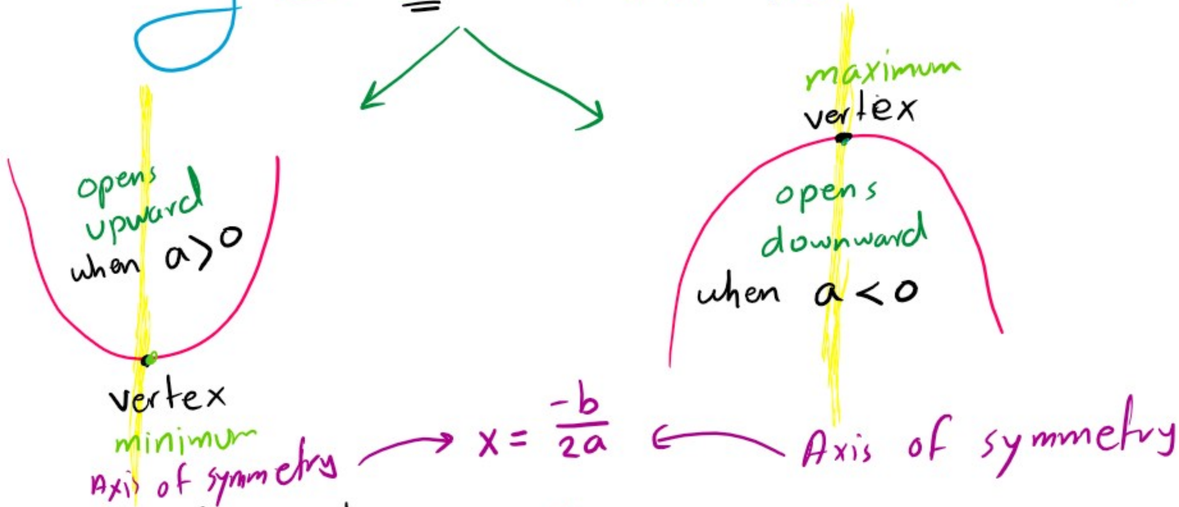
2.2 Part 1
Thursday, December 16, 2021 9:54 AM

Parabola (Quadratic Function)

$$y = ax^2 + bx + c \qquad a \neq 0$$



$x = \frac{-b}{2a}$

Q. How to find vertex

A. vertex is $(x, y) = \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

Q. When the parabola $(y = ax^2 + bx + c)$ opens up/down?

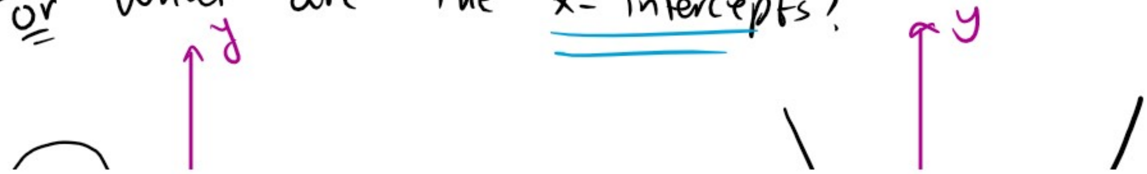
A: opens up when $a > 0 \Rightarrow$ vertex is minimum
opens down " $a < 0 \Rightarrow$ " " maximum

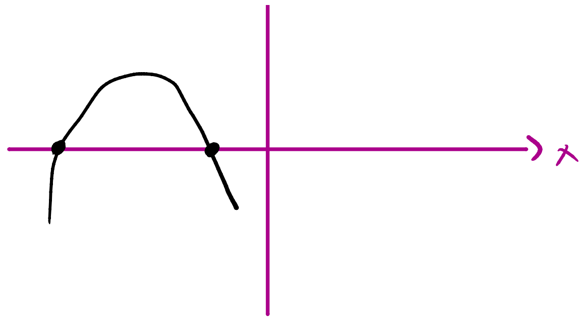
Q. How to find axis of symmetry?

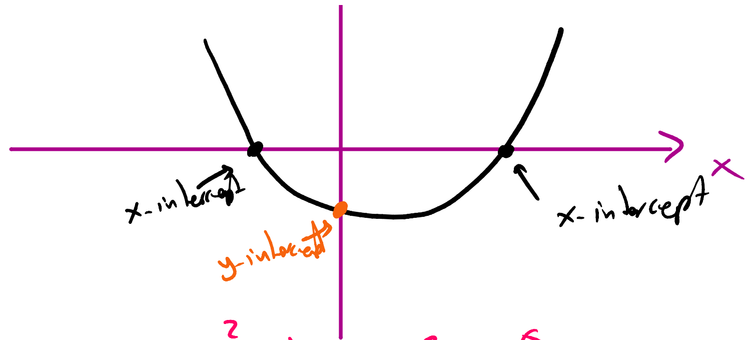
Axis of symmetry $x = -\frac{b}{2a}$

Q. When paraba intersects x-axis?
or What are the x-intercepts?

$y = ax^2 + bx + c$







x-intercept $\Rightarrow$ $y = 0$ $\Rightarrow$ $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a} \qquad \Delta = b^2 - 4ac$$

