CHAPTER 4

Thermal Sensors

INSTRUCTIONAL OBJECTIVES

The objectives of this and the following two chapters stress the understanding required for application of measurement and instrumentation sensors. After you have read this chapter, you should be able to

- Define thermal energy, the relation of temperature scales to thermal energy, and temperature scale calibrations.
- Transform a temperature reading among the Kelvin, Rankine, Celsius, and Fahrenheit temperature scales.
- Design the application of an RTD temperature sensor to specific problems in temperature measurement.
- Design the application of a thermistor to specific temperature measurement problems.
- Design the application of a thermocouple to specific temperature measurement problems.
- Explain the operation of a bimetal strip for temperature measurement.
- Explain the operation of a gas thermometer and a vapor pressure thermometer.
- Develop the design of a system to measure temperature using a solid-state temperature sensor.

4.1 INTRODUCTION

Process control is a term used to describe any condition, natural or artificial, by which a physical quantity is regulated. There is no more widespread evidence of such control than that associated with temperature and other thermal phenomena. In our natural surroundings, some of the most remarkable techniques of temperature regulation are found in the bodily functions of living creatures. On the artificial side, humans have been vitally concerned with temperature control since the first fires were struck for warmth. Industrial temperature regulation has always been of paramount importance and becomes even more so with the advance of technology. In this chapter, we will be concerned first with developing an understanding of the principles of thermal energy and temperature, and then with developing a working knowledge of the various thermal sensors employed for temperature measurement.

4.2 DEFINITION OF TEMPERATURE

The materials that surround us and, indeed, of which we are constructed are composed of assemblages of atoms. Each of the 92 natural elements of nature is represented by a particular type of atom. The materials that surround us typically are not pure elements, but combinations of atoms of several elements that form molecules. Thus, helium is a natural element consisting of a particular type of atom; water, however, is composed of molecules, with each molecule consisting of a combination of two hydrogen atoms and one oxygen atom. In presenting a physical picture of thermal energy, we need to consider the physical relations or interaction of elements and molecules in a particular material as either solid, liquid, or gas. These statements actually refer to how the molecules of the material are interacting and to the thermal energy of the molecules.

4.2.1 Thermal Energy

Solid In any solid material, the individual atoms or molecules are strongly attracted and bonded to each other, so that no atom is able to move far from its particular location, or *equilibrium position.* Each atom, however, is capable of vibration about its particular location. We introduce the concept of thermal energy by considering the molecules' vibration.

Consider a particular solid material in which molecules are exhibiting no vibration; that is, the molecules are at rest. Such a material is said to have zero thermal energy, $W_{TH} = 0$. If we add energy to this material by placing it on a heater, for example, this energy starts the molecules vibrating about their equilibrium positions. We may say that the material now has some finite thermal energy, $W_{TH} > 0$.

Liquid If more and more energy is added to the material, the vibrations become more and more violent as the thermal energy increases. Finally, a condition is reached where the bonding attractions that hold the molecules in their equilibrium positions are overcome and the molecules "break away" and move about in the material. When this occurs, we say the material has *melted* and become a liquid. Now, even though the molecules are still attracted to one another, the thermal energy is sufficient to cause the molecules to move about and to no longer maintain the rigid structure of the solid. Instead of vibrating, one considers the molecules as randomly sliding about each other, and the average speed with which they move is a measure of the thermal energy imparted to the material.

Gas Further increases in thermal energy of the material intensify the velocity of the molecules until finally, the molecules gain sufficient energy to escape completely from the attraction of other molecules. Such a condition is manifested by *boiling* of the liquid. When the material consists of such unattached molecules moving randomly throughout a containing volume, we say the material has become a gas. The molecules still collide with each other and the walls of the container, but otherwise move freely throughout the container. The average speed of the molecules is again a measure of the thermal energy imparted to the molecules of the material.

Not all materials undergo these transitions at the same thermal energy, and indeed some not at all. Thus, nitrogen can be solid, liquid, and gas, but paper will experience a breakdown of its molecules before a liquid or gaseous state can occur. The whole subject of thermal sensors is associated with the measurement of the thermal energy of a material or an environment containing many different materials.

4.2.2 Temperature

If we are to measure thermal energy, we must have some sort of units by which to classify the measurement. The original units used were "hot" and "cold." These were satisfactory for their time but are inadequate for modern use. The proper unit for energy measurement is the *joules* of the sample in the SI system, but this would depend on the size of the material because it would indicate the total thermal energy contained. A measurement of the average thermal energy per molecule, expressed in joules, would be better to define thermal energy. We say "would be" because it is not traditionally used. Instead, special sets of units, whose origins are contained in the history of thermal energy measurements, are employed to define the *average energy per molecule* of a material. We will consider the four most common units. In each case, the name used to describe the thermal energy per molecule of a material is related by the statement that the material has a certain *degree of temperature;* the different sets of units are referred to as *temperature scales.*

Calibration To define the temperature scales, a set of *calibration points* is used; for each, the average thermal energy per molecule is well defined through equilibrium conditions existing between solid, liquid, or gaseous states of various pure materials. Thus, for example, a state of equilibrium exists between the solid and liquid phase of a pure substance when the *rate of phase change* is the same in either direction: liquid to solid, and solid to liquid. Some of the standard calibration points are

1. Oxygen: liquid/gas equilibrium
2. Water: solid/liquid equilibrium
3. Water: liquid/gas equilibrium
4. Gold: solid/liquid equilibrium

The various temperature scales are defined by the assignment of numerical values of temperatures to the list and additional calibration points. Essentially, the scales differ in two respects: (1) the location of the zero of temperature, and (2) the size of one unit of measure; that is, the average thermal energy per molecule represented by one unit of the scale.

The SI definition of the kelvin unit of temperature is in terms of the triple point of water. This is the state at which an equilibrium exists between the liquid, solid, and gaseous state of water maintained in a closed vessel. This system has a temperature of 273.16 K.

TABLE 4.1
Temperature scale calibration points

Calibration Point	Temperature K	°R	°F	°C
Zero thermal energy	0	0	−459.6	−273.15
Oxygen: liquid/gas	90.18	162.3	−297.3	−182.97
Water: solid/liquid	273.15	491.6	32	0
Water: liquid/gas	373.15	671.6	212	100
Gold: solid/liquid	1336.15	2405	1945.5	1063

Absolute Temperature Scales An absolute temperature scale is one that assigns a zero temperature to a material that has no thermal energy, that is, no molecular vibration. There are two such scales in common use: the Kelvin scale in kelvin (K) and Rankine scale in degrees Rankine (°R). These temperature scales differ only by the quantity of energy represented by one unit of measure; hence, a simple proportionality relates the temperature in °R to the temperature in K. Table 4.1 shows the values of temperature in kelvin and degrees Rankine at the calibration points introduced earlier. From this table we can determine the transformation of temperature between the water liquid/solid point and water liquid/gas point is 100 K and 180°R, respectively. Because these two numbers represent the same difference of thermal energy, it is clear that 1 K must be larger than 1°R by the ratio of the two numbers:

$$(1\,\mathrm{K}) = \frac{180}{100}(1^\circ\mathrm{R}) = \frac{9}{5}(1^\circ\mathrm{R})$$

Thus, the transformation between scales is given by

$$T(\mathrm{K}) = \frac{5}{9}T(^\circ\mathrm{R}) \tag{4.1}$$

where $T(\mathrm{K})$ = temperature in K
$T(^\circ\mathrm{R})$ = temperature in °R

EXAMPLE 4.1 A material has a temperature of 335 K. Find the temperature in °R.

Solution

$$T(^\circ\mathrm{R}) = \frac{9}{5}T(\mathrm{K})$$

$$T(^\circ\mathrm{R}) = \frac{9}{5}(335\,\mathrm{K}) = \mathbf{603^\circ R}$$

Relative Temperature Scales The *relative* temperature scales differ from the absolute scales only in a shift of the zero axis. Thus, when these scales indicate a zero of temperature, the thermal energy of the sample is not zero. These two scales are the Celsius (related to the kelvin) and the Fahrenheit (related to the Rankine), with temperature indicated by °C and °F, respectively. Table 4.1 shows various calibration points of these scales. The quantity of energy represented by 1°C is the same as that indicated by 1 K, but the zero has been shifted in the Celsius scale, so that

$$T(°\mathrm{C}) = T(\mathrm{K}) - 273.15 \tag{4.2}$$

Similarly, the size of 1°F is the same as the size of 1°R, but with a scale shift, so that

$$T(°\mathrm{F}) = T(°\mathrm{R}) - 459.6 \tag{4.3}$$

To transform from Celsius to Fahrenheit, we simply note that the two scales differ by the size of the degree, just as in K and °R, and a scale shift of 32 separates the two; thus,

$$T(°\mathrm{F}) = \frac{9}{5}T(°\mathrm{C}) + 32 \tag{4.4}$$

Relation to Thermal Energy It is possible to relate temperature to actual thermal energy in joules by using a constant called *Boltzmann's constant.* Although not true in all cases, it is a good approximation to state that the average thermal energy, W_{TH}, of a molecule can be found from the absolute temperature in K from

$$W_{\mathrm{TH}} = \frac{3}{2}kT \tag{4.5}$$

where $k = 1.38 \times 10^{-23}$ J/K is Boltzmann's constant. Thus, it is possible to determine the average thermal speed or velocity, v_{TH}, of a gas molecule by equating the kinetic energy of the molecule to its thermal energy

$$\frac{1}{2}mv_{\mathrm{TH}}^2 = W_{\mathrm{TH}} = \frac{3}{2}kT$$

and

$$v_{\mathrm{TH}} = \sqrt{\frac{3\,kT}{m}} \tag{4.6}$$

where m is the molecule mass in kilograms.

EXAMPLE 4.2 Given temperature of 144.5°C, express this temperature in **(a)** K and **(b)** °F.

Solution

a.
$$T(\mathrm{K}) = T(°\mathrm{C}) + 273.15$$
$$T(\mathrm{K}) = 144.5 + 273.15$$
$$T(\mathrm{K}) = \mathbf{417.65\ K}$$

b. $$T(°\text{F}) = \frac{9}{5}T(°\text{C}) + 32$$

$$T(°\text{F}) = \frac{9}{5}(144.5°\text{C}) + 32$$

$$T(°\text{F}) = \mathbf{292.1°F}$$

EXAMPLE 4.3

A sample of oxygen gas has a temperature of 90°F. If its molecular mass is 5.3×10^{-26} kg, find the average thermal speed of a molecule.

Solution

We first convert 90°F to K and then use Equation (4.6) to find the speed.

$$T(°\text{R}) = T(°\text{F}) + 459.6$$

$$T(°\text{R}) = 90°\text{F} + 459.6$$

$$T(°\text{R}) = 549°\text{R}$$

$$T(\text{K}) = \frac{5}{9}T(°\text{R}) = \frac{5}{9}(549.6\text{ K})$$

$$T(\text{K}) = 305.33\text{ K}$$

Then the speed is

$$v_{\text{TH}} = \sqrt{\frac{3\,kT}{m}}$$

$$v_{\text{TH}} = \left[\frac{(3)(1.38 \times 10^{-23}\text{ J/K})(305.33\text{ K})}{5.3 \times 10^{-26}\text{ kg}}\left(1\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2\text{J}}\right)\right]^{1/2}$$

$$v_{\text{TH}} = \mathbf{488.37\ m/s}$$

4.3 METAL RESISTANCE VERSUS TEMPERATURE DEVICES

One of the primary methods for electrical measurement of temperature involves changes in the electrical resistance of certain materials. In this, as well as other cases, the principal measurement technique is to place the temperature-sensing device in contact with the environment whose temperature is to be measured. The sensing device then takes on the temperature of the environment. Thus, a measure of its resistance indicates the temperature of the device and the environment. Time response becomes very important in these cases because the measurement must wait until the device comes into thermal equilibrium with the environment. The two basic devices used are the *resistance-temperature detector* (RTD), based on the variation of metal resistance with temperature, and the *thermistor*, based on the variation of semiconductor resistance with temperature.

4.3.1 Metal Resistance versus Temperature

A metal is an assemblage of atoms in the solid state in which the individual atoms are in an equilibrium position with superimposed vibration induced by the thermal energy. The chief

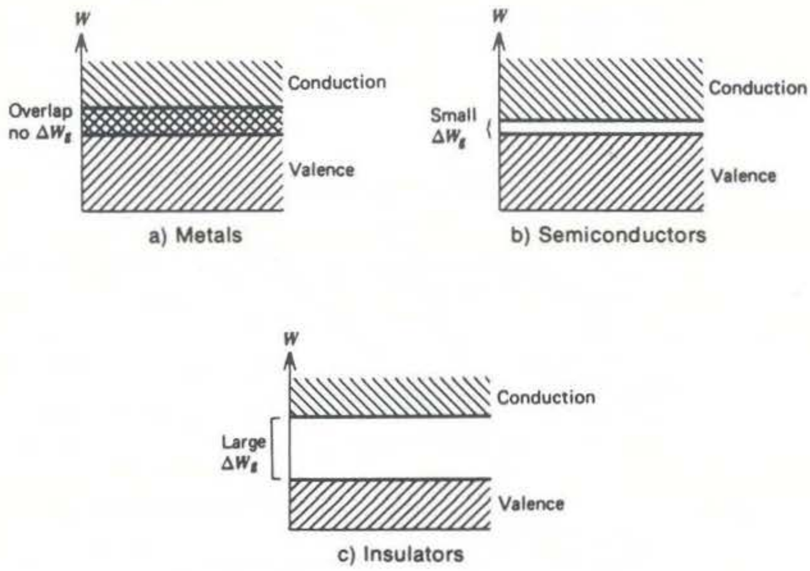


FIGURE 4.1
Energy bands for solids. Only conduction-band electrons are free to carry current.

characteristic of a metal is the fact that each atom gives up one electron, called its *valence electron,* that can move freely throughout the material; that is, it becomes a *conduction electron.* We say, then, for the whole material, that the valence band of electrons and the conduction band of electrons in the material overlap in energy, as shown in Figure 4.1a. Contrast this with a semiconductor, where a small gap exists between the top electron energy of the valence band and the bottom electron energy of the conduction band, as shown in Figure 4.1b. Similarly, Figure 4.1c shows that an insulator has a large gap between valence and conduction electrons. When a current is to be passed through a material, it is the conduction band electrons that carry the current.

