1.5 : Random Variable of Discrete .

example :

Experiment : Toss a coin

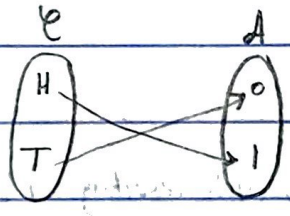
$\mathcal{C}$: sample space of experiment.

$\mathcal{C}$: $\{H, T\}$

$P(H) = \frac{1}{2}$

$P(T) = \frac{1}{2}$ } Fair coin

$H \rightarrow 1$, $T \rightarrow 0$



A : space (of Random Variable) . $A = \{0, 1\}$.

X : Random Variable .

$X : \mathcal{C} \rightarrow A$

$X(H) = 1$

$X(T) = 0$ } $\rightarrow$ $X(c) = x$ $\rightarrow$ observation .

$c \in \mathcal{C}$

$x \in A$

Random variable, sample point

Note : - A : discrete set

- X : discrete Random variable .

Def : consider a random experiment with a sample space $\mathcal{C}$. A function X which assigns to each $c \in \mathcal{C}$ one and only one real number $X(c) = x$ is called a Random variable.

→ The space of X is the set of real numbers $\mathcal{A} = \{x : x = X(c),\ c \in \mathcal{C}\}$.

Def : let $A \subset \mathcal{A}$

$$P_X(A) = P_r(X \in A) = P(C) \quad \text{where } C = \{c : c \in \mathcal{C} \text{ and } X(c) \in A\}.$$

- $P_X(A)$ is called the induced probability.

RMK : P_X is a probability set function.

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example 1 : cosider a seq. of indep. flips of a coin, each resulting in (H) or (T). بدي اضل أكرر التجربة حتى يطلع عندي جواب صحيح (H)

→ $P(H) = \frac{1}{2}$, $P(T) = \frac{1}{2}$

→ Random variable (X) : # of flip until you get a Head.

→ $\mathcal{C} = \{ H, TH, TTH, \ldots \}$ ---- نحصل على H من أول مرة او من ثاني مرة او ----

→ $\mathcal{A} = \{ 1, 2, 3, \ldots \}$ " "

so :

$$P_r(X=1) = P(H) = \left(\tfrac{1}{2}\right)^1 = \tfrac{1}{2}$$

$$P_r(X=2) = P(TH) = \tfrac{1}{2} \cdot \tfrac{1}{2} = \left(\tfrac{1}{2}\right)^2$$

$$P_r(X=3) = P(TTH) = \tfrac{1}{2} \cdot \tfrac{1}{2} \cdot \tfrac{1}{2} = \left(\tfrac{1}{2}\right)^3$$

$\vdots$

$$P_r(X=x) = \left(\tfrac{1}{2}\right)^x, \quad x = 1, 2, 3, \ldots$$

عبارة عن نقاط
finite
infinite

Def: let X be a Random variable with one dimensional space $\mathcal{A}$, suppose $\mathcal{A}$ a discrete set (consists of a countable number of points). let $f(x)$ be a function such that $f(x) > 0 \ \forall x \in \mathcal{A}$ and $\sum_{\mathcal{A}} f(x) = 1$.

Whenever a probability set function $P(A)$, $A \subset \mathcal{A}$ can be expressed in terms of $f(x)$ by: $P(A) = P_r(x \in A) = \sum_{A} f(x)$, Then X is called a Random variable of the discrete type and $f(x)$ is called the probability density function (p.d.f) of X.

Back to example 1: (Geometric distribution).

$\ldots$: $p(X = x) = \left(\frac{1}{2}\right)^x, \quad x = 1, 2, 3, \ldots$

$$f(x) = P_r(X = x) = \begin{cases} \left(\frac{1}{2}\right)^x, & x = 1, 2, \ldots \\ 0, & \text{elsewhere.} \end{cases}$$

$\rightarrow f(x) = \left(\frac{1}{2}\right)^x > 0, \quad x \in \mathcal{A}$

$\rightarrow f(x) = 0, \quad x \notin \mathcal{A}$.

$$\rightarrow \sum_{\mathcal{A}} f(x) = \underbrace{\sum_{x=1}^{\infty} \left(\frac{1}{2}\right)^x}_{\text{Geometric series}} = \frac{\frac{1}{2}}{1 - \frac{1}{2}} = \frac{1/2}{1/2} = 1 \checkmark$$

example 2: consisting bo fuses.

Five of these fuses are choosen at Random and tested.

If all 5 blows at the correct amperage so bt accepted.

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let the Random variable X be the number of defective fuses ---.

so The p.d.f of X:

$$f(x) = \begin{cases} \dfrac{\binom{20}{x}\binom{80}{5-x}}{\binom{100}{5}}, & x = 1,2,3,4,5. \\ 0, & \text{elsewhere}. \end{cases}$$

→ The space of X is $A = \{0,1,2,3,4,5\}$.

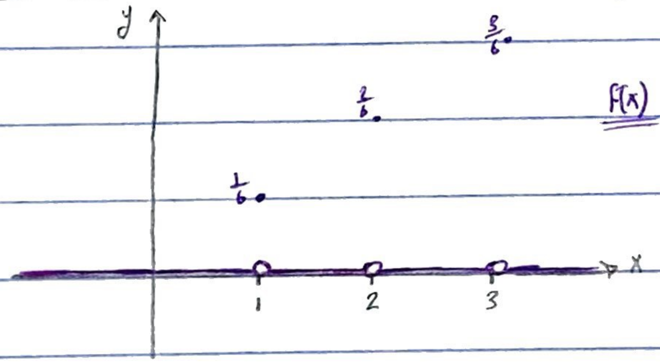
Def: let X be a Random variable with a p.d.f $f(x)$. let $A = (-\infty, x] \subset \mathcal{A}$, $P(A) = P_r(x \in A) = P_r(X \leq x)$, we say that $F(x) = P_r(X \leq x)$ is the cumulative distribution function (C.D.F) OR distribution function.

Inparticular, if X is a discrete random variable, then $F(x) = \sum_{w \leq x} f(w)$.

Example 3:

X discrete random variable with $f(x) = \begin{cases} \frac{x}{6} & , x = 1, 2, 3 \\ 0 & , \text{elsewhere} \end{cases}$

Find C.D.F $F(x)$:



f(x) بكل مكان 0 ما عدا 1,2,3

هسا بطلع F(x) كيف؟؟

$$F(x) = \begin{cases} 0 & , x < 1 \\ \frac{1}{6} & , 1 \leq x < 2 \\ \frac{3}{6} & , 2 \leq x < 2 \\ 1 & , 3 \leq x \end{cases}$$

بلش من $-\infty$ لحتى أوصل 1 بكون الاقتران قيمته صفر

هسا عند 1 لـ 2 ((1 مغلق، 2 مفتوح)) كم قيمة القفزة الموجودة $\frac{1}{6}$

هسا بدخل 2 و مفتوح عند 3 ح يكون عبارة عن $\frac{2}{6} + \frac{1}{6} = \frac{3}{6}$

هسا لحتى أوصل 3 ((داخلة)) ح يكون $\frac{3}{6} + \frac{3}{6} = 1$