

wave equation

$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$	$\psi = \sin(kx \pm \omega t)$	$k = \frac{2\pi}{\lambda}$	$\omega = \frac{2\pi}{T}$	$\frac{\omega}{k} = v$
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Phasors and addition of waves

$y_1 = A_1 e^{i\phi_1}$	$y_2 = A_2 e^{i\phi_2}$	$\delta = \phi_2 - \phi_1$
$y_r = y_1 + y_2$	$y_r = y_r e^{i\phi}$	

$ y_r = A_1^2 + A_2^2 + 2A_1 A_2 \cos \delta$	$\tan \phi = \frac{A_1 \sin \phi_1 + A_2 \sin \phi_2}{A_1 \cos \phi_1 + A_2 \cos \phi_2}$
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Spherical waves	Cylindrical waves
$\vec{E}(x, y, z, t) = \frac{\vec{E}_0}{r} e^{i(\vec{k} \cdot \vec{r} - \omega t)}$	$\vec{E}(x, y, z, t) = \frac{\vec{E}_0}{\sqrt{r}} e^{i(\vec{k} \cdot \vec{r} - \omega t)}$

Electromagnetic theory

$\vec{\nabla} \cdot \vec{D} = \rho$	$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\vec{\nabla} \cdot \vec{B} = 0$	$\vec{\nabla} \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$
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$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon_0(1 + \chi) \vec{E} = \epsilon \vec{E} = \epsilon_0 n^2 \vec{E}$	$\vec{B} = \mu_0 \vec{H} + \vec{M}$
$n = \sqrt{\frac{\epsilon}{\epsilon_0}}$	$\vec{j} = \sigma \vec{E}$

Electromagnetic wave equation

$$\nabla^2 \vec{E} = \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$v = \frac{1}{\sqrt{\mu \epsilon}} = \frac{c}{n}$	$\vec{k} \cdot \vec{E} = 0$	$\vec{K} \times \vec{E} = \omega \vec{B}$	$ \vec{B} = \frac{n}{c} \vec{E} $	$\frac{ \vec{E} }{ \vec{H} } = \sqrt{\frac{\mu}{\epsilon}} = Z$
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Energy and momentum

$$\vec{S} = \vec{E} \times \vec{H} \quad \text{pointing vector}$$

$$I = |\langle \vec{S} \rangle| = \frac{1}{2} E_0 H_0 = \frac{1}{2} c \epsilon_0 E_0^2 = c \langle u \rangle \quad I: \text{irradiance}, \langle u \rangle: \text{average energy density}$$

$\langle \vec{F} \rangle = \langle \frac{d\vec{P}}{dt} \rangle = \hat{k} q \langle v_x B \rangle$	$\langle \frac{dU}{dt} \rangle = c q \langle v_x B \rangle$	$\langle \frac{d\vec{P}}{dt} \rangle = \frac{1}{c} \langle \frac{dU}{dt} \rangle$
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Damped oscillator model

$m(\ddot{x} + \gamma \dot{x} + \omega_0^2 x) = q E_0 e^{i\omega t}$	$x_0 = \frac{q E_0}{m(\omega_0^2 - \omega^2 + i\gamma\omega)}$	$p = qx = \frac{N q^2 E_0 e^{i\omega t}}{m(\omega_0^2 - \omega^2 + i\gamma\omega)}$
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dipole moment for N oscillator per unit volume (macroscopic polarization)

$$n = 1 + \frac{N q^2}{2 m \epsilon_0 (\omega_0^2 - \omega^2 + i\gamma\omega)} = n' + i n''$$

$n' = 1 + \frac{N q^2}{2 m \epsilon_0 (\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2} \frac{\omega_0^2 - \omega^2}{\omega^2}$	$n'' = \frac{N q^2}{2 m \epsilon_0 (\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2} \frac{\gamma \omega}{\omega^2}$
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For $\omega_0 \ll \omega$:

$n' = 1 + \frac{N q^2}{2 m \epsilon_0 \omega_0 (\omega_0 - \omega)^2 + (\frac{\gamma}{2})^2} \frac{\omega_0 - \omega}{(\omega_0 - \omega)^2}$	$n'' = \frac{N q^2}{8 m \epsilon_0 \omega_0 (\omega_0 - \omega)^2 + (\frac{\gamma}{2})^2} \frac{\gamma}{(\omega_0 - \omega)^2}$
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Refraction