As electrons move throughout the material, they collide with the stationary atoms or molecules of the material. When a thermal energy is present in the material and the atoms vibrate, the conduction electrons tend to collide even more with the vibrating atoms. This impedes the movement of electrons and absorbs some of their energy; that is, the material exhibits a *resistance* to electrical current flow. Thus, metallic resistance is a function of the vibration of the atoms and thus of the temperature. As the temperature is raised, the atoms vibrate with greater amplitude and frequency, which causes even more collisions with electrons, further impeding their flow and absorbing more energy. From this argument, we can see that metallic resistance should increase with temperature, and it does.

The graph in Figure 4.2 shows the effect of increasing resistance with temperature for several metals. To compare the different materials, the graph shows the relative resistance versus temperature. For a specific metal of high purity, the curve of relative resistance versus temperature is highly repeatable, and thus either tables or graphs can be used to determine the temperature from a resistance measurement using that material. It is possible to

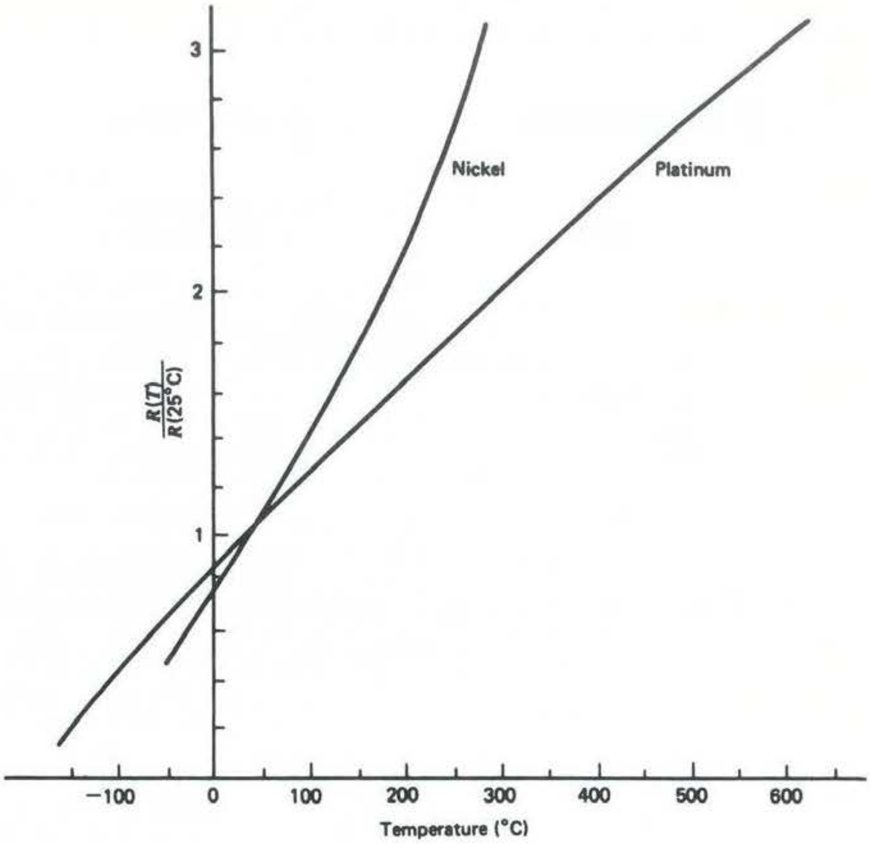


FIGURE 4.2
Metal resistance increases almost linearly with temperature.

express the resistance of a particular metal sample at a constant temperature (T) analytically using the equation

$$R = \rho \frac{l}{A} \, (T = \text{constant}) \tag{4.7}$$

where R = sample resistance(Ω)
l = length (m)
A = cross-sectional area(m^2)
ρ = resistivity($\Omega \cdot \mathrm{m}$)

In Equation (4.7), the principal increase in resistance with temperature is due to changes in the resistivity (ρ) of the metal with temperature. If the resistivity of some metal is known as a function of temperature, then Equation (4.7) can be used to determine the resistance of any particular sample of that material at the same temperature. In fact, curves such as those in Figure 4.2 are curves of resistivity versus temperature because, for example,

$$\frac{R(T)}{R(25°)} = \frac{\rho(T)l/A}{\rho(25°)l/A} = \frac{\rho(T)}{\rho(25°)} \quad \textbf{(4.8)}$$

The use of Equation (4.7), resistance versus temperature graphs, or resistance versus temperature tables is practical only when high accuracy is desired. For many applications, we can use an analytical approximation of the curves, for which we simply insert the temperature and quickly calculate the resistance, as described in Section 4.3.2.

4.3.2 Resistance versus Temperature Approximations

The curves of Figure 4.2 cover a large span of temperature, from about −150°C to about 600°C, or a nearly 750°C span. By examining the curves you can see that for smaller ranges of temperature, say the 100°C span between 100° and 200°C, the curves are nearly linear. This observation leads to development of a linear approximation of the sensor resistance versus temperature.

Linear Approximation A linear approximation means that we may develop an equation for a straight line that approximates the resistance versus temperature $(R - T)$ curve over some specified span. In Figure 4.3, we see a typical $R - T$ curve of some ma-

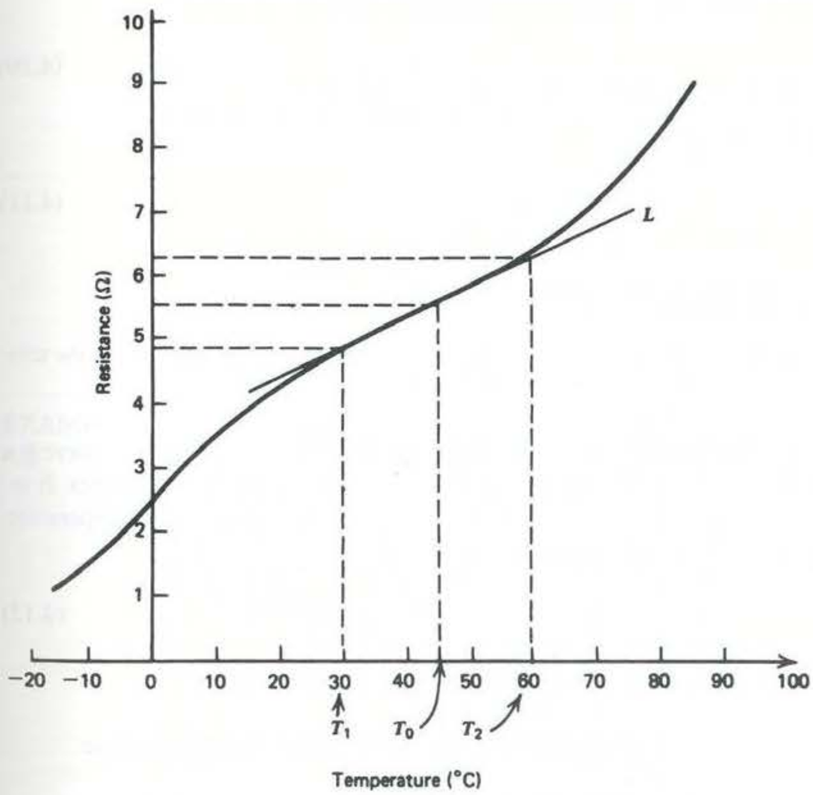


FIGURE 4.3
Line L represents a linear approximation of resistance versus temperature between T_1 and T_2.

terial. A straight line has been drawn between the points of the curve that represent temperature, T_1 and T_2 as shown, and T_0 represents the midpoint temperature. The equation of this straight line is the linear approximation to the curve over the span T_1 to T_2. The equation for this line is typically written as

$$R(T) = R(T_0)[1 + \alpha_0 \Delta T] \qquad T_1 < T < T_2 \tag{4.9}$$

where $R(T)$ = approximation of resistance at temperature T
$R(T_0)$ = resistance at temperature T_0
$\Delta T = T - T_0$
α_0 = fractional change in resistance per degree of temperature at T_0

The reason for using α_0 as the fractional slope of the $R - T$ curve is that this same constant can be used for cases of other physical dimensions (length and cross-sectional area) of the same kind of wire. Note that α_0 depends on the midpoint temperature T_0, which simply says that a straight-line approximation over some other span of the curve would have a different slope.

The value of α_0 can be found from values of resistance and temperature taken either from a graph, as given in Figure 4.2, or from a table of resistance versus temperature, as given in Problem 4.9 (see end of chapter). In general, then,

$$\alpha_0 = \frac{1}{R(T_0)} \cdot (\text{slope at } T_0) \tag{4.10}$$

or, for example, from Figure 4.3,

$$\alpha_0 = \frac{1}{R(T_0)} \cdot \left(\frac{R_2 - R_1}{T_2 - T_1} \right) \tag{4.11}$$

where R_2 = resistance at T_2
R_1 = resistance at T_1

The quantity α_0 has units of *inverse* temperature degrees and therefore depends on the temperature scale being used. Thus, the units of α_0 are typically $1/°C$ or $1/°F$.

Quadratic Approximation A quadratic approximation to the $R - T$ curve is a more accurate representation of the $R - T$ curve over some span of temperatures. It includes both a linear term, as before, and a term that varies as the square of the temperature. Such an analytical approximation is usually written as

$$R(T) = R(T_0)[1 + \alpha_1 \Delta T + \alpha_2 (\Delta T)^2] \tag{4.12}$$

where $R(T)$ = quadratic approximation of the resistance at T
$R(T_0)$ = resistance at T_0
α_1 = linear fractional change in resistance with temperature
$\Delta = T - T_0$
α_2 = quadratic fractional change in resistance with temperature

Values of α_1 and α_2 are found from tables or graphs, as indicated in the following examples, using values of resistance and temperature at three points. As before, both α_1 and α_2 depend on the temperature scale being used and have units of $1/°C$ and $(1/°C)^2$ if Celsius temperature is used, and $1/°F$ and $(1/°F)^2$ if the Fahrenheit scale is used.

The following examples show how these approximations are formed.

EXAMPLE 4.4

A sample of metal resistance versus temperature has the following measured values:

T(°F)	$R(\Omega)$
60	106.0
65	107.6
70	109.1
75	110.2
80	111.1
85	111.7
90	112.2

Find the linear approximation of resistance versus temperature between 60° and 90°F.

Solution

Since 75°F is the midpoint, this will be used for T_0 so that $R(T_0) = 110.2\ \Omega$. Then the slope can be found from Equation (4.11):

$$\alpha_0 = \frac{1}{110.2}\frac{(112.2 - 106.0)}{(90 - 60)} = 0.001875/°F$$

Thus, the linear approximation for resistance is

$$R(T) = 110.2[1 + 0.001875\,(T - 75)]\Omega$$

EXAMPLE 4.5

Find the quadratic approximation of resistance versus temperature for the data given in Example 4.4 between 60° and 90°F.

Solution

Again, since 75°F is the midpoint, we will use this for T_0, and therefore $R(T_0) = 110.2\ \Omega$. To find α_1 and α_2, two equations can be set up using the endpoints of the data, namely, $R(60°F)$ and $R(90°F)$.

$$112.2 = 110.2\,[1 + \alpha_1(60 - 75) + \alpha_2(60 - 75)^2]$$
$$106.0 = 110.2\,[1 + \alpha_1(90 - 75) + \alpha_2(90 - 75)^2]$$

Adding these two equations eliminates α_1 so that we can solve for α_2:

$$\alpha_2 = -44.36 \times 10^{-6}/(°F)^2$$

This can be used in either equation to find the value of $\alpha_1 = 0.001875/°F$. Thus, the quadratic approximation for the resistance versus temperature is

$$R(T) = 110.2\,[1 + 0.001875(T - 75) - 44.36 \times 10^{-6}(T - 75)^2]$$

EXAMPLE 4.6 By what percentage do the predictions of the linear and quadratic approximations vary from the actual values at 60°F and 85°F?

Solution

From Example 4.4, the predictions of the linear model can be found as

$$R(60°\text{F}) = 110.2\,[1 + 0.001875(60 - 75)]$$
$$= 107.1\ \Omega$$

This is an error from the actual value of 106 Ω of +1%. At 85°F, we find $R(85°\text{F}) = 112.3\ \Omega$, which is in error by 0.54%.

The quadratic model from Example 4.5 finds the resistances to be

$$R(60°\text{F}) = 110.2\,[1 + 0.001875(60 - 75) - 44.36 \times 10^{-6}(60 - 75)^2]$$
$$= 106.0\ \Omega$$

This is a zero error, because the endpoint was one of those used to determine the constants. For 85°F, we find that $R(85°\text{F}) = 111.8\ \Omega$, which is an error of +0.09%.

Clearly, the quadratic approximation provides a much better approximation of the resistance versus temperature.

4.3.3 Resistance-Temperature Detectors

A *resistance-temperature detector* (RTD) is a temperature sensor that is based on the principles discussed in the preceding sections; that is, metal resistance increasing with temperature. Metals used in these devices vary from platinum, which is very repeatable, quite sensitive, and very expensive, to nickel, which is not quite as repeatable, more sensitive, and less expensive.

Sensitivity An estimate of RTD *sensitivity* can be noted from typical values of α_0, the linear fractional change in resistance with temperature. For platinum, this number is typically on the order of 0.004/°C, and for nickel a typical value is 0.005/°C. Thus, with platinum, for example, a change of only 0.4 Ω would be expected for a 100-Ω RTD if the temperature is changed by 1°C. Usually, a specification will provide calibration information either as a graph of resistance versus temperature or as a table of values from which the sensitivity can be determined. For the same materials, however, this number is relatively constant because it is a function of resistivity.

Response Time In general, RTD has a response time of 0.5 to 5 s or more. The slowness of response is due principally to the slowness of thermal conductivity in bringing the device into thermal equilibrium with its environment. Generally, time constants are specified either for a "free air" condition (or its equivalent) or an "oil bath" condition (or its equivalent). In the former case, there is poor thermal contact and hence slow response, and in the latter, good thermal contact and fast response. These numbers yield a range of response times, depending on the application.

Construction An RTD, of course, is simply a length of wire whose resistance is to be monitored as a function of temperature. The construction is typically such that the wire is wound on a form (in a coil) to achieve small size and improve thermal conductivity to decrease response time. In many cases, the coil is protected from the environment by a *sheath* or protective tube that inevitably increases response time but may be necessary in hostile environments. A loosely applied standard sets the resistance at multiples of 100 Ω for a temperature of 0°C.

