

APPENDIX C

Answers to Selected Exercises

CHAPTER 1

1.1 (a) $\{x : x = 0, 1, 2, 3, 4\};$
 $\{x : x = 2\}.$

(b) $\{x : 0 < x < 3\};$
 $\{x : 1 \leq x < 2\}.$

1.2 (a) $\{x : 0 < x < \frac{5}{8}\}.$

1.7 (a) $\{x : 0 < x < 3\}.$

(b) $\{(x, y) : 0 < x^2 + y^2 < 4\}.$

1.8 (a) $\{x : x = 2\}.$

(b) Null set.

(c) $\{(x, y) : x^2 + y^2 = 0\}.$

1.9 $\frac{80}{81}; 1.$

1.10 $\frac{11}{16}; 0; 1.$

1.11 $\frac{8}{3}; 0; \pi/2.$

1.12 $\frac{1}{2}; 0; \frac{2}{9}.$

1.13 $\frac{1}{6}; 0.$

1.15 10.

1.18 $\frac{1}{4}, \frac{1}{13}, \frac{1}{52}, \frac{4}{13}.$

1.19 $\frac{31}{32}, \frac{3}{64}, \frac{1}{32}, \frac{63}{64}.$

1.20 0.3.

1.21 $e^{-4}; 1 - e^{-4}; 1.$

1.22 $\frac{1}{2}.$

1.26 (a) $\binom{6}{4} / \binom{16}{4}.$

(b) $\binom{10}{4} / \binom{16}{4}.$

1.27 $1 - \binom{990}{5} / \binom{1000}{5}.$

1.29 (b) $1 - \binom{10}{3} / \binom{20}{3}.$

1.34 (a) $\frac{1}{7}.$ (b) $\frac{5}{56}.$

(c) $\left[\binom{3}{x} / \binom{8}{x} \right] [5/(8-x)].$

1.37 $\frac{1}{3}.$

1.38 $\frac{9}{20}; \frac{2}{3}.$

1.39 $\frac{5}{14}.$

1.40 $\frac{3}{7}, \frac{4}{7}.$

1.42 (a) 0.18. (b) 0.72.

(c) 0.88.

1.45 0.1029 for (a), (b), (c), (d).
(e) 0.4116.

1.46 $\frac{1}{4}, \frac{3}{4}$.

1.47 $\frac{9}{13}, \frac{1}{13}, \frac{1}{13}, \frac{1}{13}, \frac{1}{13}$.

1.48 (a) $\frac{1}{2}$. (b) $\frac{1}{21}$.

1.49 $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$.

$$\mathbf{1.51} \quad (\text{a}) \frac{\binom{13}{x} \binom{39}{5-x}}{\binom{52}{5}},$$

$x=0, 1, \dots, 5$.

$$(\text{b}) \frac{\binom{39}{5} + \binom{13}{1} \binom{39}{4}}{\binom{52}{5}}.$$

1.54 (a) $\frac{1}{10}$, $x=1, 2, \dots, 9$.
(b) $\frac{4}{10}$.

1.56 $\frac{6}{36}$, $x=0$;
 $\frac{12-2x}{36}$, $x=1, 2, 3, 4, 5$.

1.59 $\frac{3}{4}$.

1.61 $\frac{5}{8}, \frac{7}{8}, \frac{3}{8}$.

1.63 $e^{-2} - e^{-3}$.

1.64 $\frac{1}{27}, 1, \frac{2}{9}, \frac{25}{36}$.

1.66 (a) 1. (b) $\frac{2}{3}$. (c) 2.

1.69 (a) 0, $x < 0$; $1 - (1-x)^3$,
 $0 \leq x < 1$; $1, 1 \leq x$;
 $1 - \sqrt[3]{\frac{3}{4}}, 1 - \sqrt[3]{\frac{1}{2}}$.

1.71 (a) $\frac{1}{4}$. (b) 0. (c) $\frac{1}{4}$. (d) 0.

1.72 0, $y < 0$; y^2 , $0 \leq y < 1$; 1,
 $1 \leq y$, $2y$, $0 < y < 1$;
0 elsewhere.

1.74 $\frac{1}{2}, \frac{1}{4}$.

1.76 0, $x < 0$; $1 - e^{-x/2}$, $0 \leq x$.
 $\frac{1}{2} e^{-x/2}$, $0 < x$; 0 elsewhere.