$r_{\perp} = \frac{E_{0r}}{E_{0i}} = \frac{n_i \cos \theta_i - n_t \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t}$	$t_{\perp} = \frac{E_{0t}}{E_{0i}} = \frac{2 n_i \cos \theta_i}{n_i \cos \theta_i + n_t \cos \theta_t}$
$r_{\parallel} = \frac{E_{0r}}{E_{0i}} = \frac{n_i \cos \theta_t - n_t \cos \theta_i}{n_i \cos \theta_t + n_t \cos \theta_i}$	$t_{\parallel} = \frac{E_{0t}}{E_{0i}} = \frac{2 n_i \cos \theta_i}{n_i \cos \theta_t + n_t \cos \theta_i}$

$\theta_{Broster} = \tan^{-1} \frac{n_t}{n_i}$	$\theta_{Critical} = \sin^{-1} \frac{n_t}{n_i}$
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Reflected power	Transmittance power	
$R = \frac{I_r A_r}{I_i A_i} = r^2$	$T = \frac{I_t A_t}{I_i A_i} = \frac{n_t \cos \theta_t}{n_i \cos \theta_i} t^2$	$I = \frac{n \epsilon_0 c_0}{2} E_0 ^2$

Lenses

Refracting prisms	Spherical	Convex lens
$\frac{n_{prism}}{n_0} = \frac{\sin \frac{1}{2}(A + D)}{\sin \frac{1}{2}A}$	$\frac{n_1}{s_{01}} + \frac{n_2}{s_{i1}} = \frac{n_2 - n_1}{R_1}$	$\frac{1}{s_{01}} + \frac{1}{s_{i1}} = \left(\frac{n_l}{n_m} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{1}{f}$

Mirror

$-\frac{h_0}{h_i} = \frac{d_0}{d_i}$	$\frac{1}{d_0} + \frac{1}{d_i} = \frac{2}{R} = \frac{1}{f}$	$magnification = \frac{h_i}{h_0}$
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<u>Simple telescope</u>	<u>Compound lens</u>
$M = \frac{\theta'}{\theta} = \frac{f_0}{f_e}$	$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$

Ray tracing

<u>Translation</u>	<u>Refraction</u>
$\begin{bmatrix} x_2 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ \gamma_1 \end{bmatrix}$	$\begin{bmatrix} x_2 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_2}{n_1} \end{bmatrix} \begin{bmatrix} x_1 \\ \gamma_1 \end{bmatrix}$

Curved lens

<u>n_1 to n_2</u>	<u>n_2 to n_3</u>
$\begin{bmatrix} x_2 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{n_2 - n_1} & \frac{0}{n_2} \\ \frac{n_1}{n_2 R_1} & \frac{n_1}{n_2} \end{bmatrix} \begin{bmatrix} x_1 \\ \gamma_1 \end{bmatrix}$	$\begin{bmatrix} x_3 \\ \gamma_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{n_3 - n_2} & \frac{0}{n_3} \\ \frac{n_2}{n_3 R_2} & \frac{n_2}{n_3} \end{bmatrix} \begin{bmatrix} x_2 \\ \gamma_2 \end{bmatrix}$

Thin lens

$$\begin{bmatrix} x_3 \\ \gamma_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{n_3 R_2} + \frac{n_1 - n_2}{n_3 R_1} & \frac{0}{n_3} \\ \frac{n_1}{n_3} & \frac{n_1}{n_3} \end{bmatrix} \begin{bmatrix} x_1 \\ \gamma_1 \end{bmatrix}$$

For $n_1 = n_3 = 1$ and $R_1 = -R_2$	$\begin{bmatrix} x_3 \\ \gamma_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{f} & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ \gamma_1 \end{bmatrix}$	$f = \frac{R}{2(n-1)}$
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Imaging by thin lens

$$\begin{bmatrix} x_i \\ \gamma_i \end{bmatrix} = \begin{bmatrix} 1 - \frac{s_i}{f} & s_0 + s_i - \frac{s_0 s_i}{f} \\ -\frac{1}{f} & 1 - \frac{s_0}{f} \end{bmatrix} \begin{bmatrix} x_0 \\ \gamma_0 \end{bmatrix}$$

$$s_0 + s_i - \frac{s_0 s_i}{f} = 0 \rightarrow \frac{1}{s_0} + \frac{1}{s_i} = \frac{1}{f}$$