Signal Conditioning In view of the very small fractional changes of resistance with temperature (0.4%), the RTD is generally used in a *bridge* circuit. Figure 4.4 illustrates the essential features of such a system. The *compensation line* in the R_3 leg of the bridge is required when the lead lengths are so long that thermal gradients along the RTD leg may cause changes in line resistance. These changes show up as false information, suggesting changes in RTD resistance. By using the compensation line, the same resistance changes also appear on the R_3 side of the bridge and cause no net shift in the bridge null.

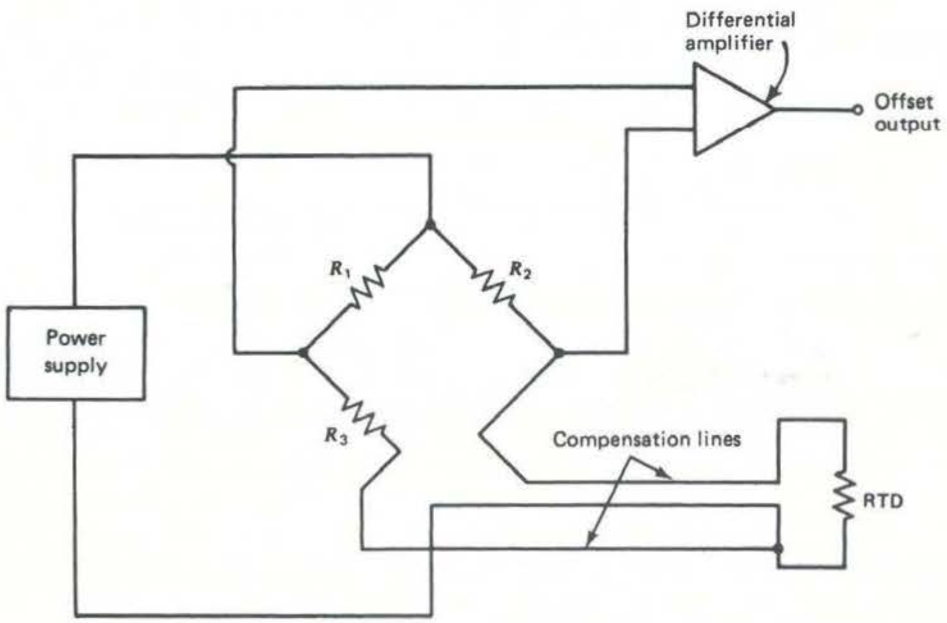


FIGURE 4.4
Note the compensation lines in this typical RTD signal-conditioning circuit.

Dissipation Constant Because the RTD is a resistance, there is an I^2R power dissipated by the device itself that causes a slight heating effect, a *self-heating*. This may also cause an erroneous reading or even upset the environment in delicate measurement conditions. Thus, the current through the RTD must be kept sufficiently low and constant to avoid self-heating. Typically, a *dissipation constant* is provided in RTD specifications. This number relates the power required to raise the RTD temperature by one degree of temperature. Thus, a 25-mW/°C dissipation constant shows that if I^2R power losses in the RTD equal 25 mW, the RTD will be heated by 1°C.

The dissipation constant is usually specified under two conditions: free air and a well-stirred oil bath. This is because of the difference in capacity of the medium to carry heat away from the device. The self-heating temperature rise can be found from the power dissipated by the RTD, and the dissipation constant from

$$\Delta T = \frac{P}{P_D} \tag{4.13}$$

where ΔT = temperature rise because of self-heating in°C
P = power dissipated in the RTD from the circuit in W
P_D = dissipation constant of the RTD in W/°C

EXAMPLE 4.7 An RTD has $\alpha_0 = 0.005/°\text{C}$, $R = 500\ \Omega$, and a dissipation constant of $P_D = 30\,\text{mW}/°\text{C}$ at 20°C. The RTD is used in a bridge circuit such as that in Figure 4.4, with $R_1 = R_2 = 500\ \Omega$ and R_3 a variable resistor used to null the bridge. If the supply is 10 V and the RTD is placed in a bath at 0°C, find the value of R_3 to null the bridge.

Solution

First we find the value of the RTD resistance at 0°C without including the effects of dissipation. From Equation (4.9), we get

$$R = 500[1 + 0.005(0 - 20)]\Omega$$
$$R = 450\ \Omega$$

Except for the effects of self-heating, we would expect the bridge to null with R_3 equal to 450 Ω also. Let's see what self-heating does to this problem. First, we find the power dissipated in the RTD from the circuit, assuming the resistance is still 450 Ω. The power is

$$P = I^2R$$

and the current I to three significant figures is found from

$$I = \frac{10}{500 + 450} = 0.011\ \text{A}$$

so that the power is

$$P = (0.011)^2(450) = 0.054\,\text{W}$$

We get the temperature rise from Equation (4.13):

$$\Delta T = \frac{0.054}{0.030} = 1.8°\text{C}$$

Thus, the RTD is not actually at the bath temperature of 0°C, but at a temperature of 1.8°C. We must find the RTD resistance from Equation (4.9) as

$$R = 500[1 + 0.005(1.8 - 20)]\Omega$$
$$R = 454.5\,\Omega$$

Thus, the bridge will null with $R_3 = \mathbf{454.5\,\Omega}$.

Range The effective range of RTDs principally depends on the type of wire used as the active element. Thus, a typical platinum RTD may have a range of −100° to 650°C, whereas an RTD constructed from nickel might typically have a specified range of −180° to 300°C.

4.4 THERMISTORS

The thermistor represents another class of temperature sensor that measures temperature through changes of material resistance. The characteristics of these devices are very different from those of RTDs and depend on the peculiar behavior of semiconductor resistance versus temperature.

4.4.1 Semiconductor Resistance versus Temperature

In contrast to metals, electrons in semiconductor materials are bound to each molecule with sufficient strength that no conduction electrons are contributed from the valence band to the conduction band. We say that a gap of energy, ΔW_g, exists between valence and conduction electrons, as shown in Figure 4.1b. Such a material behaves as an insulator because there are no conduction electrons to carry current through the material. This is true only when no thermal energy is present in the sample—that is, at a temperature of 0 K. When the temperature of the material is increased, the molecules begin to vibrate. In the case of a semiconductor, such vibration provides additional energy to the valence electrons. When such energy equals or exceeds the gap energy, ΔW_g, some of these electrons become free of the molecules. Thus, the electron is now in the conduction band and is free to carry current through the bulk of the material. As the temperature is further increased, more and more electrons gain sufficient energy to enter the conduction band. It is then clear that the semiconductor becomes a better conductor of

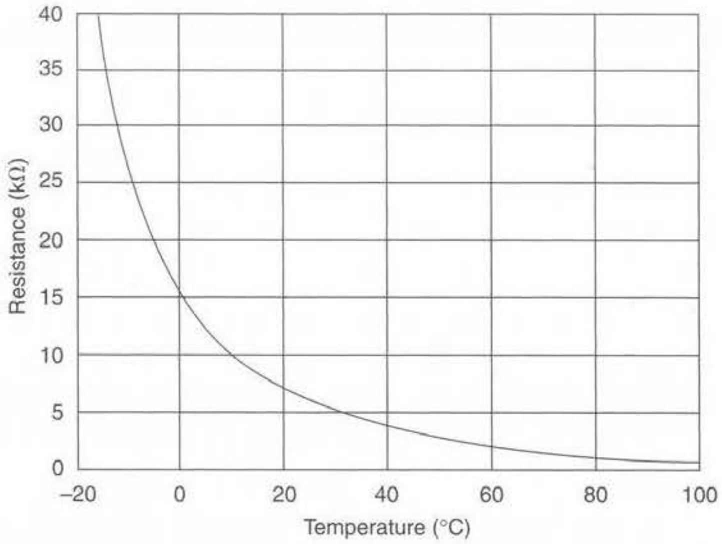


FIGURE 4.5
Thermistor resistance versus temperature is highly nonlinear and usually has a negative slope.

current as its temperature is increased—that is, as its resistance decreases. From this discussion, we form a picture of the resistance of a semiconductor material decreasing from very large values at low temperature to smaller resistance at high temperature. This is just the opposite of a metal. An important distinction, however, is that the change in semiconductor resistance is highly nonlinear, as shown in Figure 4.5. The reason semiconductors (but not insulators and other materials) behave this way is that the energy gap between conduction and valence bands is small enough to allow thermal excitation of electrons across the gap.

It is important to note that the effect just described requires that the thermal energy provide sufficient energy to overcome the band gap energy, ΔW_g. In general, a material is classified as a semiconductor when the gap energy is typically $0.01 - 4\,\text{eV}(1\,\text{eV} = 1.6 \times 10^{-19}\text{J})$. That this is true is exemplified by a consideration of silicon, a semiconductor that has a band gap of $\Delta W_g = 1.107\,\text{eV}$. When heated, this material passes from insulator to conductor. The corresponding thermal energies that bring this about can be found using Equation (4.5) and the joules-to-eV conversion, thus:

$$\begin{array}{ll} \text{For } T = 0\,\text{K} & W_{\text{TH}} = 0.0\ \text{eV} \\ \text{For } T = 100\,\text{K} & W_{\text{TH}} = 0.013\,\text{eV} \\ \text{For } T = 300\,\text{K} & W_{\text{TH}} = 0.039\ \text{eV} \end{array}$$

With average thermal energies as high as 0.039 eV, sufficient numbers of electrons are raised to the conduction level for the material to become a conductor. In *true* insulators, the gap energy is so large that temperatures less than destructive to the material cannot provide sufficient energy to overcome the gap energy.

4.4.2 Thermistor Characteristics

A thermistor is a temperature sensor that has been developed from the principles just discussed regarding semiconductor resistance change with temperature. The particular semiconductor material used varies widely to accommodate temperature ranges, sensitivity, resistance ranges, and other factors. The devices are usually mass-produced for a particular configuration, and tables or graphs of resistance versus temperature are provided for calibration. Variation of individual units from these nominal values is indicated as a net percentage deviation or a percentage deviation as a function of temperature.

Sensitivity The sensitivity of the thermistors is a significant factor in their application. Changes in resistance of 10% per °C are not uncommon. Thus, a thermistor with a nominal resistance of $10\,\text{k}\Omega$ at some temperature may change by $1\,\text{k}\Omega$ for a 1°C change in temperature. When used in null-detecting bridge circuits, sensitivity this large can provide for control, in principle, to less than 1°C in temperature.

Construction Because the thermistor is a bulk semiconductor, it can be fabricated in many forms. Thus, common forms include discs, beads, and rods, varying in size from a bead 1 mm in diameter to a disc several centimeters in diameter and several centimeters thick. By variation of doping and use of different semiconducting materials, a manufacturer can provide a wide range of resistance values at any particular temperature.

Range The temperature range of thermistors depends on the materials used to construct the sensor. In general, there are three range limitation effects: (1) melting or deterioration of the semiconductor, (2) deterioration of encapsulation material, and (3) insensitivity at higher temperatures.

The semiconductor material may melt or otherwise deteriorate as the temperature is raised. This condition generally limits the upper temperature to less than 300°C. At the low end, the principal limitation is that the thermistor resistance becomes very high, into the $\text{M}\Omega$s, making practical applications difficult. For the thermistor shown in Figure 4.5, if extended, the lower limit is about −80°C, where its resistance has risen to over $3\,\text{M}\Omega$! Generally, the lower limit is −50 to −100°C.

In most cases, the thermistor is encapsulated in plastic, epoxy, Teflon, or some other inert material. This protects the thermistor itself from the environment. This material may place an upper limit on the temperature at which the sensor can be used.

At higher temperatures, the slope of the R-T curve of the thermistor goes to zero. The device then is unable to measure temperature effectively because very little change in resistance occurs. You can see this occurring for the thermistor resistance versus temperature curve of Figure 4.5.

Response Time The response time of a thermistor depends principally on the quantity of material present and the environment. Thus, for the smallest bead thermistors in an oil bath (good thermal contact), a response of 1/2 s is typical. The same thermistor in still air will respond with a typical response time of 10 s. When encapsulated, as in Teflon or other materials, for protection against a hostile environment, the time response is increased by the poor thermal contact with the environment. Large disc or rod thermistors may have response times of 10 s or more, even with good thermal contact.

Signal Conditioning Because a thermistor exhibits such a large change in resistance with temperature, there are many possible circuit applications. In many cases, however, a bridge circuit is used because the nonlinear features of the thermistor make its use difficult as an actual measurement device. Because these devices are resistances, care must be taken to ensure that power dissipation in the thermistor does not exceed the limits specified or even interfere with the environment for which the temperature is being measured. *Dissipation constants* are quoted for thermistors as the power in milliwatts required to raise a thermistor's temperature 1°C above its environment. Typical values vary from 1 mW/°C in free air to 10 mW/°C or more in an oil bath.

EXAMPLE 4.8

A thermistor is to monitor room temperature. It has a resistance of 3.5 kΩ at 20°C with a slope of −10%/°C. The dissipation constant is $P_D = 5\,\text{mW}/°\text{C}$. It is proposed to use the thermistor in the divider of Figure 4.6 to provide a voltage of 5.0 V at 20°C. Evaluate the effects of self-heating.

Solution

It is easy to see that the design seems to work. At 20°C, the thermistor resistance will be 3.5 kΩ, and the divider voltage will be

$$V_D = \frac{3.5\,\text{k}\Omega}{3.5\,\text{k}\Omega + 3.5\,\text{k}\Omega} 10 = 5\,\text{V}$$

Let us now consider the effect of self-heating. The power dissipation in the thermistor will be given by

$$P = \frac{V^2}{R_{\text{TH}}} = \frac{(5)^2}{3.5\,\text{k}\Omega} = 7.1\,\text{mW}$$

The temperature rise of the thermistor can be found from Equation (4.13):

$$\Delta T = \frac{P}{P_D} = \frac{7.1\text{ mW}}{5\text{ mW}/°\text{C}} = 1.42°\text{C}$$

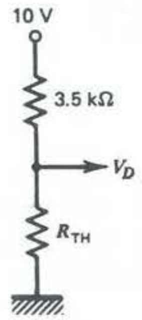


FIGURE 4.6
Divider circuit for Example 4.8.

