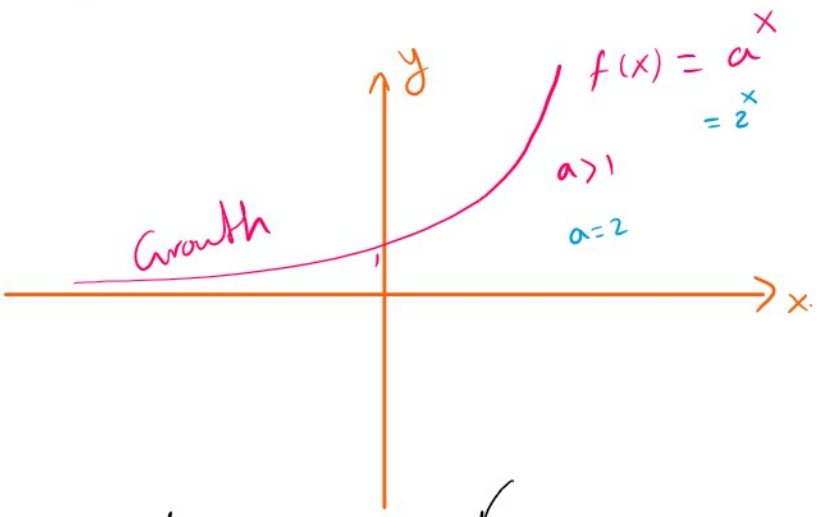
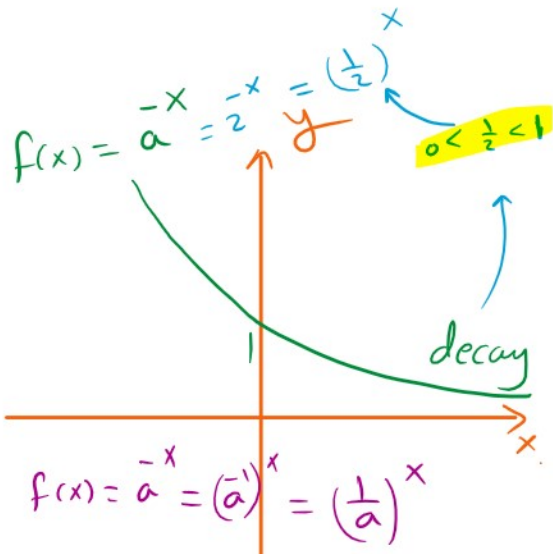
5.3 Solution of Exponential Equations

Monday, January 17, 2022 7:57 PM

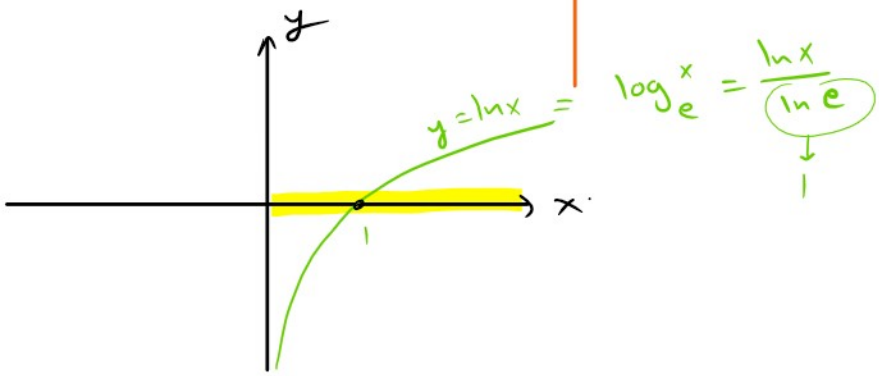
Exponential function $f(x) = a^x$





$f(x) = a^{-x} = (a^{-1})^x = (\frac{1}{a})^x$

Properties of $\ln x$



① Domain $x > 0$

② $\ln a = \ln b \Rightarrow$

$a = b$

Exp $\ln 5 = \ln x$

$x = 5$

③ $a^y = a^x \Rightarrow y = x$

④ $\ln xy = \ln x + \ln y$, $x > 0$, $y > 0$

$\ln \frac{x}{y} = \ln x - \ln y$, $x > 0$, $y > 0$

(5) $\ln \frac{x}{y} = \ln x - \ln y$, $x>0, y>0$

(6) $\ln x^r = r \ln x$ r any real number, $x>0$

(7) $\ln 1 = 0$

Exp Solve the following exponential equation:

(1) $4(25)^{2x} = 20 \Rightarrow \frac{4}{4}(25)^{2x} = \frac{20}{4}$

$(25)^{2x} = 5$

$\ln(25)^{2x} = \ln 5$

$2x \ln 25 = \ln 5$

$\frac{2x \ln 25}{2 \ln 25} = \frac{\ln 5}{2 \ln 25}$

$x = \frac{\ln 5}{2 \ln 5} = \frac{1}{4}$

$x = \frac{1}{4}$

Sim

$\left(\frac{2}{5}\right)^{2x} = 5$

$5^{4x} = 5^1$

$4x = 1$

$x = \frac{1}{4}$

3x

$x = \frac{1}{4}$

(2) $8^{3x} = 216$

$\ln 8^{3x} = \ln 216$

$3x \ln 8 = \ln 216$

$\frac{3x \ln 8}{3 \ln 8} = \frac{\ln 216}{3 \ln 8}$

$8 = 2 \cdot 2 \cdot 2 = 2^3$

2	216
2	108
2	54
3	27
3	9
3	3
	1

$$x = \frac{\ln 216}{3 \ln 8} = \frac{\ln 2^3 \cdot 3^3}{3 \ln 2^3}$$

$$= \frac{3 \ln 2 + 3 \ln 3}{9 \ln 2} = \frac{3 \ln 2 + 3 \ln 3}{9 \ln 2}$$

$\frac{A+B}{C} = \frac{A}{C} + \frac{B}{C}$

$$= \frac{3 \ln 2}{9 \ln 2} + \frac{3 \ln 3}{9 \ln 2}$$

$$= \frac{1}{3} + \frac{\ln 3}{3 \ln 2}$$

(3) $25000 = 10{,}000\ 5^{2x}$

$\frac{25000}{10.000} = 5^{2x}$

$\frac{25000}{10,000} = 5$

2

$\frac{5}{2} = 5^{2x}$

$\ln \frac{5}{2} = 2x \ln 5$

$\frac{\ln \frac{5}{2}}{2\ln 5} = \frac{2x \ln 5}{2\ln 5}$

$\frac{\ln 5 - \ln 2}{2\ln 5} = x$

$x = \frac{\ln 5}{2\ln 5} - \frac{\ln 2}{2\ln 5}$

$$x = \frac{1}{2} - \frac{\ln 2}{2\ln 5}$$

(4) $\log_5 (x+2) = 2$

$\Rightarrow 5^2 = x + 2$

$25 = x + 2$

$-2 \quad -2$

$23 = x$

بس كده

$\log_5 (23+2)$

$\log_5 25$

$\log_5 5^2 = 2 \log_5 5^1$

$= (2)(1)$

$= 2$

(5) $\ln (x+2) + \ln x = \ln (x+12)$

شرط ضروري

$x+2 > 0$

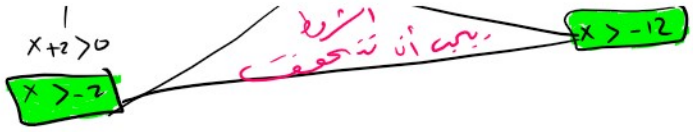
$x > -2$

$x > 0$

الشرط يجب أن تتحقق

$x + 12 > 0$

$x > -12$



$\ln a = \ln b$

$a = b$

$\ln(x+2)(x) = \ln(x+12)$

$(x+2)(x) = (x+12)$

$x^2 + 2x = x + 12$

$-x \quad -x$

$x^2 + x = 12$

$-12 \quad -12$

$x^2 + x - 12 = 0$

$(x+a)(x+b) = 0$

$ab = -12$

$a+b = 1$

$a = 4$

$b = -3$

$(x+4)(x-3) = 0$

$x+4=0$ or $x-3=0$

$-4 \quad -4$ $+3 \quad +3$

$x = -4$

$x = 3$

Check

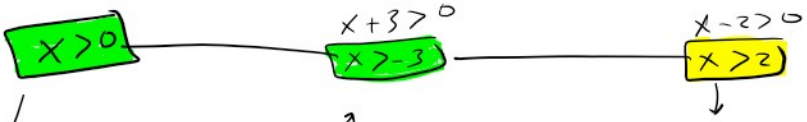
$\ln(x+2) + \ln x = \ln(x+12)$

