15.1 (Q15) $I = \iint_R xy\cos y\, dA$, R: $-1 \le x \le 1$
$0 \le y \le \pi$

$$I = \int_0^{\pi} \boxed{\int_{-1}^{1} xy\cos y\, dx}\, dy = \int_0^{\pi} \left(\frac{x^2}{2} y\cos y \Big]_{-1}^{1} \right) dy$$

$$= \int_0^{\pi} \left(\frac{1}{2} y\cos y - \frac{1}{2} y\cos y \right) dy = \int_0^{\pi} 0\, dy = 0$$

(Q18) $I = \iint_R xy\, e^{xy^2}\, dA$, R: $0 \le x \le 2$, $0 \le y \le 1$

$$I = \int_0^2 \left(\int_0^1 xy\, e^{xy^2}\, dy \right) dx$$

, Let $u = xy^2$
$\frac{du}{dy} = 2xy$
$\therefore \frac{du}{2} = xy\, dy$

$$= \int_0^2 \left(\int_0^x \frac{e^u}{2}\, du \right) dx$$

$$= \int_0^2 \left(\frac{1}{2} e^u \Big]_0^x \right) dx = \frac{1}{2} \int_0^2 (e^x - 1)\, dx$$

$$= \frac{1}{2} \left[e^x - x \right]_0^2 = \frac{1}{2}\left(e^2 - 2 - 1\right) = \frac{e^2}{2} - \frac{3}{2}.$$

$\left(\frac{2}{3}\right)(102)$

15.1

(Q19) $I = \iint_R \frac{x y^3}{x^2+1}\, dA \quad , \quad R: 0 \le x \le 1 \;,\; 0 \le y \le 4$

$$I = \int_0^1 \left(\int_0^4 \left(\frac{x}{x^2+1}\, y^3 \right) dy \right) dx$$

$$= \int_0^1 \left[\frac{x}{x^2+1} \left(\frac{y^4}{4} \right) \right]_0^4 dx = \frac{32}{2} \int_0^1 \frac{2x}{x^2+1}\, dx$$

$$= 32 \left(\ln (x^2+1) \Big|_0^1 \right) = 32 \ln 2$$

(Q27) Find the Volume of the region bounded above by the Surface $z = 2 \sin x \cos y$ and below by the rectangle $R: 0 \le x \le \frac{\pi}{2} \;,\; 0 \le y \le \frac{\pi}{4}$.

Sol: Volume $= \iint_R z\, dA = \int_0^{\frac{\pi}{4}} \left(\int_0^{\frac{\pi}{2}} 2 \sin x \cos y\, dx \right) dy$

$$= \int_0^{\frac{\pi}{4}} \left(-2 \cos x \cos y \right)_0^{\frac{\pi}{2}} dy = \int_0^{\frac{\pi}{4}} (\overset{\cos\frac{\pi}{2}=0}{0} + 2 \overset{\cos 0 = 1}{\cos y})\, dy$$

$$= 2 \int_0^{\frac{\pi}{4}} \cos y\, dy = 2 \left(\sin y \right)_0^{\frac{\pi}{4}} = 2 \cdot \frac{\sqrt{2}}{2} = \sqrt{2}.$$