x-intercepts

$$x_1 = \frac{-b + \sqrt{\Delta}}{2a}$$

$$x_2 = \frac{-b - \sqrt{\Delta}}{2a}$$

→ roots
→ zeros of y

$\boxed{\Delta \geq 0}$ ✓

Q: How to find y-intercept?

$y = ax^2 + bx + c$

A: y-intercept $\Rightarrow$ $\underline{\underline{x = 0}}$ $\Rightarrow$ $y = c$

y-intercept

$y = 0 + 0 + c$

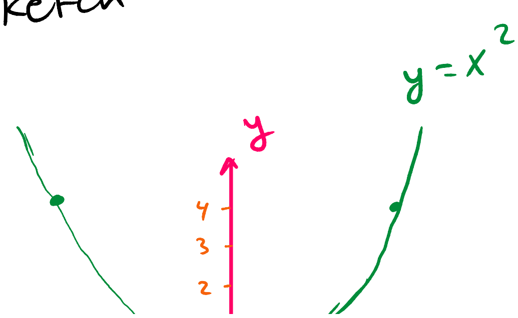
$\boxed{y = c}$

Exp $y = x^2$ Find vertex, axis of symmetry, intercepts and sketch

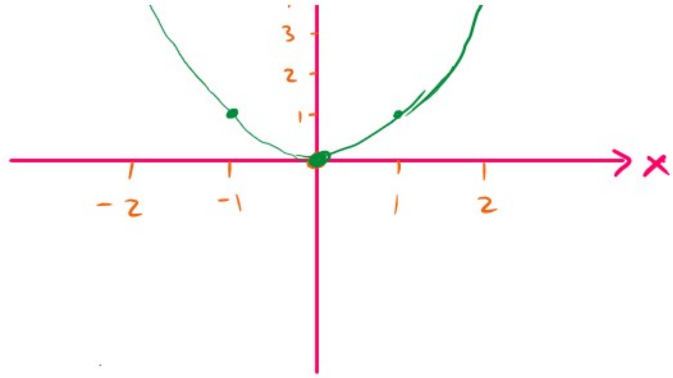
solution 1

S1

x	y
-2	4



x	y
-2	4
-1	1
0	0
1	1
2	4



S2

$y = x^2$

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$$\boxed{y = ax^2 + bx + c}$$

$a = 1$, $b = 0$, $c = 0$

$a > 0$

$\Downarrow$

opens up

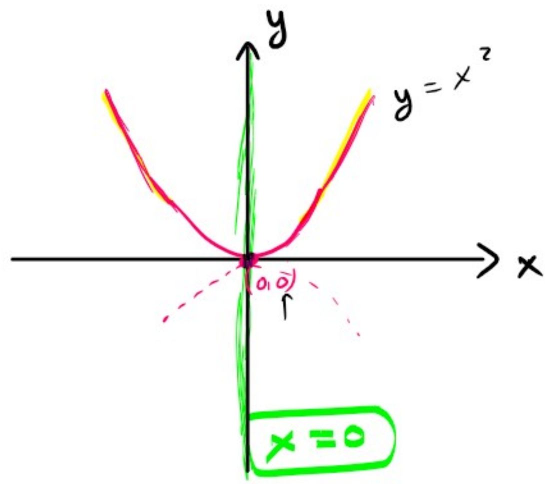
vertex minimum

① vertex $= (x, y) = \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$= \left(\frac{-0}{2(1)}, f(0)\right)$

$= (0, 0^2)$

$= (0,0)$



axis of symmetry $x = -\frac{b}{2a} = 0 \Rightarrow \boxed{x = 0}$

intercepts $\rightarrow$ x $\Rightarrow$ $\Delta = b^2 - 4ac$

$= 0^2 - 4(1)(0)$

$= 0$ ✓

$$x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-0 \pm \sqrt{0}}{2(1)} = \frac{0}{2} = 0$$