1.79 $1/3\sqrt{y}$, $0 < y < 1$; $1/6\sqrt{y}$,
 $1 < y < 4$; 0 elsewhere.

1.80 2; 86.4; -160.8.

1.81 3; 11; 27.

1.83 (a) $\frac{3}{4}$.

(b) $\frac{1}{4}, \frac{1}{2}$.

1.85 \$7.80.

1.88 $\frac{7}{3}$.

1.89 (a) 1.5, 0.75. (b) 0.5, 0.05.

(c) 2; does not exist.

1.90 $e^t/(2-e^t)$, $t < \ln 2$; 2; 2.

1.99 10; 0; 2; -30.

1.101 $-\frac{2\sqrt{2}}{5}, \frac{2\sqrt{2}}{5}$.

1.103 $1/2p; \frac{3}{2}; \frac{5}{2}; 5; 50$.

1.105 $\frac{31}{12}, \frac{167}{144}$.

1.110 $\frac{5}{8}, \frac{37}{192}$.

1.114 0.84.

CHAPTER 2

2.1 $\frac{15}{64}; 0; \frac{1}{2}; \frac{1}{2}$.

2.2 $\frac{1}{4}$.

2.6 ze^{-z} , $0 < z < \infty$;
0 elsewhere.

2.7 $-\ln z$, $0 < z < 1$;
0 elsewhere.

2.10 $5x_2^4$, $0 < x_2 < 1$;
0 elsewhere.

2.11 $(3x_1 + 2)/(6x_1 + 3)$;
 $(6x_1^2 + 6x_1 + 1)/(2)(6x_1 + 3)^2$.

2.13 $3x_2/4$; $3x_2^2/80$.

2.18 (b) $1/e$.

2.20 (a) 1. (b) -1. (c) 0.

2.21 (a) $7/\sqrt{804}$.

2.31 $\frac{5}{81}$.

2.32 $\frac{7}{8}$.

2.36 $\frac{1}{2}$.

2.38 (a) $\frac{1}{8}, 0$.

2.39 $1 - (1-y)^{12}$, $0 \leq y < 1$;
 $12(1-y)^{11}$, $0 < y < 1$.

2.40 $g(y) = [y^3 - (y-1)^3]/6^3$,
 $y = 1, 2, 3, 4, 5, 6$.

2.42 $b_2 = \sigma_1(\rho_{12} - \rho_{13}\rho_{23})/$
 $[\sigma_2(1 - \rho_{23}^2)]$;
 $b_3 = \sigma_1(\rho_{13} - \rho_{12}\rho_{23})/$
 $[\sigma_3(1 - \rho_{23}^2)]$.

CHAPTER 3

- 3.1 $\frac{40}{81}$.
 3.4 $\frac{147}{512}$.
 3.6 5.
 3.8 $\frac{3}{16}$.
 3.10 $\frac{65}{81}$.
 3.13 $(\frac{1}{3})(\frac{2}{3})^{x-3}$, $x = 3, 4, 5, \dots$.
 3.14 $\frac{5}{72}$.
 3.17 $\frac{1}{6}$.
 3.18 $\frac{24}{625}$.
 3.20 $\frac{11}{6}$; $x_1/2$; $\frac{11}{6}$.
 3.21 $\frac{25}{4}$.
 3.22 0.09.
 3.25 $4^x e^{-4}/x!$, $x = 0, 1, 2, \dots$.
 3.26 0.84.
 3.31 2.
 3.33 (a) $\exp[-2 + e^{1/2}(1 + e^{1/2})]$.
 (b) $\mu_1 = 1$, $\mu_2 = 2$,
 $\sigma_1^2 = 1$, $\sigma_2^2 = 2$,
 $\rho = \sqrt{2}/2$.
 (c) $y/2$.
 3.34 0.05.
 3.35 0.831, 12.8.
 3.36 0.90.
 3.37 $\chi^2(4)$.
 3.39 $3e^{-3y}$, $0 < y < \infty$.
 3.40 2, 0.95.
 3.45 $\frac{11}{16}$.
 3.46 $\chi^2(2)$.
 3.49 0.067; 0.685.
 3.51 71.3, 189.7.
 3.52 $\sqrt{\ln 2/\pi}$.
 3.57 0.774.
 3.58 $\sqrt{2/\pi}$; $(\pi - 2)/\pi$.
 3.59 0.90.
 3.60 0.477.
 3.61 0.461.
 3.62 $N(0, 1)$.
 3.63 0.433.
 3.64 0; 3.
 3.69 $N(0, 2)$.
 3.70 (a) 0.574.
 (b) 0.735.