<u>magnification</u>	<u>angular magnification</u>
$\beta_l = 1 - \frac{s_i}{f}$	$\beta_\gamma = 1 - \frac{s_0}{f}$

$\beta_l \beta_\gamma = 1$	$x_i \gamma_i = x_0 \gamma_0$
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Thick lens and lenses system

$$M = \begin{bmatrix} \frac{1}{n_2 - n_3} & \frac{0}{n_3} \\ \frac{n_2}{n_3 R_2} & \frac{n_2}{n_3} \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{n_1 - n_2} & \frac{0}{n_2} \\ \frac{n_1}{n_2 R_1} & \frac{n_1}{n_2} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$A = 1 + \frac{n_1 - n_2}{n_2 R_1} d$	$D = \frac{n_1}{n_3} (1 + \frac{n_2 - n_3}{n_2 R_2} d)$
$B = \frac{n_1}{n_2} d$	$C = \frac{1}{n_3} (\frac{n_2 - n_3}{R_2} + \frac{n_1 - n_2}{R_1} + \frac{n_1 - n_2}{n_2 R_1} \frac{n_2 - n_3}{R_2} d)$

Thin lens

Take: $n_1 = n_3$ and $d = 0$

$A = 1$	$B = 0$	$C = \frac{1}{n_1} (\frac{n_2 - n_1}{R_2} + \frac{n_1 - n_2}{R_1})$	$D = 1$
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At $-R_1 = R_2 = -R$	At $n_1 = 1$
$C = -\frac{2}{n_1} (\frac{n_2 - n_1}{R})$	$C = -\frac{1}{f}$

Fourier optics

$f(t) = \frac{a_0}{2} + \sum_{l=1}^{\infty} a_l \cos(l\omega t) + \sum_{l=1}^{\infty} b_l \sin l\omega t$		
$a_0 = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) dt$	$a_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \cos(n\omega t) dt$	$b_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \sin(n\omega t) dt$

$f(t) = \sum_{-\infty}^{\infty} \alpha_l e^{il\omega t}$	$\alpha_l = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{-il\omega t} dt$
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Fourier integral

$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(\omega) e^{i\omega t} d\omega$	$f(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$
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Dirac delta function

$\delta(t - t_0) = \begin{cases} f(t) & t = t_0 \\ 0 & t \neq t_0 \end{cases}$	$\int_{-\infty}^{\infty} \delta(t - t_0) dt = 1$	$\int_{-\infty}^{\infty} f(t) \delta(t - t_0) dt = f(t_0)$
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Comb function

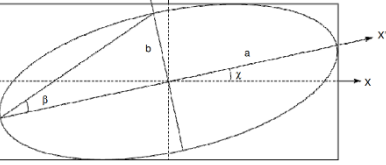
$comb(t) = \sum_{-N}^N \delta(t - t_n)$	$t_n = nt_0$
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$$F(\omega) = \int_{-\infty}^{\infty} comb(t) e^{-i\omega t} dt = \sum_{-N}^N e^{-i\omega t_n}$$

Polarization

$E_x(z, t) = E_{0x} \cos(\tau + \delta_x)$	$E_y(z, t) = E_{0y} \cos(\tau + \delta_y)$
$\frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} - 2 \frac{E_x E_y}{E_{0x} E_{0y}} \cos(\delta) = \sin^2(\delta)$	$I = I_0 \cos^2 \theta$
Linear ($\delta = 0$ or π)	Right/left circular ($\delta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$, $E_{0x} = E_{0y} = E_0$)
$\left(\frac{E_x}{E_{0x}} + \frac{E_y}{E_{0y}}\right)^2 = 0$	$\left(\frac{E_x}{E_{0x}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2 = 1$