But this means the thermistor resistance is really given by

$$R_{\text{TH}} = 3.5\,\text{k}\Omega - 1.42^\circ\text{C}(0.1/^\circ\text{C})\,(3.5\,\text{k}\Omega)$$
$$= 3.0\,\text{k}\Omega$$

and so the divider voltage is actually $V_D = 4.6$ V. The actual temperature of the environment is 20°C, but the measurement indicates that this is not so. Clearly, the system is unsatisfactory.

This example shows the importance of including dissipation effects in resistive-temperature transducers. The real answer to this problem involves a new design that reduces the thermistor current to a value giving perhaps 0.1°C of self-heating.

4.5 THERMOCOUPLES

In previous sections, we considered the change in material resistance as a function of temperature. Such a resistance change is considered a variable parameter property in the sense that the measurement of resistance, and thereby temperature, requires external power sources. There exists another dependence of electrical behavior of materials on temperature that forms the basis of a large percentage of all temperature measurement. This effect is characterized by a voltage-generating sensor in which an electromotive force (emf) is produced that is proportional to temperature. Such an emf is found to be almost linear with temperature and very repeatable for constant materials. Devices that measure temperature on the basis of this thermoelectric principle are called *thermocouples* (TCs).

4.5.1 Thermoelectric Effects

The basic theory of the thermocouple effect is found from a consideration of the electrical and thermal transport properties of different metals. In particular, when a temperature differential is maintained across a given metal, the vibration of atoms and motion of electrons is affected so that a difference in potential exists across the material. This potential difference is related to the fact that electrons in the hotter end of the material have more thermal energy than those in the cooler end, and thus tend to drift toward the cooler end. This drift varies for different metals at the same temperature because of differences in their thermal conductivities. If a circuit is closed by connecting the ends through another conductor, a current is found to flow in the closed loop.

The proper description of such an effect is to say that an emf has been established in the circuit and is causing the current to flow. Figure 4.7a shows a pictorial representation of this effect, called the *Seebeck effect*, in which two different metals, *A* and *B*, are used to close the loop with the connecting junctions at temperatures T_1 and T_2. We could not close the loop with the same metal because the potential differences across each leg would be the same, and thus no net emf would be present. The emf produced is proportional to the difference in temperature between the two junctions. Theoretical treatments of this problem involve the thermal activities of the two metals.

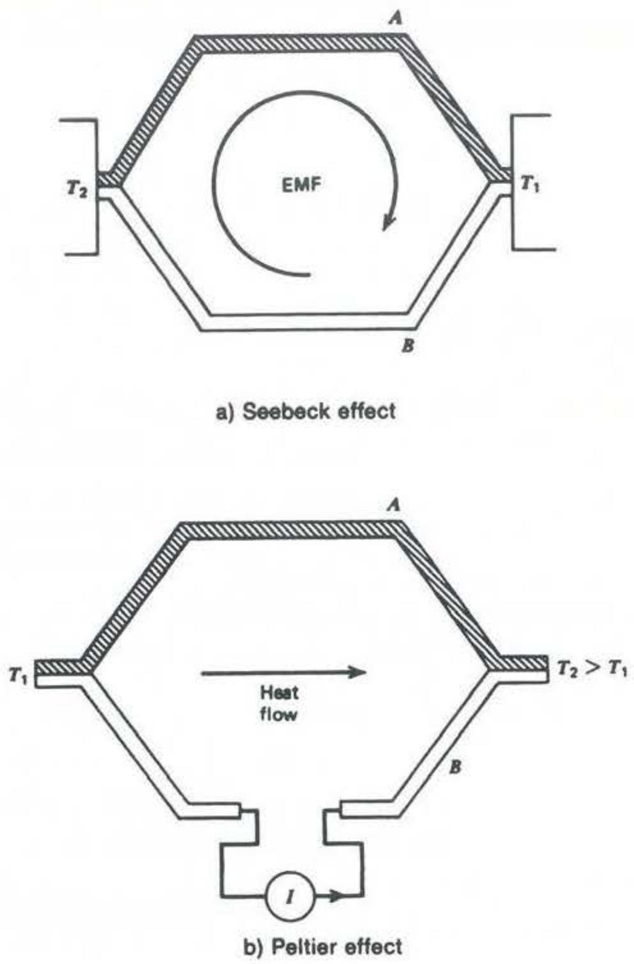


FIGURE 4.7
The Seebeck and Peltier effects refer to the relation between emf and temperature in a two-wire system.

Seebeck Effect Using solid-state theory, the aforementioned situation may be analyzed to show that its emf can be given by an integral over temperature

$$\varepsilon = \int_{T_1}^{T_2} (Q_A - Q_B)\, dT$$

where ε = emf produced in volts
T_1, T_2 = junction temperatures in K
Q_A, Q_B = thermal transport constants of the two metals

This equation, which describes the Seebeck effect, shows that the emf produced is proportional to the *difference* in temperature and, further, to the difference in the metallic thermal transport constants. Thus, if the metals are the same, the emf is zero, and if the temperatures are the same, the emf is also zero.

In practice, it is found that the two constants, Q_A and Q_B, are nearly independent of temperature and that an approximate linear relationship exists as

$$\varepsilon = \alpha(T_2 - T_1)$$

where α = constant in V/K

T_1, T_2 = junction temperatures in K

However, the small but finite temperature dependence of Q_A and Q_B is necessary for accurate considerations.

EXAMPLE 4.9 Find the Seebeck emf for a material with $\alpha = 50\,\mu\text{V}/°\text{C}$ if the junction temperatures are 20°C and 100°C.

Solution

The emf can be found from

$$\varepsilon = \alpha(T_2 - T_1)$$
$$\varepsilon = (50\,\mu\text{V}/°\text{C})(100°\text{C} - 20°\text{C})$$
$$\varepsilon = \mathbf{4\ mV}$$

Peltier Effect An interesting and sometimes useful extension of the same thermoelectric properties occurs when the reverse of the Seebeck effect is considered. In this case, we construct a closed loop of two different metals, A and B, as before. Now, however, an external voltage is applied to the system to cause a current to flow in the circuit, as shown in Figure 4.7b. Because of the different electrothermal transport properties of the metals, it is found that one of the junctions will be *heated* and the other *cooled;* that is, the device is a refrigerator! This process is referred to as the *Peltier effect.* Some practical applications of such a device, such as cooling small electronic parts, have been employed.

4.5.2 Thermocouple Characteristics

To use the Seebeck effect as the basis of a temperature sensor, we need to establish a definite relationship between the measured emf of the thermocouple and the unknown temperature. We see first that one temperature must already be known because the Seebeck voltage is proportional to the *difference* between junction temperatures. Furthermore, every connection of different metals made in the thermocouple loop for measuring devices, extension leads, and so on will contribute an emf, depending on the difference in metals and various junction temperatures. To provide an output that is definite with respect to the temperature to be measured, an arrangement such as shown in Figure 4.8a is used. This shows that the measurement junction, T_M, is exposed to the environment whose temperature is to be measured. This junction

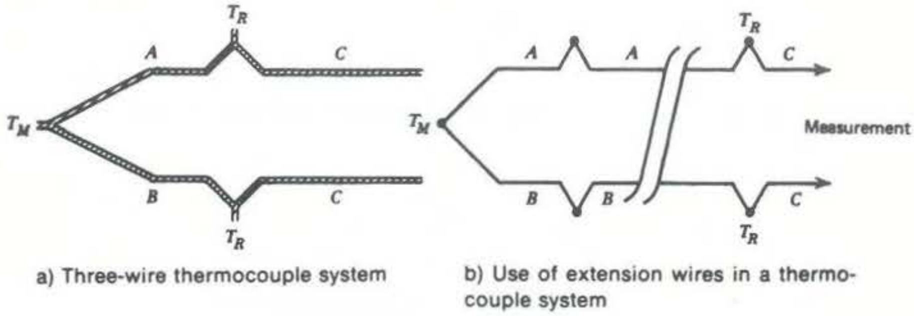


FIGURE 4.8
Practical measurements with a thermocouple system often employ extension wires to move the reference to a more secure location.

is formed of metals A and B as shown. Two other junctions are then formed to a common metal, C, which then connects to the measurement apparatus. The "reference" junctions are held at a common, known temperature T_R, the reference junction temperature. When an emf is measured, such problems as voltage drops across resistive elements in the loop must be considered. In this arrangement, an open-circuit voltage is measured (at high impedance) that is then a function of only the temperature difference $(T_M - T_R)$ and the type of metals A and B. The voltage produced has a magnitude dependent on the absolute magnitude of the temperature difference and a polarity dependent on which temperature is larger, reference or measurement junction. Thus, it is not necessary that the measurement junction have a higher temperature than the reference junctions, but both magnitude and sign of the measured voltage must be noted.

To use the thermocouple to measure a temperature, the reference temperature must be known, and the reference junctions must be held at the same temperature. The temperature should be constant, or at least not vary much. In most industrial environments, this would be difficult to achieve if the measurement junction and reference junction were close. It is possible to move the reference junctions to a remote location without upsetting the measurement process by the use of *extension wires,* as shown in Figure 4.8b. A junction is formed with the measurement system, but to wires of the same type as the thermocouple. These wires may be stranded and of different gauges, but they must be of the same type of metal as the thermocouple. The extension wires now can be run a significant distance to the actual reference junctions.

Thermocouple Types Certain standard configurations of thermocouples using specific metals (or alloys of metals) have been adopted and given letter designations; examples are shown in Table 4.2. Each type has its particular features, such as range, linearity, inertness to hostile environments, sensitivity, and so on, and is chosen for specific applications accordingly. In each type, various sizes of conductors may be employed for specific cases, such as oven measurements, highly localized measurements, and so on. The curves of voltage versus temperature in Figure 4.9 are shown for a reference temperature

TABLE 4.2
Standard thermocouples

Type	Materials[a]	Normal Range
J	Iron-constantan[b]	−190°C to 760°C
T	Copper-constantan	−200°C to 371°C
K	Chromel-alumel	−190°C to 1260°C
E	Chromel-constantan	−100°C to 1260°C
S	90% platinum + 10% rhodium-platinum	0°C to 1482°C
R	87% platinum + 13% rhodium-platinum	0°C to 1482°C

[a]First material is more positive when the measurement temperature is more than the reference temperature.
[b]Constantan, chromel, and alumel are registered trade names of alloys.

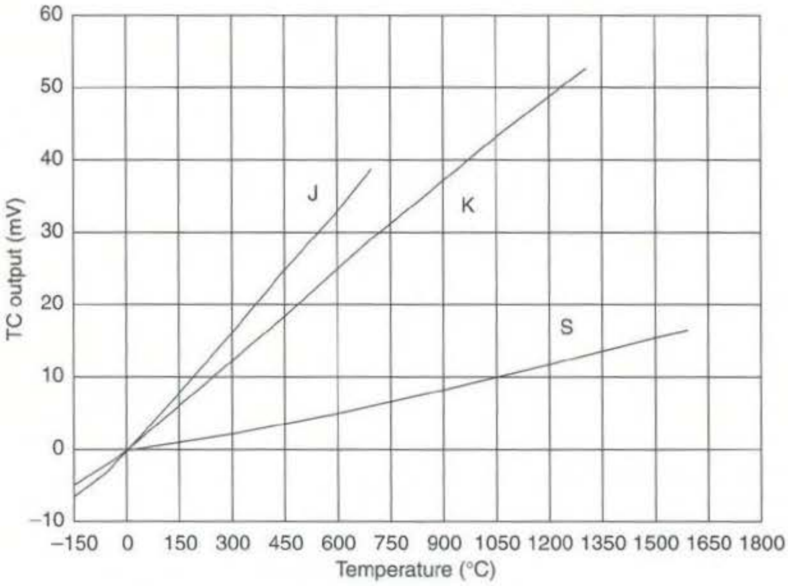


FIGURE 4.9
These curves of thermocouple voltage versus temperature for a 0°C reference show the different sensitivities and nonlinearities of three types.

of 0°C and for several types of thermocouples. We wish to note several important features from these curves.

First, we see that the type J and K thermocouples are noted for their rather large slope, that is, high sensitivity—making measurements easier for a given change in temperature. We note that the type S thermocouple has much less slope and is appropriately less sensitive. It has the significant advantages of a much larger possible range of measurement, including very high temperatures, and is highly inert material. Another important feature is that these curves are not exactly linear. To take advantage of the inherent accuracy possible with these devices, comprehensive tables of voltage versus temperature have been determined for many types of thermocouples. Such tables are found in Appendix 3.

Thermocouple Polarity The voltage produced by a TC is differential in the sense that it is measured between the two metal wires. As noted in the footnote to Table 4.2, by convention the description of a TC identifies how the polarity is interpreted. A type J thermocouple is called iron-constantan. This means that if the reference temperature is less than the measurement junction temperature, the iron will be more positive than the constantan. Thus, a type J with a 0°C reference will produce +5.27 mV for a measurement junction of 100°C, meaning that the iron is more positive than the constantan. For a measurement junction of −100°C, the polarity changes, and the voltage will be −4.63 mV, meaning that the iron is less positive than the constantan.

Thermocouple Tables The thermocouple tables simply give the voltage that results for a particular type of thermocouple when the reference junctions are at a particular reference temperature, and the measurement junction is at a temperature of interest. Referring to the tables, for example, we see that for a type J thermocouple at 210°C with a 0°C reference, the voltage is

$$V(210°\text{C}) = 11.34\ \text{mV} \qquad (\text{type J, 0°C ref})$$

Conversely, if we measure a voltage of 4.768 mV with a type S and a 0°C reference, we find from the table

$$T(4.768\ \text{mV}) = 555°\text{C} \qquad (\text{type S, 0°C ref})$$

In most cases, the measured voltage does not fall exactly on a table value. When this happens, it is necessary to *interpolate* between table values that bracket the desired value. In general, the value of temperature can be found using the following interpolation equation:

$$T_M = T_L + \left[\frac{T_H - T_L}{V_H - V_L}\right](V_M - V_L) \tag{4.14}$$

The measured voltage, V_M, lies between a higher voltage, V_H, and a lower voltage, V_L, which are in the tables. The temperatures corresponding to these voltages are T_H and T_L, respectively, as shown in Example 4.10.

EXAMPLE 4.10 A voltage of 23.72 mV is measured with a type K thermocouple at a 0°C reference. Find the temperature of the measurement junction.