$\ln(3+2) + \ln 3 = \ln(3+12)$

$\ln 5 + \ln 3 = \ln 15$

$\ln 5.3 = \ln 15$

$\ln 15 = \ln 15$ ✓



(6) $\log_4 x - \log_4 (x+3) = \log_4 (x-2)$

$\log_4 \frac{x}{x+3} = \log_4 (x-2)$

$\log_a x = \log_a y$ means $x = y$

$\frac{x}{x+3} = \frac{x-2}{1}$

$x = (x-2)(x+3)$

$x = x^2 + 3x - 2x - 6$

$x = x^2 + x - 6$

$-x \quad -x$

$0 = x^2 - 6 \Rightarrow \sqrt{x^2} = \sqrt{6} \Rightarrow x = \pm\sqrt{6}$

$0 = (x - \sqrt{6})(x + \sqrt{6})$

$x - \sqrt{6} = 0$ or $x + \sqrt{6} = 0$

$+\sqrt{6} \quad +\sqrt{6} \qquad -\sqrt{6} \quad -\sqrt{6}$

$x = \sqrt{6}$ $\qquad x = -\sqrt{6}$

$6 > 4$

$\sqrt{6} > \sqrt{4}$

$\sqrt{6} > 2$

$x > 0$

(7) $\ln x - \ln 2 = 3$

$\ln \frac{x}{2} = 3$

$3 = (3)(1)$

$= (3)\ln e$

$= \ln e^3$

$\ln \frac{x}{2} = \ln e^3$

$\frac{x}{2} = e^3$

$x = 2e^3$

Exp Suppose the demand for commodity q is given by $p = 40(2)^{-\frac{q}{2}}$ where p is price in dollars

(1) At what price per unit will the demand equal 4 units?

(1) If ... equal 4 units ?

Find p when $q = 4$

$$p = 40\,(2)^{-\frac{q}{2}} = 40\,(2)^{-\frac{4}{2}} = 40\,(2)^{-2}$$

$$= 40\,\frac{1}{2^2} = \frac{40}{4} = 10 \text{ \$}$$

(2) How many units will be demanded if the price is \$20

Find q when $p = 20$

$$p = 40\,(2)^{-\frac{q}{2}}$$

$$\frac{20}{40} = \frac{40\,(2)^{-\frac{q}{2}}}{40}$$

$$\frac{1}{2} = (2)^{-\frac{q}{2}}$$

$$2^{-1} = 2^{-\frac{q}{2}}$$

$$-1 = -\frac{q}{2}$$

$$q = 2$$

Exp Suppose the demand function for a commodity

Exp Suppose the demand function for a commodity is given by $p = 100\, e^{-\frac{x}{10}}$ where p is the price per unit when x units are sold

(1) Find the total revenue

Revenue = (Price)(number of units sold)

$$R(x) = p \quad x$$

$$= \left(100\, e^{-\frac{x}{10}}\right) x$$

$$R(x) = 100\, x\, e^{-\frac{x}{10}}$$

(2) Find total revenue if 30 units were demanded and supplied

Find $R(30) = 100\,(30)\, e^{-\frac{30}{10}}$

$$= 3000\, e^{-3}$$

$$= \frac{3000}{e^3}$$

≈ 149.4 $