- 3.71 (a) 0.264. (b) 0.440.
 (c) 0.433. (d) 0.642.

3.73 $\rho = \frac{4}{5}$.

3.74 (38.2, 43.4).

CHAPTER 4

- 4.2 $\frac{405}{1024}$.
 4.3 0.405.
 4.6 $\frac{16}{15}$.
 4.7 $\frac{1}{8}$.
 4.9 $(n+1)/2$; $(n^2-1)/12$.
 4.10 $a + b\bar{x}$; $b^2 s_x^2$.
 4.11 $\chi^2(2)$.
 4.14 $\frac{1}{2}$, $0 < y < 1$;
 $1/2y^2$, $1 < y < \infty$.
 4.15 y^{15} , $0 \leq y < 1$; $15y^{14}$,
 $0 < y < 1$.
 4.16 $\frac{4}{7}$.
 4.17 $\frac{1}{3}$, $y = 3, 5, 7$.
 4.19 $(\frac{1}{2})^{\sqrt{y}}$, $y = 1, 8, 27, \dots$.
 4.20
- | y_1 | $g_1(y_1)$ |
|-------|-----------------|
| 1 | $\frac{1}{36}$ |
| 2 | $\frac{4}{36}$ |
| 3 | $\frac{6}{36}$ |
| 4 | $\frac{4}{36}$ |
| 6 | $\frac{12}{36}$ |
| 9 | $\frac{9}{36}$ |
- 4.25 $\frac{1}{27}$, $0 < y < 27$.
 4.32 $y_1 e^{-y_1}$, $0 < y_1 < \infty$.
 4.34 $(2y_1)(4y_2^3)$, $0 < y_1 < 1$,
 $0 < y_2 < 1$.
 4.35 $\alpha/(\alpha + \beta)$;
 $\alpha\beta/[(\alpha + \beta + 1)(\alpha + \beta)^2]$.
 4.36 (a) 20. (b) 1260. (c) 495.
 4.37 $\frac{10}{243}$.
 4.40 0.05.
 4.43 1/4.74, 3.33.
 4.48 $(1/\sqrt{2\pi})^3 y_1^2 e^{-y_1^2/2} \sin y_3$,
 $0 \leq y_1 < \infty$, $0 \leq y_2 < 2\pi$,
 $0 \leq y_3 \leq \pi$.

4.49 $y_2 y_3^2 e^{-y_3}$, $0 < y_1 < 1$,
 $0 < y_2 < 1$, $0 < y_3 < \infty$.

4.53 $1/(2\sqrt{y})$, $0 < y < 1$.

4.54 $e^{-y_1/2}/(2\pi\sqrt{y_1 - y_2^2})$,
 $-\sqrt{y_1} < y_2 < \sqrt{y_1}$,
 $0 < y_1 < \infty$.

4.56 $1 - (1 - e^{-3})^4$.

4.57 $\frac{1}{8}$.

4.62 $\frac{5}{16}$.

4.63 $48z_1 z_2^3 z_3^5$, $0 < z_1 < 1$,
 $0 < z_2 < 1$, $0 < z_3 < 1$.

4.64 $\frac{7}{12}$.

4.69 $\frac{1}{4}$.

4.70 $6uv(u+v)$,
 $0 < u < v < 1$.

4.75

y	$g(y)$
2	$\frac{1}{36}$
3	$\frac{2}{36}$
4	$\frac{3}{36}$
5	$\frac{4}{36}$
6	$\frac{5}{36}$
7	$\frac{6}{36}$
8	$\frac{5}{36}$
9	$\frac{4}{36}$
10	$\frac{3}{36}$
11	$\frac{2}{36}$
12	$\frac{1}{36}$

4.76 0.24.

4.79 0.159.

4.82 0.159.

4.88 0.818.

4.91 (b) -1 or 1 .

(c) $Z_i = \sigma_i Y_i + \mu_i$.

4.92 $\sum_{i=1}^n a_i b_i = 0$.

4.94 6.41.

4.95 $n = 16$.

4.97 $(n-1)\sigma^2/n$;
 $2(n-1)\sigma^4/n^2$.

4.98 0.90.

4.100 0.945.

4.102 0.618.

4.103 0.78.