Solution

From the table, we find that $V_M = 23.72$ lies between $V_L = 23.63$ mV and $V_H = 23.84$ mV with corresponding temperatures of $T_L = 570$°C and $T_H = 575$°C, respectively. The junction temperature is found from Equation (4.14):

$$T_M = 570°C + \frac{(575°C - 570°C)}{(23.84 - 23.63 \text{ mV})}(23.72 \text{ mV} - 23.63 \text{ mV})$$

$$T_M = 570°C + \frac{5°C}{0.21}(0.09 \text{ mV})$$

$$T_M = \mathbf{572.1°C}$$

The reverse situation occurs when the voltage for a particular temperature, T_M, which is not in the table, is desired. Again, an interpolation equation can be used, such as

$$V_M = V_L + \left[\frac{V_H - V_L}{T_H - T_L}\right](T_M - T_L) \tag{4.15}$$

where all terms are as defined for Equation (4.14).

EXAMPLE 4.11 Find the voltage of a type J thermocouple with a 0°C reference if the junction temperature is −172°C.

Solution

We do not let the signs bother us but merely apply the interpolation relation directly. From the Appendix 3 tables, we see that the junction temperature lies between a high (algebraically) $T_H = -170°C$ and a low $T_L = -175°C$. The corresponding voltages are $V_H = -7.12$ mV and $V_L = -7.27$ mV. The TC voltage will be

$$V_M = -7.27 \text{ mV} + \frac{-7.12 + 7.27}{-170 + 175}(-172°C + 175°C)$$

$$V_M = -7.27 \text{ mV} + \frac{0.15 \text{ mV}}{5°C}(3°C)$$

$$V_M = \mathbf{-7.18 \text{ mV}}$$

Change of Table Reference It has already been pointed out that thermocouple tables are prepared for a particular junction temperature. It is possible to use these tables with a TC that has a different reference temperature by an appropriate shift in the table scale. The key point to remember is that the voltage is proportional to the difference between the reference and measurement junction temperature. Thus, if a new reference is greater than the table reference, all voltages of the table will be less for this TC. The amount less will be just the voltage of the new reference as found on the table.

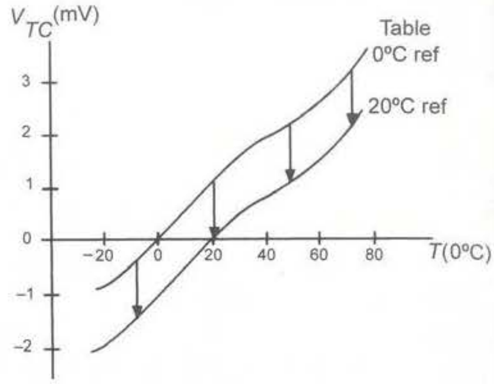
Consider a type J thermocouple with a 30°C reference. The tables show that a type J thermocouple with a 0°C reference produces 1.54 mV at 30°C. This, then, is the correction factor that will be applied to any voltage expected when the reference is 30°C. Consider a temperature of 400°C.

$$V(400°C) = 21.85 \text{ mV} \qquad \text{(Type J, 0°C ref)}$$

and

$$V(30°C) = 1.54 \text{ mV} \qquad \text{(Type J, 0°C ref)}$$

FIGURE 4.10
A change of reference from 0°C to 20°C is equivalent to sliding the TC curve down in voltage.



The correction factor is subtracted because the difference between 400° and 30°C is less than the difference between 400° and 0°C, and the voltage depends upon this difference. Therefore,

$$V(400°\text{C}) = 20.31 \text{ mV} \qquad (\text{Type J, } 30°\text{C ref})$$

To avoid confusion on the reference, all TC voltages will henceforth be represented with a subscript of the type and reference. Thus, V_{J0} means a type J with a 0°C reference, and V_{J30} means a type J with a 30°C reference.

To consider a couple more temperatures, verify the following:

$$V_{J30}(150°\text{C}) = 8.00 - 1.54 = 6.46 \text{ mV}$$
$$V_{J30}(-90°\text{C}) = -4.21 - 1.54 = -5.75 \text{ mV}$$

In the last case, the magnitude of the voltage is larger because the difference between −90°C and 30°C is greater than the difference between −90°C and 0°C, and the voltage depends on this difference.

In summary, tables of TC voltage versus temperature are given for a specific reference temperature. These tables can be used to relate voltage and temperature for a different reference temperature by using a voltage correction factor. This correction factor is simply the voltage that the new reference would produce from the tables. The correction factor is algebraically subtracted from table values, if the new reference is less than the table reference, and added if the new reference is less than the table reference.

Figure 4.10 illustrates the correction process graphically. The curve for a 0°C reference is assumed to be the table values. You can see, then, that for a reference of 20°C, the curve is simply reduced everywhere by the correction voltage of 1.02 mV. In effect, the original curve slides down by 1.02 mV.

Note that you cannot simply add or subtract the new reference temperature as a correction factor. Correction is always applied to voltages.

EXAMPLE 4.12 A type J thermocouple with a 25°C reference is used to measure oven temperature from 300° to 400°C. What output voltages correspond to these temperatures?

Solution

Since the reference is not 0°C, a correction factor must be applied to the table voltages. That correction factor is simply the table voltage for a temperature of 25°C—that is, the reference. From the table, we find

$$V_{J0}(25°C) = 1.28 \text{ mV}$$

This, then, is the correction factor. From the tables, we now find the voltages for 300° and 400°C,

$$V_{J0}(300°C) = 16.33 \text{ mV}$$
$$V_{J0}(400°C) = 21.85 \text{ mV}$$

Now, since the reference is actually closer to these temperatures than to the table reference, we expect the voltages to be smaller. Therefore, the correction factor is subtracted to find the actual voltages:

$$V_{J25}(300°C) = 16.33 - 1.28 \text{ mV} = 15.05 \text{ mV}$$
$$V_{J25}(400°C) = 21.85 - 1.28 \text{ mV} = 20.57 \text{ mV}$$

EXAMPLE 4.13

A type K thermocouple with a 75°F reference produces a voltage of 35.56 mV. What is the temperature?

Solution

First, the reference temperature must be converted to Celsius, $T(°C) = 5(75 - 32)/9 = 23.9°C$. So we get $V_{K23.9} = 35.56$ mV and now need to determine the temperature. The voltage correction factor is determined using the 0°C tables and interpolation. The reference falls between 20° and 25°C, so

$$V_{K0}(23.9°C) = 0.8 + \frac{(1.00 - 0.80)}{(25 - 20)}(23.9 - 20) = 0.96 \text{ mV}$$

Since the reference is greater than the table reference, we would expect the table voltages to be larger for the same temperature. Thus, the correction is added to the measured voltage:

$$V_{K0}(T) = 35.56 + 0.96 = 36.52 \text{ mV}$$

From the tables, this voltage lies between 36.35 mV at 875°C and 36.55 mV at 880°C. Using interpolation,

$$T = 875 + \frac{(880 - 875)}{(36.55 - 36.35)}(36.52 - 36.35) = 879.3°C$$

The correction factors are not just minor adjustments. In this last example, if you ignored the correction factor and used 35.56 mV directly in the tables, the erroneously indicated temperature would be 850.5°C, or a 28.8°C error.

4.5.3 Thermocouple Sensors

The use of a thermocouple for a temperature sensor has evolved from an elementary process with crudely prepared thermocouple constituents into a precise and exacting technique.

Sensitivity A review of the tables shows that the range of thermocouple voltages is typically less than 100 mV. The actual sensitivity strongly depends on the type of signal conditioning employed and on the TC itself. We see from Figure 4.9 the following worst and best case of sensitivity:

- Type J: 0.05 mV/°C (typical)
- Type S: 0.006 mV/°C (typical)

Construction A thermocouple by itself is, of course, simply a welded or even twisted junction between two metals, and in many cases, that is the construction. There are cases, however, where the TC is sheathed in a protective covering or even sealed in glass to protect the unit from a hostile environment. The size of the TC wire is determined by the application and can range from #10 wire in rugged environments to fine #30 AWG wires or even 0.02-mm microwire in refined biological measurements of temperature.

Range The thermocouple temperature sensor has the greatest range of all the types considered. The tables in Appendix 3 show that the general-purpose, type J thermocouple is usable from −150° to 745°C. The type S is usable up to 1765°C. Other special types have ranges above and below these.

Time Response Thermocouple time response is simply related to the size of the wire and any protective material used with the sensor. The time response equates to how long it takes the TC system to reach thermal equilibrium with the environment.

Large, industrial TCs using thick wire or encased in stainless steel sheathing may have time constants as high as 10 to 20 s. However, a TC made from very small-gauge wire can have a time constant as small as 10 to 20 ms.

Often, the time constant is specified under conditions of good thermal contact and poor thermal contact as well, so that you can account for the environment.

Signal Conditioning The key element in the use of thermocouples is that the output voltage is very small, typically less than 50 mV. This means that considerable amplification will be necessary for practical application. In addition, the small signal levels make the devices susceptible to electrical noise. In most cases, the thermocouple is used with a high-gain differential amplifier.

Reference Compensation A problem with the practical use of thermocouples is the necessity of knowing the reference temperature. Because the TC voltage is proportional to the difference between the measurement and reference junction temperatures, variations of the reference temperature show up as direct errors in the measurement temperature determination. The following techniques are employed for reference junction compensation:

1. *Controlled temperature reference block* In some cases, particularly when many thermocouples are in use, extension wires bring all reference junctions to a temperature-controlled box in the control room. Then, a local control system maintains this box at a precisely controlled temperature so that the reference is regulated. Readouts of temperature from the TC voltage take into account this known reference temperature.
2. *Reference compensation circuits* The modern approach to reference correction is supplied by specialized integrated circuits (ICs) that add or subtract the correction factor directly to the TC output. These ICs, which are called *cold junction compensators* or *ice point compensators,* are actually temperature sensors themselves that measure the reference junction temperature. The ICs include circuitry that provides a scaled correction voltage, depending on the type of TC being used. Figure 4.11 shows a block diagram of how the compensator is used. The actual reference junctions are at the connection to the IC, so the IC temperature is the reference temperature.
3. *Software reference correction* In computer-based measurement systems, the reference junction temperature can be measured by a precision thermistor or another IC temperature sensor and provided as an input to the computer. Software routines then can provide necessary corrections to the thermocouple temperature signal that is also an input to the computer.

Noise Perhaps the biggest obstacle to the use of thermocouples for temperature measurement in industry is their susceptibility to electrical noise. First, the voltages generated generally are less than 50 mV and often are only 2 or 3 mV, and in the industrial environment it is common to have hundreds of millivolts of electrical noise generated by large electrical machines in any electrical system. Second, a thermocouple constitutes an excellent antenna for pickup of noise from electromagnetic radiation in the radio, TV, and

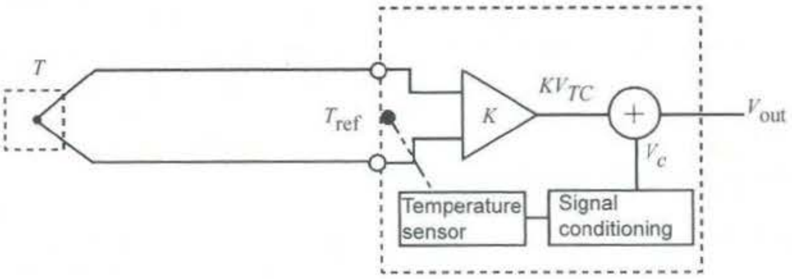


FIGURE 4.11
Automatic reference correction is now common in TC systems. Frequently thermistors or other solid-state temperature sensors are used for the reference measurement.

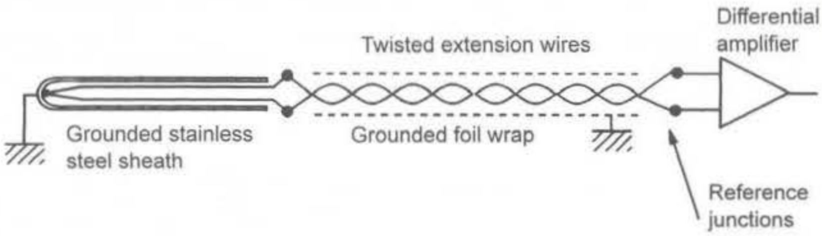


FIGURE 4.12
Since TC voltages are small, great care must be taken to protect against electrical noise by using shielding, twisting, and differential amplification.

microwave bands. In short, a bare thermocouple may have many times more noise than temperature signal at a given time.

To use thermocouples effectively in industry, a number of noise reduction techniques are employed. The following three are the most popular:

1. The extension or lead wires from the thermocouple to the reference junction or measurement system are twisted and then wrapped with a grounded foil sheath.
2. The measurement junction itself is grounded at the point of measurement. The grounding is typically to the inside of the stainless steel sheath that covers the actual thermocouple.
3. An instrumentation amplifier that has excellent common-mode rejection is employed for measurement.

Figure 4.12 shows a typical arrangement for measurement with a thermocouple. Note that the junction itself is grounded through the stainless steel sheath. The differential amplifier must have very good common-mode rejection to aid in the noise-rejection process.

The advantage to grounding the measurement junction is that the noise voltage will be distributed equally on each wire of the TC. Then the differential amplifier will, at least partially, cancel this noise because the voltage on these lines is subtracted.

Twisting is done to decouple the wires from induced voltages from varying electric and magnetic fields that permeate our environment. In principle, equal voltages are induced in each loop of the twisted wires but of opposite phase, so they cancel.

4.6 OTHER THERMAL SENSORS

The sensors discussed in the previous sections cover a large fraction of the temperature measurement techniques used in process control. There remain, however, numerous other devices or methods of temperature measurement that may be encountered. Pyrometric methods involve measurement of temperature by the electromagnetic radiation that is emitted in proportion to temperature. This technique is discussed in detail in Chapter 6.

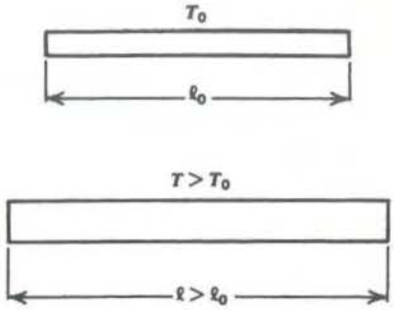


FIGURE 4.13
A solid object experiences a physical expansion in proportion to temperature. Here the effect is highly exaggerated.

4.6.1 Bimetal Strips

This type of temperature sensor has the characteristics of being relatively inaccurate, having hysteresis, having relatively slow time response, and being low in cost. Such devices are used in numerous applications, particularly where an ON/OFF cycle rather than smooth or continuous control is desired.