$$\boxed{x = 0}$$

$\boxed{x=0}$

y-intercept is $\boxed{y=c} \Rightarrow \boxed{y=0}$

Remark: The optimum value occurs at vertex

optimum value → Maximum Value, Mimimum Value

Exp $y = -\frac{1}{2}x^2$ Find vertex, optimum, axis of symmetry, sketch

$y = ax^2 + bx + c$

$a = -\frac{1}{2}$, $b = 0$, $c = 0$

opens down since $a<0$ ∩

↓

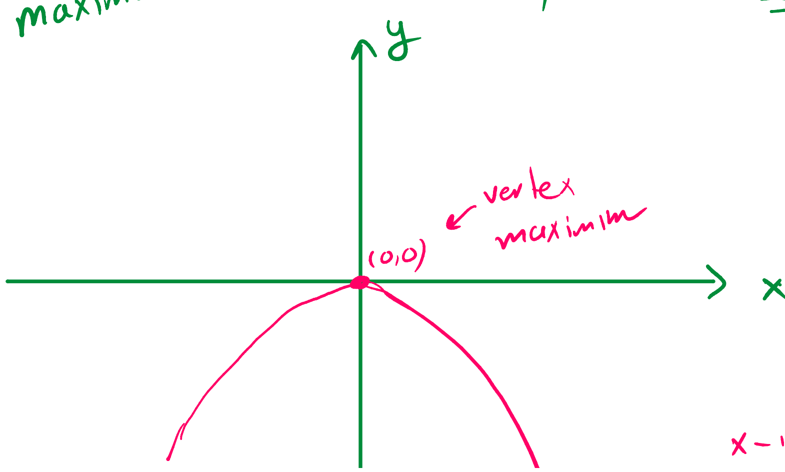
vertex is maximum

vertex $= (x,y) = \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

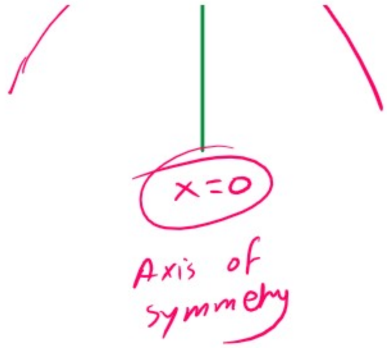
$= (0, f(0))$

$= (0, -\frac{1}{2}0^2)$

$= (0,0)$



x-intercept $x=0$



x-intercept $x = 0$

y-intercept $y = 0$

$y = -\frac{1}{2}x^2$

→ x-intercept $\Rightarrow y = 0 \Rightarrow -\frac{1}{2}x^2 = 0$

$x^2 = 0$

$\boxed{x = 0}$

→ y-intercept $\Rightarrow x = 0 \Rightarrow y = -\frac{1}{2}(0)^2$

$\boxed{y = 0}$

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③ $f(x) = 4x - 2x^2 - 4$

$y = ax^2 + bx + c$

$f(1) = 4(1) - 2(1)^2 - 4 = \cancel{4} - 2 - \cancel{4} = -2$

$a = -2$ → opens down → vertex maximum

$b = 4$

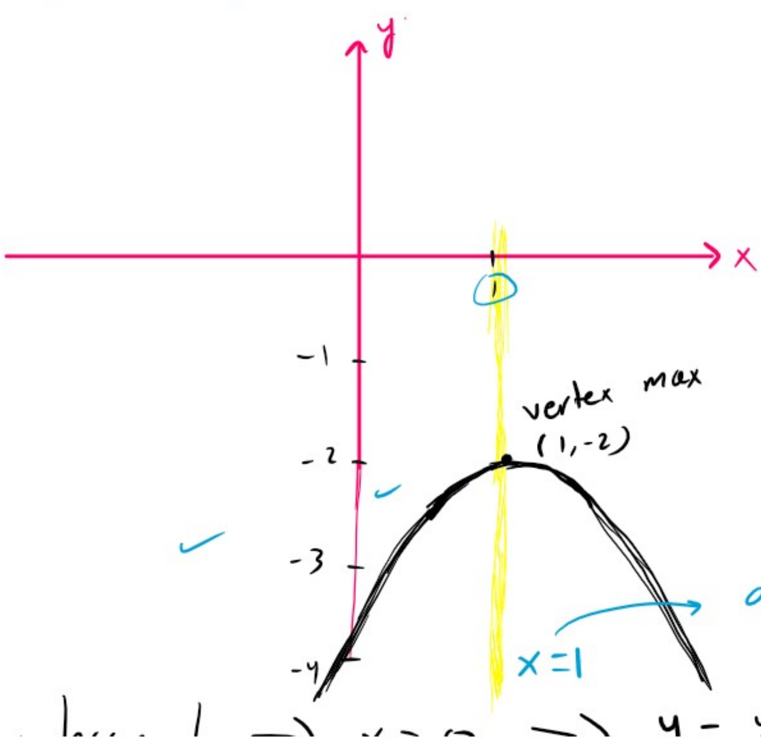
$c = -4$

$\text{vertex} = (x, y) = \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$= \left(\cancel{-}\frac{4}{\cancel{2(-2)}}, \right.$

