1.5 Solutions of Systems of Linear Equations

we will study two methods to solve linear systems:

[1] Substitution Method

[2] Elimination Method

Exp solve the following linear systems by substitution

[1] $\begin{cases} 4x + 3y = 11 \\ 2x - 5y = -1 \end{cases}$

$2x - 5y = -1$

$2x = 5y - 1$

$\boxed{4x = 10y - 2}$

$4x + 3y = 11$

$10y - 2 + 3y = 11$

$13y = 13$

$\boxed{y = 1}$

$4x = 10y - 2$

$= 10(1) - 2$

$4x = 8$

$\boxed{x = 2}$

Check

$4(2) + 3(1) \stackrel{?}{=} 11$

$11 = 11$ ✓

$2(2) - 5(1) \stackrel{?}{=} -1$

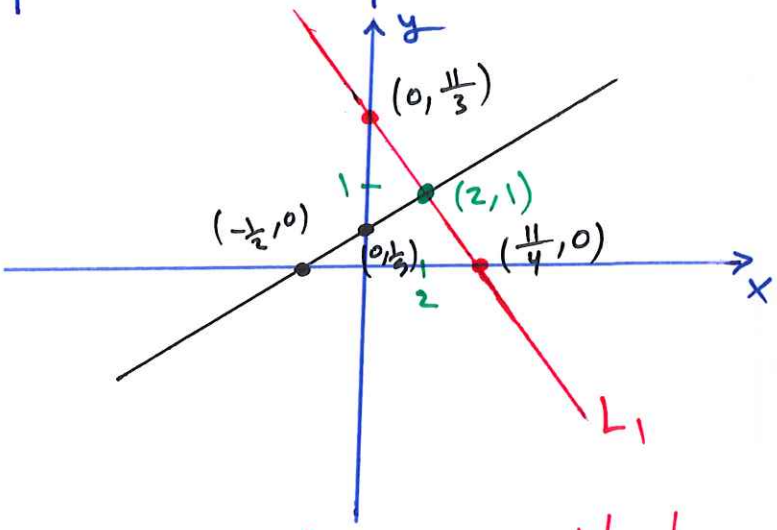
$-1 = -1$ ✓

- sketch the two lines

L_1: $4x + 3y = 11$

$x = 0 \Rightarrow y = \frac{11}{3}$ is y-intercept

$y = 0 \Rightarrow x = \frac{11}{4}$ is x-intercept



L_2: $2x - 5y = -1$

$x = 0 \Rightarrow y = \frac{1}{5}$ is y-intercept

$y = 0 \Rightarrow x = \frac{-1}{2}$ is x-intercept

This is consistent system since it has one solution $(x, y) = (2, 1)$

- Find slopes of these lines

L_1: $4x + 3y = 11 \Rightarrow 3y = 11 - 4x$

$\Rightarrow y = \frac{11}{3} - \frac{4}{3}x$

$\Rightarrow m_1 = -\frac{4}{3}$

L_2: $2x - 5y = -1 \Rightarrow -5y = -2x - 1$

$y = \frac{2}{5}x + \frac{1}{5}$

$\Rightarrow m_2 = \frac{2}{5}$

Hence, L_1 not parallel to L_2 since $m_1 \neq m_2$

(2) $\begin{cases} x + y = 1 \\ 2x + 2y = 4 \end{cases}$

$x + y = 1$

$\boxed{y = 1 - x}$

$2x + 2y = 4$

$2x + 2(1 - x) = 4$

$2x + 2 - 2x = 4$

$2 = 4$ ← impossible ✗

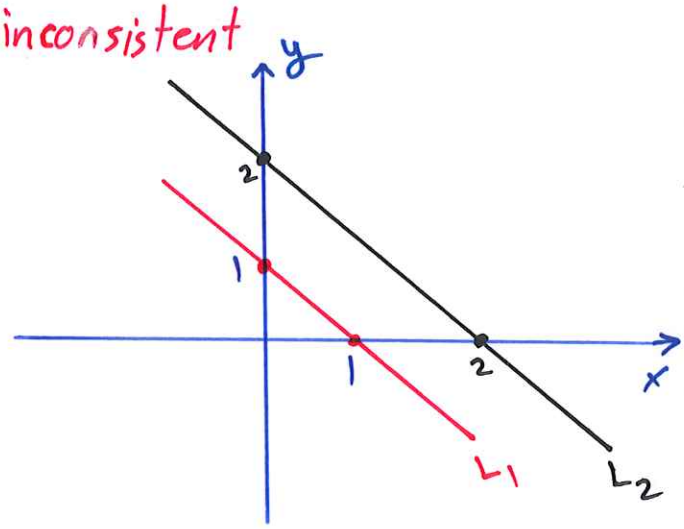
- This system has no solution
- This system is called inconsistent