General elliptical polarization

	$E'_x = E_x \cos(\aleph) + E_y \sin(\aleph)$ $\tan(2\aleph) = \frac{2E_{0x}E_{0y} \cos \delta}{E_{0x}^2 + E_{0y}^2}$	$E'_y = -E_x \sin(\aleph) + E_y \cos(\aleph)$ $\tan(\beta) = \pm \frac{b}{a}$
$\widetilde{E}_x(z, t) = E_{0x} e^{i(kz - \omega t + \delta_x)}$	$\widetilde{E}_y(z, t) = E_{0y} e^{i(kz - \omega t + \delta_y)}$	
$\vec{E} = [E_{0x} e^{i(\delta_x)} \hat{x} + E_{0y} e^{i(\delta_y)} \hat{y}] e^{i(kz - \omega t)} = \vec{E}_0 e^{i(kz - \omega t)}$	$\vec{E}_0 = \begin{bmatrix} \widetilde{E}_{0x} \\ \widetilde{E}_{0y} \end{bmatrix} = \begin{bmatrix} E_{0x} e^{i\delta_x} \\ E_{0y} e^{i\delta_y} \end{bmatrix}$	
Transformation of polarization	$\begin{pmatrix} E'_x \\ E'_y \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$	

Jones vector	$\begin{pmatrix} E_{0x} \\ E_{0y} \end{pmatrix} = \begin{pmatrix} E_{0x} e^{i\phi_x} \\ E_{0y} e^{i\phi_y} \end{pmatrix} = \begin{pmatrix} E_{0x} \\ E_{0y} e^{i\delta} \end{pmatrix}$	$\begin{pmatrix} E_{0x} \\ E_{0y} \end{pmatrix} = \frac{1}{\sqrt{E_{0x}^2 + E_{0y}^2}} \begin{pmatrix} E_{0x} \\ E_{0y} e^{i\delta} \end{pmatrix}$
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Stoke's parameters

$\vec{E} = iE_{0x} \cos(kz - \omega t + \phi_x) + jE_{0y} \cos(kz - \omega t + \phi_y)$		
$S_0 = \langle E_{0x}^2 \rangle + \langle E_{0y}^2 \rangle$	$S_1 = \langle E_{0x}^2 \rangle - \langle E_{0y}^2 \rangle$	$S_2 = 2\langle E_{0x} E_{0y} \cos \delta \rangle$
$S_3 = 2\langle E_{0x} E_{0y} \sin \delta \rangle$	$S_0^2 = S_1^2 + S_2^2 + S_3^2$ (for δ time independent)	
Degree of polarization	$m = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0}$	

Interference

Two point sources	$\vec{E}_1 = \vec{E}_{01} e^{i(\vec{k} \cdot \vec{r}_1 - \omega t + \phi_1)}$	$\vec{E}_2 = \vec{E}_{02} e^{i(\vec{k} \cdot \vec{r}_2 - \omega t + \phi_2)}$	$\vec{E}_p = \vec{E}_1 + \vec{E}_2$
$I_p \propto \langle \vec{E}_p ^2 \rangle$	$I_p = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \alpha$, $\alpha = \vec{k} \cdot (\vec{r}_2 - \vec{r}_1) + (\phi_2 - \phi_1)$		

Young's Experiment

Maxima	Two adjacent maxima	$\delta = k(r_2 - r_1) = \frac{2\pi x h}{\lambda D}$
$\delta_m = 2m\pi$	$X_m - X_{m-1} = \frac{\lambda D}{h}$	

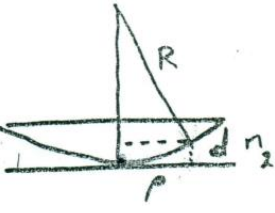
Multiple sources

$\vec{E_p} = E_0 e^{i(\vec{k} \cdot \vec{r} - \omega t + \phi_1)} \frac{\sin(\frac{N}{2}\phi)}{\sin \frac{\phi}{2}}$	$r = r_1 + \frac{N-1}{2}\phi$	$\phi = \frac{2\pi}{\lambda} d \sin \theta$		
Total phase difference	$\Phi = \frac{2\pi}{\lambda} a \sin \theta = (N-1)\phi \cong N\phi \text{ (constant)}$			
$ E_p = NE_0 \frac{\sin \alpha}{\alpha}$	$\alpha = \frac{\pi}{\lambda} a \sin \theta$	$I_p = N^2 E_0^2 \frac{\sin^2 \alpha}{\alpha^2}$		
Minimum	$\alpha = m\pi$	$a \sin \theta = m\lambda$	$a \frac{X}{D} = m\lambda$	$X_m = m \frac{\lambda D}{a}$
Maximum	Width of central maxima		Maxima	
	$w = 2X_1 = \frac{2\lambda D}{a}$		$\frac{dI_p}{d\alpha} = 0, \tan \alpha = \alpha$	

