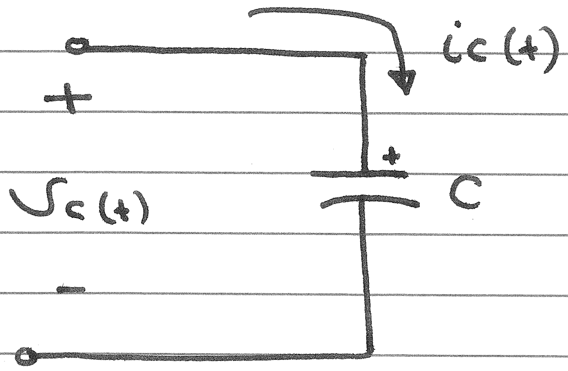
Chapter 7

"Response of First-Order RL and RC Circuits"

Capacitors and Inductors

- Resistors are passive elements which dissipate energy only.
- Two important passive Linear circuit elements : Capacitor , and Inductor
- Capacitors and Inductors do not dissipate but store energy , which can be retrieved at a Later time
- Capacitors and Inductors are called Storage elements.
- $W_c(t) = \frac{1}{2} C V_c^2(t)$
- $W_L(t) = \frac{1}{2} L i_L^2(t)$

Some of the important characteristics of a Capacitor



$$i_c(t) = C \frac{dv_c(t)}{dt}$$

$$v_c(t) = v_c(0^-) + \frac{1}{C}\int_0^t i_c(t)\,dt \quad , \text{ for } t \geq 0$$

1. The current through a capacitor is zero if the voltage across it is not changing with time. A Capacitor is therefore an open Circuit to dc.

2. A finite amount of energy can be stored in a Capacitor even if the current through the Capacitor is Zero.

3. The Capacitor never dissipate energy, but only store it.

$$v_c(t) = v_c(0^-) + \frac{1}{C}\int_{0^-}^{t} i_c(t)\,dt \quad , \text{ for } t \geq 0$$

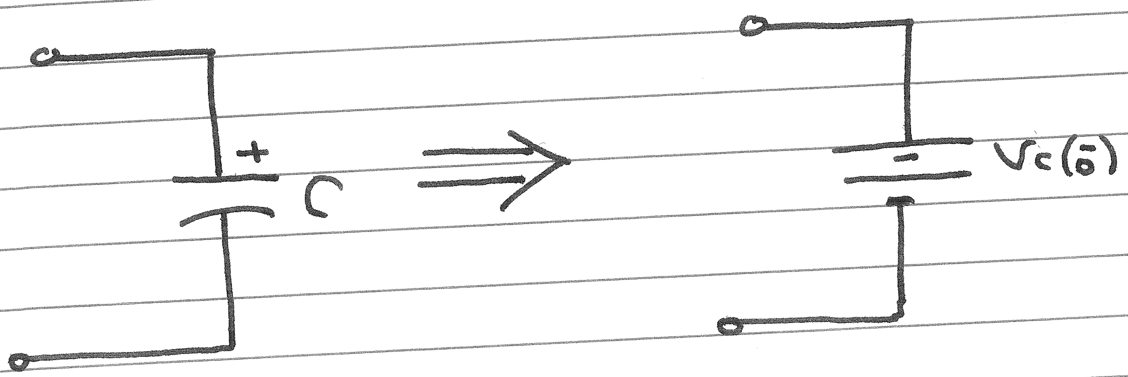
at $t = 0^+$

$$v_c(0^+) = v_c(0^-) + \frac{1}{C}\int_{0^-}^{0^+} i_c(t)\,dt$$

$$v_c(0^+) = v_c(0^-)$$

4. It is impossible to change the voltage across a capacitor by a finite amount in zero time, for this requires an infinite current through the capacitor