Thermal Expansion We have seen that greater thermal energy causes the molecules of a solid to execute greater-amplitude and higher-frequency vibrations about their average positions. It is natural to expect that an expansion of the volume of a solid would accompany this effect, as the molecules tend to occupy more volume on the average with their vibrations. This effect varies in degree from material to material because of many factors, including molecular size and weight, lattice structure, and others. Although one can speak of a volume expansion, as described, it is more common to consider a length expansion when dealing with solids, particularly in the configuration of a rod or beam. Thus, if we have a rod of length l_0 at temperature T_0 as shown in Figure 4.13, and the temperature is raised to a new value, T, the rod will be found to have a new length, l, given by

$$l = l_0[1 + \gamma \Delta T] \tag{4.16}$$

where $\Delta T = T - T_0$ and γ is the linear thermal expansion coefficient appropriate to the material of which the rod is produced. Several different expansion coefficients are given in Table 4.3.

TABLE 4.3
Thermal expansion coefficients

Material	Expansion Coefficient
Aluminum	$25 \times 10^{-6}/°C$
Copper	$16.6 \times 10^{-6}/°C$
Steel	$6.7 \times 10^{-6}/°C$
Beryllium/copper	$9.3 \times 10^{-6}/°C$

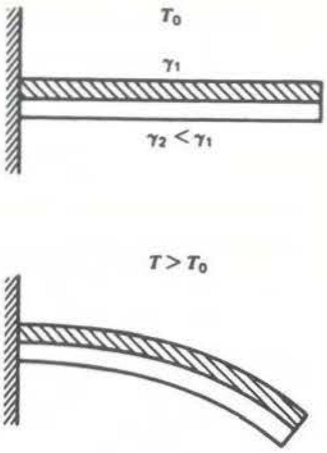


FIGURE 4.14
A bimetal strip will curve when exposed to a temperature change because of different thermal expansion coefficients. Metal thickness has been exaggerated in this view.

Bimetallic Sensor The thermal sensor exploiting the effect discussed previously occurs when two materials with grossly different thermal expansion coefficients are bonded together. Thus, when heated, the different expansion rates cause the assembly curve shown in Figure 4.14. This effect can be used to close switch contacts or to actuate an ON/OFF mechanism when the temperature increases to some appropriate setpoint. The effect also is used for temperature indicators, by means of assemblages, to convert the curvature into dial rotation.

EXAMPLE 4.14 How much will an aluminum rod of 10-m length at 20°C expand when the temperature is changed from 0° to 100°C?

Solution

First, find the length at 0°C and at 100°C; then find the difference. Using Equation (4.16) at 0°C, we get

$$l_1 = (10\text{ m})[1 + (2.5 \times 10^{-5}/°\text{C})(0°\text{C} - 20°\text{C})]$$
$$l_1 = 9.995\text{ m}$$

and at 100°C,

$$l_2 = (10\text{ m})[1 + (2.5 \times 10^{-5}/°\text{C})(100°\text{C} - 20°\text{C})]$$
$$l_2 = 10.02\text{ m}$$

Thus, the expansion is

$$l_2 - l_1 = 0.025\text{ m} = \mathbf{25\ mm}$$

4.6.2 Gas Thermometers

The operational principle of the gas thermometer is based on a basic law of gases. In particular, if a gas is kept in a container at constant volume and the pressure and temperature vary, the ratio of gas pressure and temperature is a constant:

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \tag{4.17}$$

where p_1, T_1 = absolute pressure and temperature (in K) in state 1
p_2, T_2 = absolute pressure and temperature in state 2

EXAMPLE 4.15 A gas in a closed volume has a pressure of 120 psi at a temperature of 20°C. What will the pressure be at 100°C?

Solution

First we convert the temperature to the absolute scale of Kelvin using Equation (4.2). We get 293.15 K and 373.15 K. Then we use Equation (4.17) to find the pressure in state 2.

$$p_2 = \frac{T_2}{T_1} p_1$$

$$p_2 = \frac{373.15}{293.15}(120 \text{ psi})$$

$$p_2 = \mathbf{153\ psi}$$

Because the gas thermometer converts temperature information directly into pressure signals, it is particularly useful in pneumatic systems. Such transducers are also advantageous because there are no moving parts and no electrical stimulation is necessary. For electronic analog or digital process-control applications, however, it is necessary to devise systems for converting the pressure to electrical signals. This type of sensor is often used with Bourdon tubes (see Chapter 5) to produce direct-indicating temperature meters and recorders. The gas most commonly employed is nitrogen. Time response is slow in relation to electrical devices because of the greater mass that must be heated.

4.6.3 Vapor-Pressure Thermometers

A vapor-pressure thermometer converts temperature information into pressure, as does the gas thermometer, but it operates by a different process. If a closed vessel is partially filled with liquid, then the space above the liquid will consist of evaporated vapor of the liquid at a pressure that depends on the temperature. If the temperature is raised, more liquid will vaporize, and the pressure will increase. A decrease in temperature will result in condensation of some of the vapor, and the pressure will decrease. Thus, vapor pressure depends on temperature. Different materials have different curves of pressure versus temperature, and there is no simple equation like that for a gas thermometer. Figure 4.15 shows a curve of vapor pressure versus temperature for methyl chloride, which is often employed in these sensors. The pressure available is substantial as the temperature rises. As in the case of gas thermometers, the range is not great, and response time is slow (20 s and more) because the liquid and the vessel must be heated.

EXAMPLE 4.16 Two methyl chloride vapor-pressure temperature sensors will be used to measure the temperature difference between two reaction vessels. The nominal temperature is 85°C. Find the pressure difference per degree Celsius at 85°C from the graph in Figure 4.15.

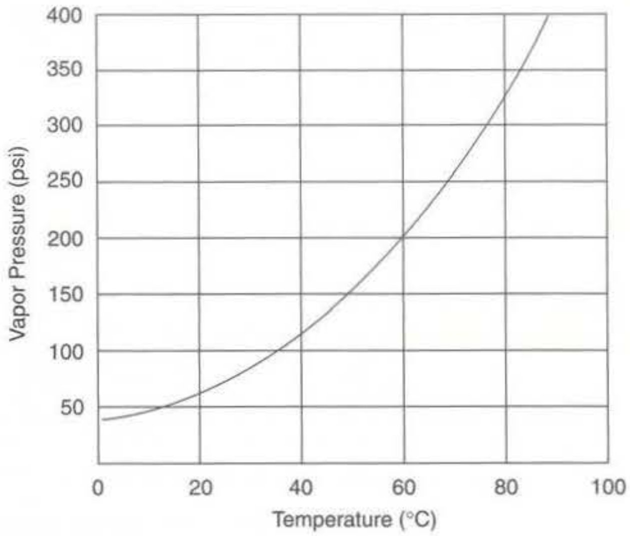


FIGURE 4.15
Vapor-pressure curve for methyl chloride.

Solution
We are just estimating the slope of the graph in the vicinity of 85°C. To do this, let us find the slope between 80°C and 90°C,

$$\frac{\Delta p}{\Delta T} = \frac{400 - 320 \text{ psi}}{90 - 80°\text{C}}$$

$$\frac{\Delta p}{\Delta T} = 8 \text{ psi}/°\text{C}$$

4.6.4 Liquid-Expansion Thermometers

Just as a solid experiences an expansion in dimension with temperature, a liquid also shows an expansion in volume with temperature. This effect forms the basis for the traditional liquid-in-glass thermometer that is so common in temperature measurement. The relationship that governs the operation of this device is

$$V(T) = V(T_0)[1 + \beta\Delta T] \tag{4.18}$$

where $V(T)$ = volume at temperature T
$V(T_0)$ = volume at temperature T_0
$\Delta T = T - T_0$
β = volume thermal expansion coefficient

In actual practice, the expansion effects of the glass container must be accounted for to obtain high accuracy in temperature indications. This type of temperature sensor is not commonly used in process-control work because further transduction is necessary to convert the indicated temperature into an electrical signal.

4.6.5 Solid-State Temperature Sensors

Many integrated circuit manufacturers now market solid-state temperature sensors for consumer and industrial applications. These devices offer voltages that vary linearly with temperature over a specified range. They function by exploiting the temperature sensitivity of doped semiconductor devices such as diodes and transistors. One common version is essentially a zener diode in which the zener voltage increases linearly with temperature.

The operating temperature of these sensors is typically in the range of −50° to 150°C. The time constant in good thermal contact varies in the range of 1 to 5 seconds, whereas in poor thermal contact it may increase to 60 seconds or more. The dissipation constant is in the range of 2 to 20 mW/°C depending on the case, conditions, and heat sinking.

One of the simplest forms of solid-state temperature sensor operates electrically like a zener diode, but the zener voltage depends upon temperature. Generally, the voltage depends upon the absolute temperature in a linear way.

Figure 4.16 shows such a temperature sensor connected to provide an output voltage that depends upon temperature. Let's assume a typical transfer function of about 12 mV/K, for example. Therefore, at a nominal room temperature of 21°C, the absolute temperature is about 293 K, so the sensor output voltage would be 3.516 V.

Generally, these devices have an accuracy of better than 1°C and often can be calibrated to provide even better accuracy. Self-heating must be considered, because they do dissipate power as given by the zener voltage times the operating current. For example, in Figure 4.16 you can see that the zener current is

$$\frac{(5 - 3.516)\ \text{V}}{510\ \Omega} = 0.0029\ \text{A}$$

So with a current of 2.9 mA, the power dissipated by the zener will be

$$(0.0029\ \text{A})(3.516\ \text{V}) = 10.2\ \text{mW}$$

In still air, the dissipation constant is typically about 5 mW/°C, so there could be a 2°C self-heating error. Clearly we try to operate the sensor at a low current to avoid such errors. For example, if we change the resistor to 2000 Ω, we can show the self-heating is reduced to about 0.5°C.

These sensors are easy to interface to control systems and computers, and are becoming popular for measurements within the somewhat limited range they offer. An important application is to provide automatic reference temperature compensation for

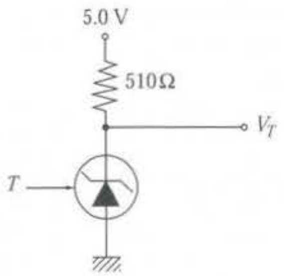


FIGURE 4.16
Some solid-state temperature sensors operate like a zener diode whose voltage depends upon temperature.

thermocouples. This is provided by connecting the sensor to the reference junction block of the TC and providing signal conditioning so that the reference corrections are automatically provided to the TC output. The following example illustrates a typical application.

EXAMPLE 4.17 A type J thermocouple is to be used in a measurement system that must provide an output of 2.00 V at 200°C. A solid-state temperature-sensor system will be used to provide reference temperature correction. The sensor has three terminals: supply voltage V_S, output voltage, V_T, and ground. The output voltage varies as 8 mV/°C.

Solution

A type J thermocouple with a 0°C reference will output 10.78 mV at 200°C. Therefore, the overall gain required will be

$$2.00 \text{ V}/0.01078 = 185.5$$

For compensation, the sensor will be physically connected to the reference block of the TC. The tables show that a type J thermocouple has a slope of approximately 50 μV/°C. Thus, the output of the sensor with temperature is $(8 \text{ mV/°C})/(50\ \mu\text{V/°C}) = 160$ times larger than the required correction. So we can provide the correction by amplifying the TC output by a gain of 160 and then adding the sensor reference correction. To make up the rest of the required gain of 185.5, we will need an amplifier with a gain of $185.5/160 = 1.159$. The output equation is given by

$$V_{\text{out}} = 1.159[160V_{TC} + V_C]$$

For the circuit we can use two differential amplifiers, one for the TC with a gain of 160 and the other to add the correction voltage and provide a gain of 1.159. We must be careful to get the polarities correct so that the correction factor is added. In Figure 4.17 the TC differential amplifier has an output of $-160V_{TC}$ since the iron is connected to the inverting side of the amplifier. This is fed to the inverting side of the second differential amplifier so the net effect provides a positive gain.

To see how well this works, we consider three cases: 50°C, 150°C, and 200°C. Suppose the actual reference temperature is 20°C. The following table shows (1) the expected output voltage if the reference was 0°C; (2) the compensation circuit output for a reference of 20°C as determined by the circuit of Figure 4.17 and the preceding equation; and (3) the percent difference.

T(°C)	V_{TC}(0°C)	V_{out}(0°C)	V_{out}(20°C)	Difference (%)
50	2.58 mV	0.479 V	0.475 V	−0.8
100	5.27 mV	0.976 V	0.974 V	−0.2
150	8.00 mV	1.484 V	1.480 V	−0.3
200	10.78 mV	1.999 V	1.995 V	−0.2

You can see that the compensated output differs from the output expected with an actual 0°C reference by less than 1%. This example illustrates the great success of using solid-state temperature sensors for reference correction when using thermocouples.

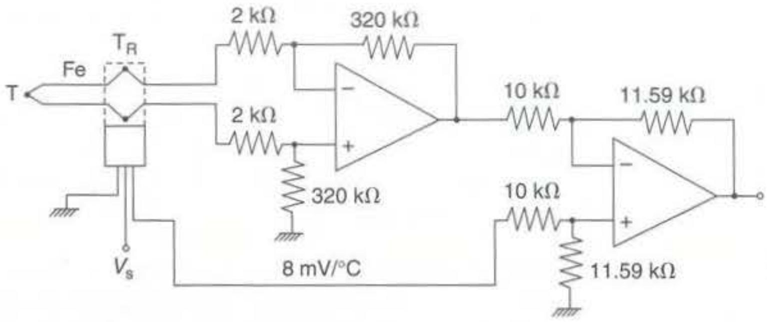


FIGURE 4.17
One possible solution to Example 4.17.

