\frac{\ln b}{\ln 10}} = \dfrac{\log_{10} x}{\log_{10} b} = \dfrac{\log x}{\log b}$

convert the base b to the base 10

Exp Evaluate

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Exp Evaluate

(1) $\log_7 15$ using the change of base e

$$\log_7 15 = \frac{\log_e 15}{\log_e 7} = \frac{\ln 15}{\ln 7} = \frac{2.71}{1.95} \approx 1.4$$

(2) $\log_7 15$ using the change of base 10

$$\log_7 15 = \frac{\log 15}{\log 7} \approx \frac{1.18}{0.845} \approx 1.4$$

(3) $\log_7 15$ using the change of base 3

$$\log_7 15 = \frac{\log_3 15}{\log_3 7} \approx 1.4$$

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