Interference in thin films

Brightness at $\delta = 2m\pi$	Brightness at normal incident ($\theta_t = 0$)	Darkness at normal incident ($\theta_t = 0$)
$2n_2 d \cos \theta_t = m\lambda_0$	$d = \frac{m\lambda_0}{2n_2} = \frac{m\lambda_n}{2}$	$d = (2m+1) \frac{\lambda_0}{4n_2}$
If equal amount of light is reflected on both sides of the film		$\frac{n_2 - n_1}{n_2 + n_1} = \frac{n_3 - n_2}{n_3 + n_2} \rightarrow n_2 = \sqrt{n_3}$

Newton's rings

	Minimum (Dark)	Maxima (Bright)
	$2kd = 2m\pi$	$\rho_m = \sqrt{\left(m + \frac{1}{2}\right) \lambda_0 R}$
$d = \frac{\rho^2}{2R}$	$\rho_m = \sqrt{\frac{m\lambda_0 R}{n_2}}$	

Michelson interferometer

Total phase difference	Circular symmetric ($\delta = 2\pi m$)		
$\delta = \frac{2\pi}{\lambda_0} \cdot 2d \cos \theta_t$	$m = \frac{2d}{\lambda_0} \cos \theta_t$	Normal incidence	$m = \frac{2d}{\lambda_0}$
Intensity	$I = 2I_0 \cos^2 kd$	At $I = \frac{I_{max}}{2}$	$\delta_1 = \frac{\pi}{2}$

Fabri-Perot (multi reflection)

$\vec{E}_t = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$	$E_t = \frac{E_1(1 - r^2 \cos \delta + ir^2 \sin \delta)}{1 - 2r^2 \cos \delta + r^4}$
$\vec{E}_1 = \vec{E}_0 t_1 t_2 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$	

$I_t = E_t \cdot E_t^* = \frac{E_1^2}{1 - 2r^2 \cos \delta + r^4} = \frac{E_1^2}{(1 - r^2)^2 + 4r^2 \sin^2 \frac{\delta}{2}}$		
At $\delta = 2m\pi$	$I_t = I_{max} = \frac{E_1^2}{(1 - r^2)^2}$	$I_t = \frac{I_{max}}{1 + \frac{4r^2}{(1 - r^2)^2} \sin^2 \frac{\delta}{2}}$
At $\delta = (2m + 1)\pi$	$I_t = I_{min} = \frac{E_1^2}{(1 + r^2)^2}$	

Visibility (v)	$v = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2r^2}{1 + r^4}$	Contrast (F)	$F = \frac{4r^2}{(1 - r^2)^2}$
$\frac{I_t}{I_{max}} = \frac{1}{1 + F \sin^2 \frac{\delta}{2}}$			
Maximum trans		At $I = \frac{I_{max}}{2}$	
$\frac{\delta}{2} = m\pi$	$2n_2 d \cos \theta_t = \frac{m\lambda_0}{2}$	$\delta_1 = 2 \sin^{-1} \frac{1}{\sqrt{F}}$	Half width (γ) $\gamma = 2\delta_1$

Resolving power

$RB = \frac{\lambda}{ \Delta\lambda } = \frac{\omega}{\Delta\omega} = \frac{\omega d}{2c} \sqrt{F} = \frac{\omega d}{\pi c} f$	$f = \frac{\pi}{2} \sqrt{F}$
Maxima, at $(\delta = 2m\pi), (\frac{\omega d}{c} = m\pi)$	$RB = mF$