4.104 $\frac{8}{3}$; $\frac{2}{9}$.

4.105 7.

4.107 2.5; 0.25.

4.109 -5 ; $60 - 12\sqrt{6}$.

4.110 $\sigma_1/\sqrt{\sigma_1^2 + \sigma_2^2}$.

4.113 0.265.

4.115 22.5, $\frac{261}{4}$.

4.116 $r_2 > 4$.

4.118 $\mu_2 \sigma_1 / \sqrt{\sigma_1^2 \sigma_2^2 + \mu_1^2 \sigma_2^2 + \mu_2^2 \sigma_1^2}$.

4.121 $5/\sqrt{39}$.

4.125 $e^{\mu + \sigma^2/2}$; $e^{2\mu + \sigma^2}(e^{\sigma^2} - 1)$.

CHAPTER 5

5.1 Degenerate at μ .

5.2 Gamma ($\alpha = 1$, $\beta = 1$).

5.3 Gamma ($\alpha = 1$, $\beta = 1$).

5.4 Gamma ($\alpha = 2$; $\beta = 1$).

5.13 0.682.

5.14 (b) 0.815.

5.17 Degenerate at μ_2
 $+(\sigma_2/\sigma_1)(x - \mu_1)$.

5.18 (b) $N(0, 1)$.

5.19 (b) $N(0, 1)$.

5.21 0.954.

5.23 0.840.

5.26 0.08.

5.28 0.267.

5.29 0.682.

5.35 $N(0, 1)$.

CHAPTER 6

6.1 (a) $\bar{X}$.

(b) $-n/\ln(X_1 X_2 \cdots X_n)$.

(c) $\bar{X}$. (d) The median.

(e) The first order statistic.

6.2 The first order statistic Y_1 ,
 $\sum_{i=1}^n (X_i - Y_1)/n$.

- 6.4 $\frac{4}{23}, \frac{11}{23}, \frac{7}{23}$.
 6.5 $Y_1 = \min(X_i)$;
 $n/\ln[(X_1 X_2 \cdots X_n)/Y_1^n]$.
 6.7 (b) $\bar{X}/(1 - \bar{X})$. (d) $\bar{X}$.
 (e) $\bar{X} - 1$.
 6.9 $1 - e^{-2/\bar{X}}$.
 6.10 Multiply by $n/(n-1)$.
 6.12 $(Y_1 + Y_n)/2$, $(Y_n - Y_1)/2$;
 $E[(Y_n - Y_1)/2] = \rho(n-1)/(n+1)$.
 6.14 (77.28, 85.12).
 6.15 24 or 25.
 6.16 (3.7, 5.7).
 6.17 160.
 6.23 $(5\bar{x}/6, 5\bar{x}/4)$.
 6.25 1692.
 6.26 3.19 to 3.61.
 6.28 3.92 to 31.50.
 6.30 $(-3.6, 2.0)$.
 6.35 135 or 136.
 6.38 $\frac{1}{4} + \frac{3}{4} \ln \frac{3}{4}$; $\frac{7}{16} + \frac{9}{8} \ln \frac{3}{4}$.
 6.39 $\frac{11}{64}$; $(31)3^8/4^9$.
 6.42 $n = 19$ or 20 .
 6.43 $K(\frac{1}{2}) = 0.062$;
 $K(\frac{1}{12}) = 0.920$.
 6.44 $n \approx 73$, $c \approx 42$.
 6.46 (a) Reject.
 (b) p -value ≈ 0.005 .
 6.49 (c) p -value ≈ 0.005 .
 6.51 23.3.
 6.52 2.91.
 6.53 $q_3 = \frac{176}{21} > 7.81$,
 reject H_0 .
 6.55 $b \leq 8$ or $32 \leq b$.
 6.56 $q_3 = \frac{22}{9} < 11.3$,
 accept H_0 .
 6.57 $6.4 < 9.49$, accept H_0 .
 6.59 $\hat{p} = (X_1 + X_2/2)/$
 $(X_1 + X_2 + X_3)$.

CHAPTER 7

- 7.4 $\frac{1}{3}, \frac{2}{3}$.
 7.5 $\delta_1(y)$.