4.7 DESIGN CONSIDERATIONS

In the design of overall process-control systems, specific requirements are set up for each element of the system. The design of the elements themselves, which constitutes subsystems, involves careful matching of the elemental characteristics to the overall system design requirements. Even in the design of monitoring systems, where no integration of subsystems is required, it is necessary to match the transducer to the measurement environment and to the required output signal. In keeping with these considerations, we can treat temperature transducer design procedures by the following steps:

1. *Identify the nature of the measurement* This includes the nominal value and range of the temperature measurement, the physical conditions of the environment where the measurement is to be made, required speed of measurement, and any other features that must be considered.
2. *Identify the required output signal* In most applications, the output will be either a standard 4- to 20-mA current or a voltage that is scaled to represent the range of temperature in the measurement. There may be further requirements related to isolation, output impedance, or other factors. In some cases, a specific digital encoding of the output may be specified.
3. *Select an appropriate sensor* Based primarily on the results of the first step, a sensor that matches the specifications of range, environment, and so forth is selected. To some extent, factors such as cost and availability will be important in the selection of a sensor. The requirements of output signals also enter into this selection, but with lower significance because signal conditioning generally provides the required signal transformations.
4. *Design the required signal conditioning* Using the signal-conditioning techniques treated in Chapters 2 and 3, the direct transduction of temperature is converted into the required output signal. The specific type of signal conditioning depends, of course, on the type of sensor employed, as well as on the nature of the specified output signal characteristics.

In one form or another, these steps are required of any temperature sensor application, although they are not necessarily performed in the sequence indicated. The primary concern of this book is to enable the reader to do sensor selection and signal conditioning associated with a particular requirement. In a particular situation where a thermal sensor is required, there is no unique design to fit the application. There are, in fact, so many different designs possible that one must adjust his or her thinking from searching for *the* solution to searching for *a* solution to a design problem. A solution is any arrangement that satisfies *all* of the specified requirements of the problem. The following examples illustrate several problems in thermal design and typical solutions.

EXAMPLE 4.18 Develop a system that turns on an alarm LED when the temperature in a chamber reaches $10 \pm 0.5°C$. When the temperature drops below about 8°C, the LED should be turned off. For a temperature sensor use a thermistor which is 10 kΩ at 10°C and a slope at 10°C of −0.5%/°C. Its dissipation constant is 5 mW/°C.

Solution

This looks like a natural for a hysteresis comparator, as discussed in Chapter 3. What we need to do is develop a measurement to give a voltage that rises with temperature to trigger the comparator at 10°C and then allow the hysteresis to leave it on until the temperature falls to 8°C. We will work with three significant figures.

The thermistor nonlinearity will not matter, as we are interested in only two specific values of temperature/resistance. The thermistor has resistances of 10 kΩ at 10°C and 11 kΩ at 8°C. The ±0.5°C requirement means that the self-heating must be kept below 0.5°C; just to be sure, let us use 0.25°C. Because the dissipation constant for this thermistor was given as 5 mW/°C, we can determine the maximum allowable power dissipated by the transducer from Equation (4.13):

$$P = P_D \Delta T = (5\ \text{mW}/°\text{C})(0.25°\text{C})$$
$$P = 1.25\ \text{mW}$$

This represents a maximum current at 10°C of

$$I = [P/R]^{1/2} = [1.25\ \text{mW}/10\ \text{k}\Omega]^{1/2}$$
$$I = 0.354\ \text{mA}$$

Since this is a maximum, we will use $I = 0.35$ mA. We will convert the resistance variation to voltage variation using a divider, such as the one shown in Figure 2.4, and then feed the divider voltage to a hysteresis comparator. A common supply is +5 V, so using this for V_S, we will use the thermistor for R_1 (so V_D will increase with decreasing resistance, which is increasing temperature). R_2 will be determined by requiring the current to be 0.35 mA. Thus, the voltage dropped across the thermistor at 10°C is

$$V_{\text{TH}} = IR_1 = (0.35\ \text{mA})(10\ \text{k}\Omega) = 3.5\ \text{V}$$

Therefore, the value of R_2 is

$$R_2 = V/I = (5 - 3.5)/0.35\ \text{mA} = 4.28\ \text{k}\Omega$$

We will simply use 4.3 kΩ, since it is a common fixed-resistance value. We now have the two voltages of interest from the divider:

$$\text{For } 10°\text{C},\ V_D = 1.50\ \text{V}$$
$$\text{For } 8°\text{C},\ V_D = 1.41\ \text{V}$$

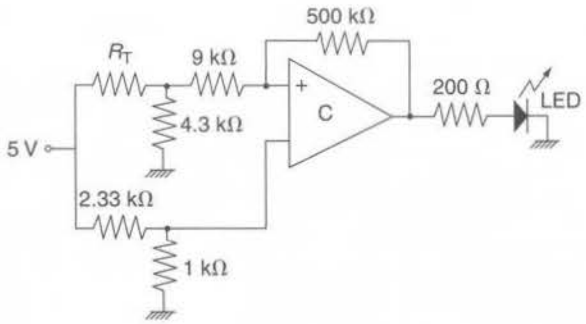


FIGURE 4.18
Circuit solution for Example 4.18. Trimmer resistors are used to obtain nonstandard resistance values.

Because the difference is 0.09 V, this must be the required hysteresis voltage. Assuming the comparator output is 5.0 V in the high state, the ratio of input to feedback resistance can be found from Equation (3.4), where $V_{ref} = 1.50$ V.

$$(R/R_f)(5.0 \text{ V}) = 0.09 \text{ V}$$
$$(R/R_f) = 0.018$$

We will pick $R_f = 500$ kΩ and then $R = 9$ kΩ. The final design is shown in Figure 4.18. The reference voltage has been achieved by a divider with a trimmer resistor. As usual, many other designs are possible.

EXAMPLE 4.19

Figure 4.19 shows an industrial process. Vapor flows through a chamber containing a liquid at 100°C. A control system will regulate the vapor temperature, so a measurement must be provided to convert 50° to 80°C into 0 to 2.0 V. The error should not exceed ± 1°C. If the liquid level rises to the tip of the transducer, its temperature will rise suddenly to 100°C. This event should cause an alarm comparator output to go high.

Solution

This is an example of a mid-range temperature measurement. Let us use an RTD, because the output over the 30° range will be substantially linear. Here are the specifications:

$$R \text{ at } 65°\text{C} = 150\ \Omega$$
$$\alpha \text{ at } 65°\text{C} = 0.004/°\text{C}$$
$$P_D = 30 \text{ mW}/°\text{C}$$

The three resistances of interest are at 50°C, 80°C, and 100°C. From the linear RTD relation for resistance [Equation (4.9)], we find

$$\text{At } 50°\text{C, RTD} = 150[1 + 0.004(50 - 65)] = 141\ \Omega$$
$$\text{At } 80°\text{C, RTD} = 150[1 + 0.004(80 - 65)] = 159\ \Omega$$
$$\text{At } 100°\text{C, RTD} = 150[1 + 0.004(100 - 65)] = 171\ \Omega$$

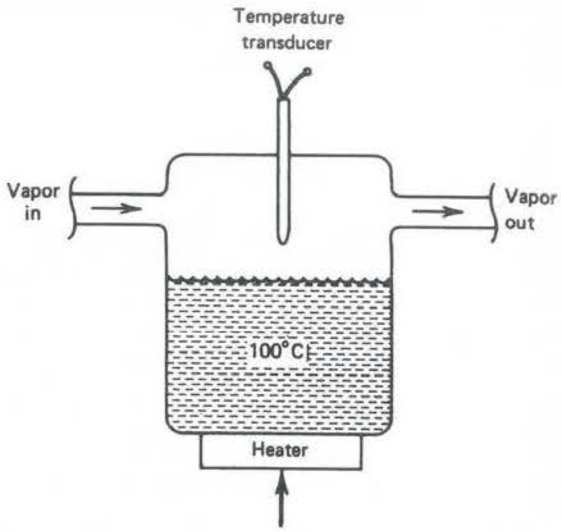


FIGURE 4.19
Vapor temperature-control process for Example 4.19.

For a 1°C error because of self-heating, we can now find the maximum current through the RTD. The maximum power is found from Equation (4.13):

$$P = P_D \Delta T = (30 \text{ mW/°C})(1\text{°C}) = 30 \text{ mW}$$

and then the maximum current from

$$I = [P/R]^{1/2} = [30 \text{ mW}/159\ \Omega]^{1/2}$$
$$I = 13.7 \text{ mA}$$

Although an op amp could be used, let us place the RTD in a bridge circuit and use the offset voltage for measurement. The small range of resistance will not cause any appreciable nonlinear effects, and the bridge can be nulled at 50°C, which will simplify the signal conditioning.

The bridge will be excited from a 5.0-V source, because this value is common. We will use the RTD as R_4 of Figure 2.4. The value of R_2 is determined by the requirement that the current be below 13.7 mA. The voltage across the RTD at 80°C will be

$$V = IR = (13.7 \text{ mA})(159\ \Omega) = 2.17 \text{ V}$$

Therefore, R_2 is found from

$$R_2 = (5 - 2.17)/13.7 \text{ mA} = 206.5\ \Omega$$

Let us use 220 Ω for R_2, because this is a standard value and will ensure that the current is low and the error condition is satisfied. To null the bridge at 50°C, we will make $R_1 = 220\ \Omega$ and use a trimmer to set R_3 to 141. The bridge is shown in Figure 4.20.

The bridge offset voltages will be found from Equation (2.8) for the endpoints and at 100°C.

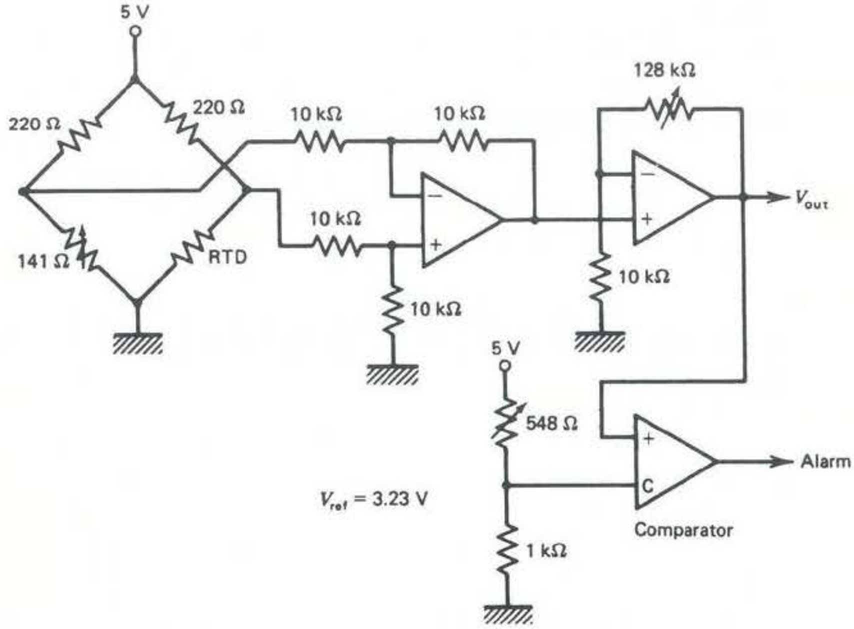


FIGURE 4.20
One possible solution for Example 4.19.

$$\text{At } 50°\text{C}, \Delta V = 5\frac{141}{220 + 141} - 5\frac{141}{220 + 141}$$
$$\Delta V = 0 \text{ (just as designed)}$$
$$\text{At } 80°\text{C}, \Delta V = 5\frac{159}{220 + 159} - 5\frac{141}{220 + 141}$$
$$\Delta V = 0.1447 \text{ V}$$
$$\text{At } 100°\text{C}, \Delta V = 5\frac{171}{220 + 171} - 5\frac{141}{220 + 141}$$
$$\Delta V = 0.2338 \text{ V}$$

All we need now is an amplifier to boost the 80°C voltage to 2.0 V. The required gain is (2/0.1447) = 13.8. Also, because the 5-V source used for the bridge is ground referenced, we must use a differential amplifier for the bridge offset voltage. Figure 4.20 shows the required amplifier with gain. The comparator reference voltage is $V_{ref} = 13.8(0.2338) = 3.23$ V.

EXAMPLE 4.20 Temperature for a plating operation must be measured for control within a range of 500° to 600°F. Develop a measuring system that scales this temperature into 0 to 5 V for input to an 8-bit ADC and a computer control system; measurement must be to within ±1°F.

Solution

The given temperature range corresponds to 260° to 315.6°C. For this high a temperature, we will use a type J thermocouple, although a platinum RTD could also be used.

The reference is assumed to be 25 ± 0.5°C to satisfy the required measurement accuracy. This can be supplied by a commercial reference, by a correction circuit, or by measuring the reference for input to the computer and software adjustment.

With a 25°C reference, the type J tables and interpolation can be used to find the voltages of the thermocouple as

$$\text{For } 260°\text{C}, V_{J25} = 12.84 \text{ mV}$$
$$\text{For } 315.6°\text{C}, V_{J25} = 15.90 \text{ mV}$$

Signal conditioning can be developed by first finding an equation for the final output from this input.

We have

$$V_{ADC} = mV_{J25} + V_0$$

using the two conditions

$$0 = m(0.01284) + V_0$$
$$5 = m(0.01590) + V_0$$

we find the values $m = 1634$ and $V_0 = -21$.

This gain is too high for a single amplifier; anyway, we need a differential amplifier on the front end for the thermocouple. Let us use a differential amplifier with a gain of 100 to produce an intermediate voltage, $V_1 = 100V_{J25}$. Then the equation becomes

$$V_{ADC} = 16.34V_1 - 21$$

or

$$V_{ADC} = 16.34(V_1 - 1.29)$$

This is simply a differential amplifier. We must be very careful to get the polarities correct. In this instance we chose to hook up the thermocouple to the first differential amplifier so that its output was negative (i.e., $-100\ V_{TC}$). When hooking it up to the second differential amplifier, we connect it to the inverting side and then −1.29 V from a divider to the noninverting side. The net result then is the required equation. The circuit is shown in Figure

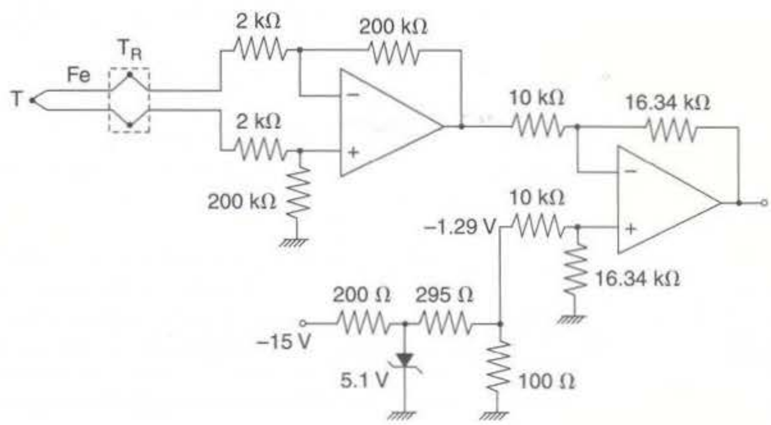


FIGURE 4.21
One possible solution for Example 4.20.