Multilayer

For 3 regions n_0, n_1, n_3		
$\begin{pmatrix} E_1 \\ H_1 \end{pmatrix} = \begin{pmatrix} \cos \delta_1 & -\frac{i \sin \delta_1}{\beta_1} \\ -i\beta_1 \sin \delta_1 & \cos \delta_1 \end{pmatrix} \begin{pmatrix} E_2 \\ H_2 \end{pmatrix}$		$\delta_1 = n_1 k_0 d \cos \theta_t$ $\beta_1 = \sqrt{\frac{\epsilon_0}{\mu_0}} n_1 \cos \theta_t$
For multi periodic layers		
$\begin{pmatrix} E_1 \\ H_1 \end{pmatrix} = M_1 M_2 \dots M_n \begin{pmatrix} E_{n+1} \\ H_{n+1} \end{pmatrix}$		$M = M_1 M_2 \dots M_n = \begin{pmatrix} M_{11} & -iM_{12} \\ -iM_{21} & M_{22} \end{pmatrix}$
Reflectivity R	$r = \frac{(\beta_0 M_{11} - \beta_s M_{22}) - i(\beta_0 \beta_s M_{12} - M_{21})}{(\beta_0 M_{11} + \beta_s M_{22}) - i(\beta_0 \beta_s M_{12} + M_{21})}$	$R = rr^*$
Transitivity T	$t = \frac{2\beta_0}{(\beta_0 M_{11} + \beta_s M_{22}) - i(\beta_0 \beta_s M_{12} + M_{21})}$	$T = tt^*$

Anti-reflecting film

For normal incidence			
$\delta = kd$	$\beta_1 = \sqrt{\frac{\epsilon_0}{\mu_0}} n_1$	$R = \frac{(n_s - n_1^2)^2}{(n_s + n_1^2)^2}$	$M = \begin{pmatrix} \cos \delta & -\frac{i \sin \delta}{\beta_1} \\ -i\beta_1 \sin \delta & \cos \delta \end{pmatrix}$
$\beta_0 = \sqrt{\frac{\epsilon_0}{\mu_0}}$	$\beta_s = \sqrt{\frac{\epsilon_0}{\mu_0}} n_s$	$R = 0$ For $n_1 = \sqrt{n_s}$	

High-reflecting films

Alternate layers of high index n_h and low n_l with thickness $d = \frac{\lambda}{4n_{hl}}$	
$M = \begin{bmatrix} -\frac{n_h}{n_l} & 0 \\ 0 & -\frac{n_l}{n_h} \end{bmatrix}$	For 2N layers (films) $M_t = M^n = \begin{bmatrix} \left(-\frac{n_h}{n_l}\right)^N & 0 \\ 0 & \left(-\frac{n_l}{n_h}\right)^N \end{bmatrix}$ $R = \left(\frac{\left(\frac{n_h}{n_l}\right)^{2N} - n_s}{\left(\frac{n_h}{n_l}\right)^{2N} + n_s} \right)^2$

Diffraction

$r = r_0 + y \sin \theta$	$u_p = \int e^{ikr} ds$
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Single and double slits

$\alpha = \frac{\pi}{\lambda} b \sin \theta$	$\beta = k \frac{a}{2} \sin \theta$	$C' = aLC e^{ikr_0}$	$I = u_p ^2 = I_m \left(\frac{\sin \beta}{\beta} \right)^2$
Single slit	$u_p = C' \frac{\sin \beta}{\beta}$	Double slits	$u_p = C' \frac{\sin \beta}{\beta} \cos \alpha$
minima	maxima	Minima	Maxima
$\beta = m\pi$	$\frac{dI}{d\beta} = 0$	$m\pi = \frac{\pi}{\lambda} b \sin \theta$	$X_m = \frac{m\lambda D}{b}$ $w_c = \frac{2\lambda D}{a}$
$\frac{\pi}{\lambda} a \sin \theta = m\pi$	$\tan \beta = \beta$	#of fringes under central max	
$X_m = \frac{m\lambda D}{a}$	$width_{central} = \frac{2\lambda D}{a}$	$n = \frac{w}{\Delta x} - 1 = 2 \frac{b}{a} - 1$	

Multi slits (diffraction grating)

$u_p = C' \frac{\sin \beta}{\beta} \frac{\sin N\alpha}{\sin \alpha}$	$I = N^2 I_0 \left(\frac{\sin \beta}{\beta} \frac{\sin N\alpha}{\sin \alpha} \right)^2$	I_0 : intensity from one slit
Maxima	$\alpha = m\pi$	$m\lambda = b \sin \theta$ Resolving power: $RB = \frac{\lambda}{\Delta\lambda} = Nm$

Circular aperture

$u_p = C e^{ikr_0} 2 \frac{J_1(p)}{p}$	$I = I_0 \left(2 \frac{J_1(p)}{p} \right)^2$	$p = kR \sin \theta$
Airy disk	$\sin \theta = \frac{1.22\lambda}{D} = \theta$	$D = 2R$