- sketch the two lines

$L_1: x + y = 1$

$x = 0 \Rightarrow y = 1$ is y-intercept

$y = 0 \Rightarrow x = 1$ is x-intercept



$L_2: 2x + 2y = 4$

$x = 0 \Rightarrow 2y = 4 \Rightarrow y = 2$ is y-intercept

$y = 0 \Rightarrow 2x = 4 \Rightarrow x = 2$ is x-intercept

- Find the slopes of these lines

$L_1: x + y = 1 \Rightarrow y = 1 - x \Rightarrow \boxed{m_1 = -1}$

$L_2: 2x + 2y = 4 \Rightarrow 2y = 4 - 2x \Rightarrow y = 2 - x \Rightarrow \boxed{m_2 = -1}$

Hence $L_1 \parallel L_2$ with different intercepts

so any parallel lines form inconsistent system

That is, they have no intersection point (no solution)

(3) $\begin{cases} -x + y = 1 \\ -2x + 2y = 2 \end{cases}$

$-x+y=1$ | $-2x+2y=2$

$y = 1+x$ | $-2x+2(1+x)=2$

$-2x+2+2x=2$

$2=2$ ✓

This system has infinitely many solutions since these Lines are Equivalent

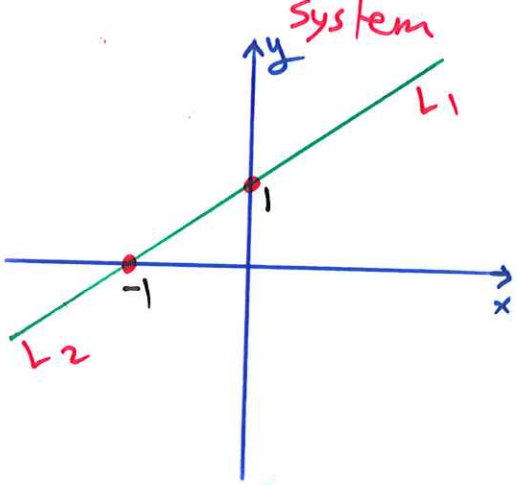
This system is called dependent system

- sketch the two lines

$L_1: -x+y=1$

$x=0 \Rightarrow y=1$ is y-intercept

$y=0 \Rightarrow x=-1$ is x-intercept



$L_2: -2x+2y=2$

$x=0 \Rightarrow 2y=2 \Rightarrow y=1$ is y-intercept

$y=0 \Rightarrow -2x=2 \Rightarrow x=-1$ is x-intercept

This system is consistent and has infinitly many solutions

- Find the slopes of these lines

$L_1: -x+y=1 \Rightarrow y=1+x \Rightarrow \boxed{m_1=1}$

$L_2: -2x+2y=2 \Rightarrow 2y=2+2x \Rightarrow y=1+x \Rightarrow \boxed{m_2=1}$

Hence, $L_1 // L_2$ with same intercepts

so any parallel lines with same intercepts form dependent system (consistent) with infinitly many solutions like $(-1,0), (0,1), (1,2), (2,3), (3,4), \ldots$

Exp Solve the following linear systems using elimination method

(1) $\begin{cases} 2x + 5y = 24 \\ -6x + 2y = 30 \end{cases}$

Multiply L_1 by 3 $\Rightarrow$ $6x + 15y = 72$

$-6x + 2y = 30$ +

$17y = 102$

$y = \frac{102}{17} = 6$

substitute $\boxed{y=6}$ in L_1 $\Rightarrow$

$2x + 5(6) = 24 \Rightarrow 2x + 30 = 24 \Rightarrow 2x = -6$

$\Rightarrow \boxed{x = -3}$

Check: L_1: $2(-3) + 5(6) \stackrel{?}{=} 24$

$-6 + 30 \stackrel{?}{=} 24$

$24 = 24$ ✓

L_2: $-6(-3) + 2(6) \stackrel{?}{=} 30$

$18 + 12 \stackrel{?}{=} 30$

$30 = 30$ ✓

So this is consistent system with one solution $(x, y) = (-3, 6)$

This means the two lines intersect in one point

(L_1 not parallel to L_2)

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(2) $\begin{cases} 4x + 3y = 4 \\ 8x + 6y = 18 \end{cases}$

Multiply L_1 by -2 $\Rightarrow$ $-8x - 6y = -8$

$8x + 6y = 18$ +

$0 = 10$ ✗

impossible

- This system is inconsistent $\Rightarrow$ it has no solution
- $L_1 \parallel L_2$ with different intercepts
- L_1 does not intersect L_2