$= (1, f(1))$

$= (1, -2)$



axis of symmetry

$y = 4(0) - 2(0)^2 - 4 = -4$

-4 $\quad x=1$

y - intercept $\Rightarrow x = 0 \Rightarrow y = 4(0) - 2(0)^2 - 4 = -4$

x - intercept $\Rightarrow y = 0 \Rightarrow 0 = 4x - 2x^2 - 4$

$\Delta = b^2 - 4ac$

$= (4)^2 - 4(-2)(-4)$

$= 16 - 4(8)$

$= 16 - 32$

$= -16$ ωL

$\Rightarrow$ no x - intercept

If $\Delta < 0$
then no x-intercepts

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③ $x^2 + y = 8 + 2x$

$\cancel{-x^2} \qquad\qquad -x^2$

$\boxed{y = ax^2 + bx + c}$

$y = \underbrace{-x^2 + 2x}_{} + 8$

$\Rightarrow f(1) = -(1)^2 + 2(1) + 8$
$= -1 + 2 + 8$
$= 9$

$\underbrace{a = -1}_{}$, $b = 2$, $c = 8$

$\downarrow$

parabola opens down

$\downarrow$

vertex maximum

Axis of symmetry

$x = -\frac{b}{2a}$

$= -\frac{2}{2(-1)}$

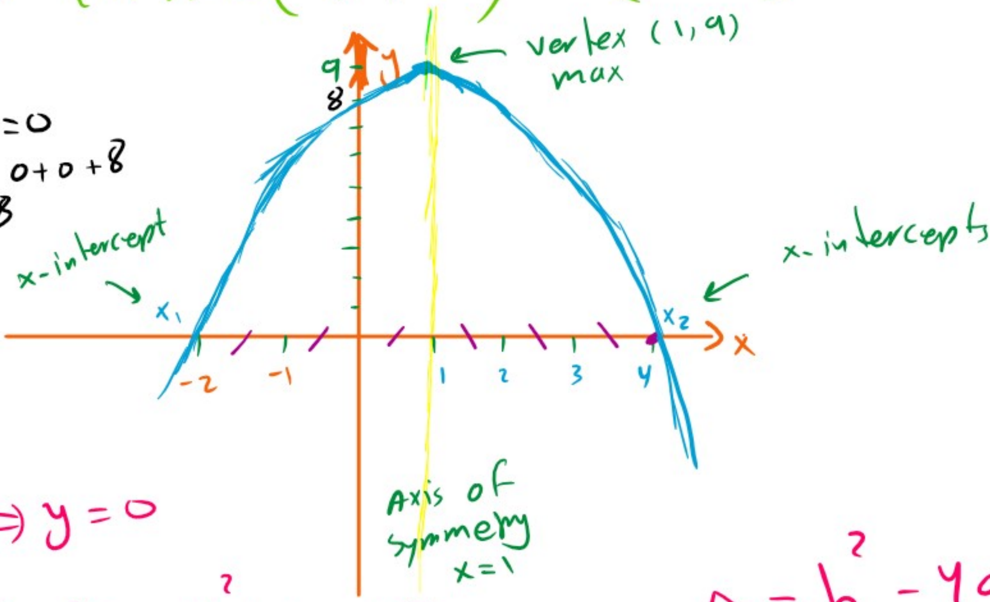
$x = 1$

vertex is $(x, y) = (1, f(1)) = (1, 9)$

vertex $(1, 9)$

vertex is $(x, y) = (1, f(1)) = (1, 9)$

y-intercept $\Rightarrow x = 0$
$\Rightarrow y = 0 + 0 + 8$
$y = 8$



x-intercepts $\Rightarrow y = 0$

$\Rightarrow 0 = -x^2 + 2x + 8$

$$D = b^2 - 4ac$$
$$= 2^2 - 4(-1)(8)$$
$$= 4 + 32$$
$$= 36 > 0$$

$\exists$ two roots

$$x_1 = \frac{-b + \sqrt{D}}{2a} = \frac{-(2) + \sqrt{36}}{2(-1)}$$
$$= \frac{-2 + 6}{-2} = \frac{4}{-2} = -2$$

$$x_2 = \frac{-b - \sqrt{D}}{2a} = \frac{-(2) - \sqrt{36}}{2(-1)} = \frac{-2 - 6}{-2} = \frac{-8}{-2} = 4$$

If $D > 0 \Rightarrow \exists$ two roots (x-intercept)

If $D < 0 \Rightarrow \exists$ no "

If $D = 0 \Rightarrow \exists$ only on x-intercept