- 7.6 $b = 0$; does not exist.
 7.7 Does not exist.
 7.17 $\prod_{i=1}^n [X_i(1 - X_i)]$.
 7.19 $60y_3^2(y_3 - y_3)/\theta^5$; $6y_3/5$;
 $\theta^2/7$; $\theta^2/35$.
 7.20 $(1/\theta^2)e^{-y_1/\theta}$,
 $0 < y_2 < y_1 < \infty$;
 $y_1/2$; $\theta^2/2$.
 7.22 $\sum X_i^2/n$; $\sum X_i/n$; $(n+1)Y_n/n$.
 7.24 X ; X .
 7.25 Y_1/n .
 7.27 $Y_1 - 1/n$.
 7.29 $Y_1 = \sum_{i=1}^n X_i$; $Y_1/4n$; yes.
 7.37 $\bar{x}$.
 7.40 $\bar{X}^2 - 1/n$.
 7.43 $\left(\frac{n-1}{n}\right)^Y \left(1 + \frac{Y}{n-1}\right)$.
 7.51 $\frac{Y_1 + Y_n}{2}$, $\frac{(n+1)(Y_n - Y_1)}{2(n-1)}$.
 7.55 Y_1 , $\sum (Y_i - Y_1)/n$.

CHAPTER 8

- 8.2 $[y\tau^2 + \mu\sigma^2/n]/(\tau^2 + \sigma^2/n)$.
 8.3 $\beta(y + \alpha)/(n\beta + 1)$.
 8.9 $\sqrt[6]{2}$ if $y_4 < 1$,
 $\sqrt[6]{2} y_4$ if $1 \leq y_4$.
 8.13 θ^2/n ; $\theta^2/n(n+2)$.
 8.15 (a) $4/\theta^2$.
 8.17 (d) $\text{var}(\hat{\theta}) = \frac{1}{nI(\theta)} = \frac{\theta^2}{5n}$.
 8.22 2.17; 2.44.
 8.25 2.20.

CHAPTER 9

- 9.4 $\sum_{i=1}^{10} x_i^2 \geq 18.3$; yes; yes.