4.21. You should think about how this design could be improved in terms of reference correction. If the reference changed to 20°C, for example, the circuit output would no longer satisfy the specification.

SUMMARY

1. The measurement and control of temperature plays an important role in the process-control industry. The class of sensors that performs this measurement consists primarily of three types: (1) the resistance-temperature detector (RTD), (2) the thermistor, and (3) the thermocouple. In this chapter, the basic operating principles and application information have been provided for these sensors. Several other sensors have been briefly described.
2. The concept of temperature is contained in the representation of a body's thermal energy as the average thermal energy per molecule, expressed in units of degrees of temperature. Four units of temperature are in common use. Two of these scales are called *absolute*, because an indication of zero units corresponds to zero thermal energy. These scales, which are designated by kelvin (K) and degrees Rankine (°R), differ only in the amount of energy represented by each unit. The amount of energy represented by 1 K corresponds to 9/5°R. Thus, we can transform temperatures using

$$T(K) = \frac{5}{9}T(^\circ\text{R}) \quad \textbf{(4.1)}$$

3. The other two scales are called *relative*, because their zero does not occur at a zero of thermal energy. The Celsius scale (°C) corresponds in degree size to the kelvin, but has a shift of the zero so that

$$T(^\circ\text{C}) = T(K) - 273.15 \quad \textbf{(4.2)}$$

4. In a similar fashion, the Fahrenheit (°F) and Rankine scales are related by

$$T(^\circ\text{F}) = T(^\circ\text{R}) - 459.6 \quad \textbf{(4.3)}$$

5. The RTD is a sensor that depends on the increase in metallic resistance with temperature. This increase is very nearly linear, and analytical approximations are used to express the resistance versus temperature as either a linear equation,

$$R(T) = R(T_0)[1 + \alpha_0 \Delta T] \quad \textbf{(4.9)}$$

or a quadratic relationship,

$$R(T) = R(T_0)[1 + \alpha_1 \Delta T + \alpha_2(\Delta T)^2] \quad \textbf{(4.12)}$$

6. When a greater degree of accuracy is desired, tables or graphs of resistance versus temperature are used. Because of the small fractional change in resistance with temperature, the RTD is usually used in a bridge circuit with a high-gain null detector.
7. The thermistor is based on the decrease of semiconductor resistance with temperature. This device has a highly nonlinear resistance versus temperature curve and is not typically used with any analytical approximations. Such transducers can exhibit a very large change in resistance with temperature, and hence make very sensitive

temperature-change detectors. Many circuit configurations are used, including bridges and operational amplifiers.

8. A thermocouple is a junction of dissimilar metal wires, usually joined to a third metal wire through two reference junctions. A voltage is developed across the common metal wires that is proportional, almost linearly, to the difference in temperature between the measurement and reference junctions. Extensive tables of temperature versus voltage for numerous types of TCs using standard metals and alloys allow an accurate determination of temperature at the reference junctions. The voltage must be measured at high impedance to avoid loading effects on the measured voltage. Thus, potentiometric, operational amplifiers, or other high-impedance techniques are employed in signal conditioning.
9. A bimetal strip converts temperature into a physical motion of metal elements. This flexing can be used to close switches or cause dial indications.
10. Gas and vapor-pressure temperature sensors convert temperature into gas pressure, which then is converted to an electrical signal or is used directly in pneumatic systems.

PROBLEMS

Section 4.1

4.1 Convert 453.1°R into K, °F, and °C.

4.2 Convert −222°F into °C, °R, and K.

4.3 Convert 150°C into K and °F.

4.4 A process temperature is found to change by 33.4°F. Calculate the change in °C. *Hint:* A change in temperature does not involve a scale shift.

4.5 A sample of hydrogen gas has a temperature of 500°C. Calculate the average molecular speed in m/s. Express this also in ft/s. *Note:* Gaseous hydrogen exists as the molecule H_2 with a mass of 3.3×10^{-27} kg.

4.6 Temperature is to be controlled in the range 350° to 550°C. What is this expressed in °F?

Section 4.2

4.7 An RTD has $\alpha(20°\text{C}) = 0.004/°\text{C}$. If $R = 106\ \Omega$ at 20°C, find the resistance at 25°C.

4.8 The RTD of Problem 4.7 is used in the bridge circuit of Figure 4.4. If $R_1 = R_2 = R_3 = 100\ \Omega$ and the supply voltage is 10.0 V, calculate the voltage the detector must be able to resolve in order to resolve a 1.0°C change in temperature.

4.9 Use the values of RTD resistance versus temperature shown in the table to find the equations for the linear and quadratic approximations of resistance between 100°C and 130°C. Assume $T_0 = 115°\text{C}$. What is the error in percent between table resistance values and those determined from the two approximations?

$T(°C)$	$R(\Omega)$
90.0	562.66
95.0	568.03
100.0	573.40
105.0	578.77
110.0	584.13
115.0	589.48
120.0	594.84
125.0	600.18
130.0	605.52

4.10 Suppose the RTD of Problem 4.7 has a dissipation constant of 25 mW/°C and is used in a circuit that puts 8 mA through the sensor. If the RTD is placed in a bath at 100°C, what resistance will the RTD have? What then is the indicated temperature?

Section 4.4

4.11 In Problem 4.8, the RTD is replaced by a thermistor with $R(20°C) = 100\ \Omega$ and an R versus T of $-10\%/°C$ near 20°C. Calculate the voltage resolution of the detector needed to resolve a 1.0°C change in temperature.

4.12 Modify the divider of Example 4.8 so that self-heating is reduced to 0.1°C. What is the divider voltage for 20°C? What is the divider voltage for 19°C and 21°C?

4.13 The thermistor of Figure 4.5 is used as shown in Figure 4.22 to convert temperature into voltage. Plot V_{out} versus temperature from 0° to 80°C. Is the result linear? What is the maximum self-heating if $P_D = 5$ mW/°C?

Section 4.5

4.14 A type J thermocouple measures 22.5 mV with a 0°C reference. What is the junction temperature?

4.15 A type S thermocouple with a 21°C reference measures 12.120 mV. What is the junction temperature?

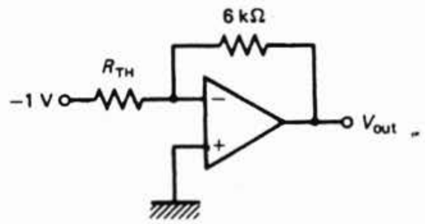
4.16 If a type J thermocouple is to measure 500°C with a −10°C reference, what voltage will be produced?

4.17 A type K thermocouple with a 0°C reference will monitor an oven temperature at about 300°C.

a. What voltage would be expected?

b. Extension wires of 1000-ft length and 0.01 Ω/ft will be used to connect to the measurement site. Determine the minimum voltage measurement input impedance if the error is to be within 0.2%.

FIGURE 4.22
Circuit for Problem 4.13.



4.18 We need to get 1.5 V from a candle flame at about 700°C. How many type K thermocouples will be required in series if the reference is assumed to be nominal room temperature, 70°F?

Section 4.6

4.19 A thermoelectric switch composed of a copper rod 10 cm long at 20°C is to touch an electrical contact at a temperature of 150°C. What distance should there be between the rod end and the contact at 20°C?

4.20 A gas thermometer has a pressure of 125 kPa at 0°C and 215 kPa at some unknown temperature. Determine the temperature in °C.

4.21 A methyl chloride vapor-pressure thermometer will be used between 70°F and 200°F. What pressure range corresponds to this temperature range?

4.22 Find a linear approximation of methyl chloride pressure versus temperature from 70° to 90°C. Find the maximum error between the approximation and actual pressure in this range.

Section 4.7

4.23 Using an RTD with $\alpha = 0.0034/°C$ and $R = 100\ \Omega$ at 20°C, design a bridge and op amp system to provide a 0.0- to 10.0-V output for a 20° to 100°C temperature variation. The RTD dissipation constant is 28 mW/°C. Maximum self-heating should be 0.05°C.

4.24 A type K thermocouple with a 0°C reference will be used to measure temperature between 200°C and 350°C. Devise a system that will convert this temperature range into an 8-bit digital word with conversion from 00H to 01H at 200°C and the change from FEH to FFH occurring at 350°C. An ADC is available with a 2.500-V internal reference.

4.25 Solve Example 4.19 using a type K thermocouple with a reference compensated to 0°C.

4.26 Solve Example 4.20 using an RTD with $R = 500\ \Omega$ and $\alpha = 0.003/°C$ at 260°C. The dissipation constant is 25 mW/°C.

4.27 You have been commissioned to design a thermistor-based digital temperature measurement system. The ADC has a 5.00-V reference and is 8 bits. The thermistor specifications are $R = 5.00\ k\Omega$ at 90°F, $P_D = 5$ mW/°C, and a slope between 90°F and 110°F of $-8\ \Omega/°C$. The design should be made so that 90°F gives an ADC output of 5AH (90_{10}) and 110°F gives 6EH(110_{10}).

4.28 Solve Example 4.19 using the RTD as the feedback element of an inverting amplifier instead of using a bridge circuit.

4.29 A type K thermocouple measurement system must provide an output of 0 to 2.5 V for a temperature variation of 500° to 700°C. A three-terminal solid-state sensor with 12 mV/°C will be used to provide reference compensation. Develop the complete circuit.

4.30 A calibrated RTD with $\alpha = 0.0041/°C$, $R = 306.5\ \Omega$ at 20°C, and $P_D =$ 30 mW/°C will be used to measure a critical reaction temperature. Temperature must be measured between 50° and 100°C with a resolution of at least 0.1°C. Devise a signal-conditioning system that will provide an appropriate digital output to a computer. Specify the requirements on the ADC and appropriate analog signal conditioning to interface to your ADC.

SUPPLEMENTARY PROBLEMS

S4.1 Figure 4.23 shows a system that is proposed to power a radio from the heat of a portable lantern. There are eight "wings" on a shield around the lantern flame. Each wing has type J thermocouples facing the flame, with the cold junctions on the outer edge of the wings. A radio in the base requires a minimum of 5.0 V at 50 mA to operate. The following assumptions are made: (1) Each thermocouple segment has a resistance of 0.05 Ω, and (2) the flame heat at the thermocouple junction is at least 300°C and nominal room temperature is 25°C.

a. Determine how many thermocouples are required and how they are distributed on the wings. Be sure to consider the internal resistance of the thermocouples.

b. What is the maximum power that can be delivered into any load at 300°C and at 400°C?

c. On a cold winter night, the ambient temperature can drop to 10°C. If the hot junctions are still at 300°C, what effect does this have on the voltage applied to the radio?

S4.2 An RTD with $\alpha = 0.0035/°C$ and $R = 300\ \Omega$ at 25°C will be used to measure the temperature of hot gas flowing in a pipe. The dissipation constant is 25 mW/°C, and the time constant is 5.5 s. Normal gas temperature is in the range of 100° to 220°C.

a. Design a system by which the temperature variation is converted into a voltage of −2.0 to +2.0 V. Keep self-heating to 0.5°C.

b. Occasionally a turbulent shock wave will propagate down the pipe, causing a sudden reduction in temperature to less than 50°C. Devise a comparator alarm that will signal such an event within 3 s of dropping below 100°C.

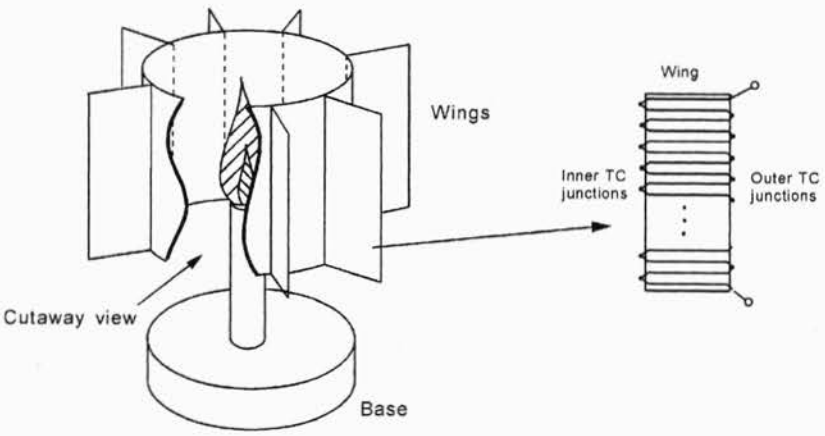


FIGURE 4.23
Electrical power from a lantern flame using thermocouples. See Problem S4.1.

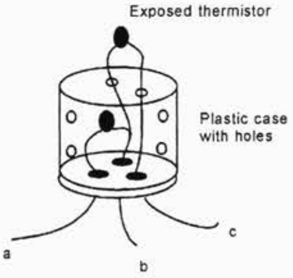


FIGURE 4.24
Use of thermistors for measurement of wind speed in Problem S4.3.

S4.3 Air-flow speed can be measured by comparing the temperatures of a shielded thermistor and one which is exposed to the moving air. A diagram of this sensor is shown in Figure 4.24. The thermistors are used in a bridge and both are operated at an elevated (no-wind) temperature of 45°C using self-heating. When wind blows, the exposed sensor will cool off and the other will not.

It has been experimentally found that the temperature difference between the thermistors are as shown in the following table for winds of 0 to 60 mph. Design a bridge and op amp system that will provide 0.0 V when the wind is 0 mph and 6.0 V when the wind is 60 mph so that voltage indicates speed directly. Conditions: (1) The thermistors have a resistance of 2.0 kΩ at 45°C with a slope of −24 Ω/°C within ±10°C, and the dissipation constant is 5.5 mW/°C, (2) use a 20-V supply voltage for the bridge, and (3) assume ambient temperature is 21°C.

Speed (mph)	ΔT(°C)	ΔR(Ω)	Bridge ΔV(V)	V_{out}(V)
0	0			0.0
10	−3.0			
20	−4.5			
30	−5.5			
40	−6.3			
50	−7.1			
60	−7.7			6.0

a. Complete the table from your design results.

b. Prepare a graph of output voltage versus wind speed.

c. What is the greatest error between voltage-indicated wind speed and actual speed? Why is there error?