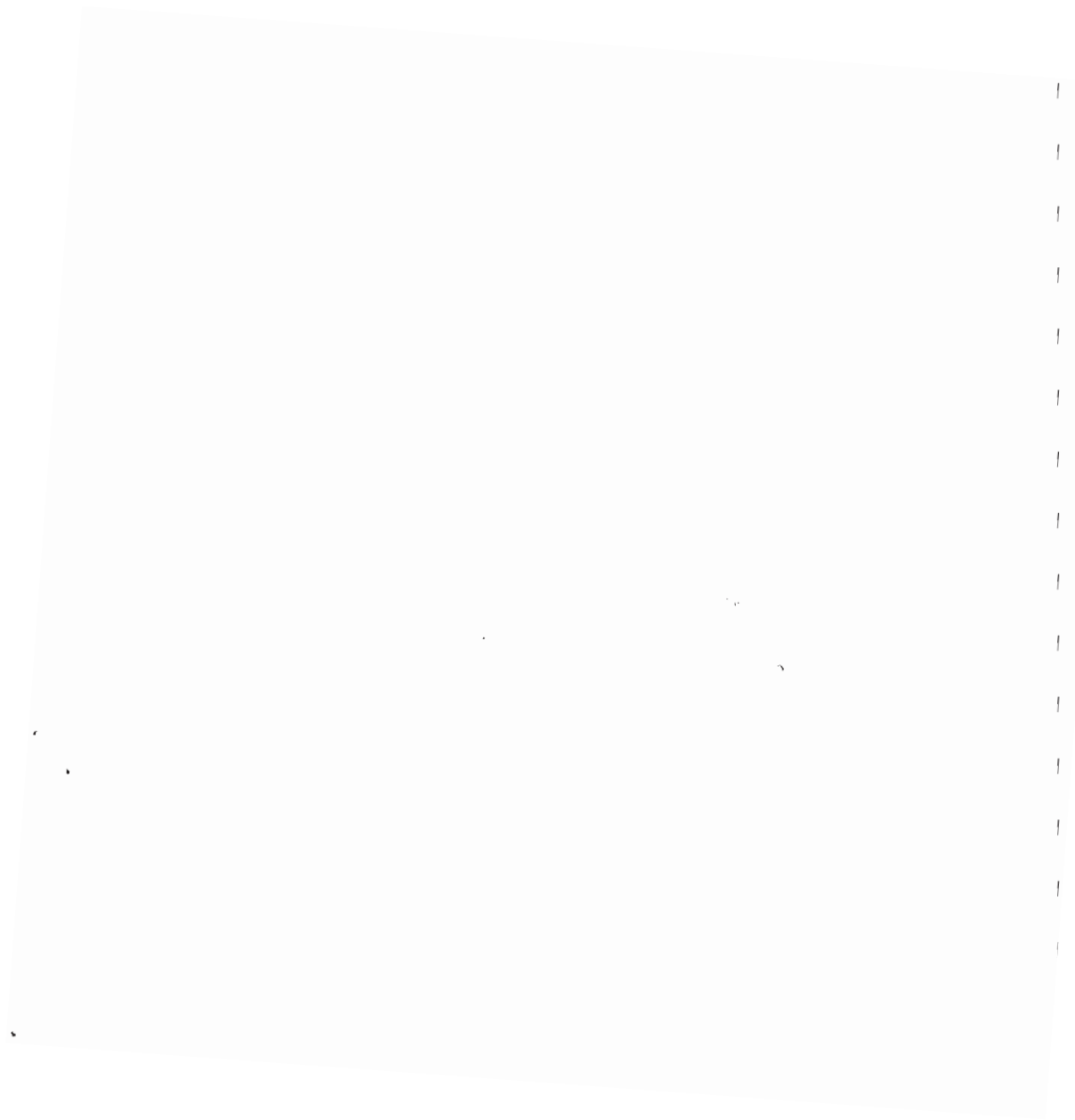
- 9.6 $3 \sum_{i=1}^{10} x_i^2 + 2 \sum_{i=1}^{10} x_i \geq c.$
- 9.7 95 or 96; 76.7.
- 9.9 38 or 39; 15.
- 9.10 0.08; 0.875.
- 9.11 $(1-\theta)^9(1+9\theta).$
- 9.12 $1, 0 < \theta \leq \frac{1}{2}; 1/(16\theta^4),$
 $\frac{1}{2} < \theta < 1; 1 - 15/(16\theta^4),$
 $1 \leq \theta.$
- 9.14 53 or 54, 5.6.
- 9.17 Reject H_0 if $\bar{x} \geq 77.564.$
- 9.18 26 or 27;
 reject H_0 if $\bar{x} \leq 24.$
- 9.19 220 or 221;
 reject H_0 if $y \geq 17.$
- 9.23 $t = 3 > 2.262$, reject $H_0.$
- 9.24 $|t| = 2.27 > 2.145$,
 reject $H_0.$
- 9.37 $c_0(n) = (14.4)$
 $\times (n \ln 1.5 - \ln 9.5);$
 $c_1(n) = (14.4)$
 $\times (n \ln 1.5 + \ln 18).$
- 9.38 $c_0(n) = (0.05n - \ln 8)/\ln 3.5;$
 $c_1(n) = (0.05n - \ln 4.5)/\ln 3.5.$
- 9.41 (b) $c = 0.18; 0.64,$
 (c) $c = 0.5; 0.16; 0.84.$
 (d) $c = 0.23; 0.06; 0.68.$
- 9.44 $(9y - 20x)/30 \leq c.$

CHAPTER 10

- 10.9 6.39.
- 10.12 $r + \theta, 2r + 4\theta.$
- 10.13 $r_2(\theta + r_1)/[r_1(r_2 - 2)].$
- 10.23 7.00, 9.98.
- 10.25 4.79, 22.82, 30.73.
- 10.26 (a) $4.483x + 6.483.$
- 10.28 $\hat{\beta} = \sum (X_i/nc_i),$
 $\sum [(X_i - \hat{\beta}c_i)^2/nc_i^2].$
- 10.32 Reject $H_0.$
- 10.44 $a_i = 0, i = 1, 2, 3, 4.$
- 10.45 $\sum_{j=1}^n a_{ij} = 0, i = 1, 2, \dots, n.$

CHAPTER 11

- 11.2 (a) $\frac{15}{16}.$ (b) $675/1024;$
 (c) $(0.8)^4.$
- 11.4 8.
- 11.6 0.954; 0.92; 0.788.
- 11.9 8.
- 11.12 (a) Beta $(n-j+1, j).$
 (b) Beta $(n-j+i-1,$
 $j-i+2).$
- 11.15 0.067.
- 11.18 Reject $H_0.$
- 11.25 0; $4(4^n - 1)/3$; no.
- 11.37 $\frac{2}{99}.$
- 11.44 $98; \frac{686}{3}.$



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