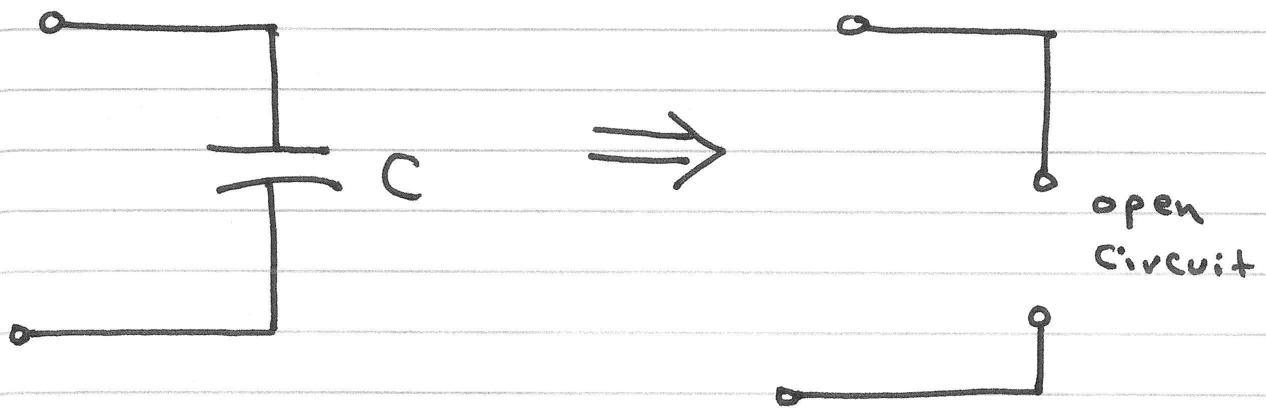
5. At $t = 0^+$



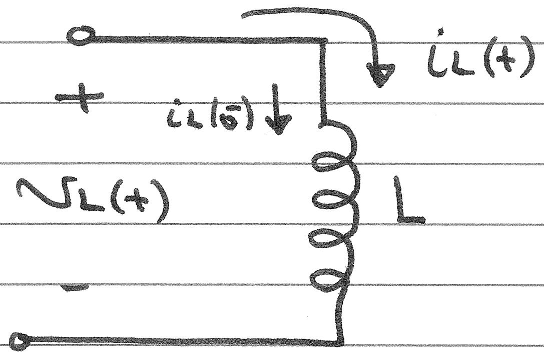
$$i_C(t) = C \frac{dv_C(t)}{dt}$$

A capacitor is therefore an open circuit to dc

At $t = 0^-$, and $t = \infty$ (After the change)



Some of the important characteristics of an inductor



$$v_L(t) = L \frac{d i_L(t)}{dt}$$

$$i_L(t) = i_L(0^-) + \frac{1}{L}\int_{0^-}^{t} v_L(t)\,dt \quad ; \text{for } t \geq 0$$

1. There is no voltage across an inductor if the Current through it is not changing with time.

   An inductor is therefore a short circuit to dc

2. A finite amount of energy Can be stored in an inductor even if the voltage across the inductor is zero.

3. The inductor never dissipate energy, but only

Store it

4. $i_L(t) = i_L(0^-) + \frac{1}{L}\int_{0^-}^{t} v_L(t)\,dt \quad ; \text{for } t \geq 0$

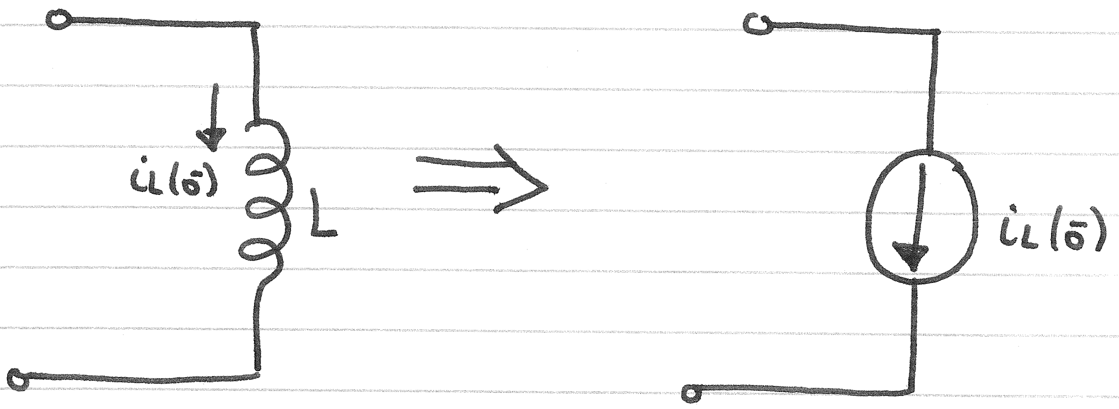
at $t = 0^+$

$$i_L(0^+) = i_L(0^-) + \frac{1}{L}\int_{0^-}^{0^+} v_L(t)\,dt$$

$\therefore \quad i_L(0^+) = i_L(0^-)$

It is impossible to change the current through an inductor by a finite amount in zero time, for this requires an infinite voltage across the inductor.

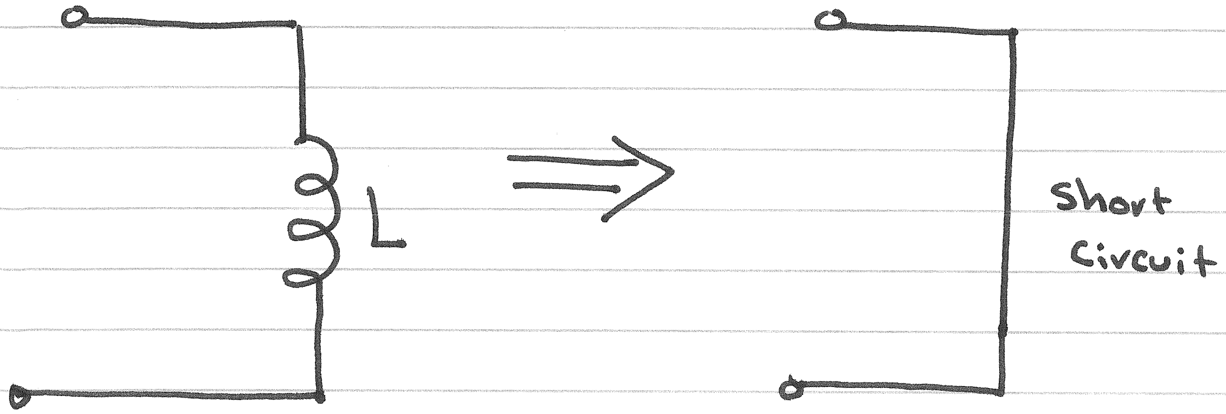
At $t = 0^+$



$$v_L(t) = L \frac{di_L(t)}{dt}$$

An inductor is therefore a short circuit to dc

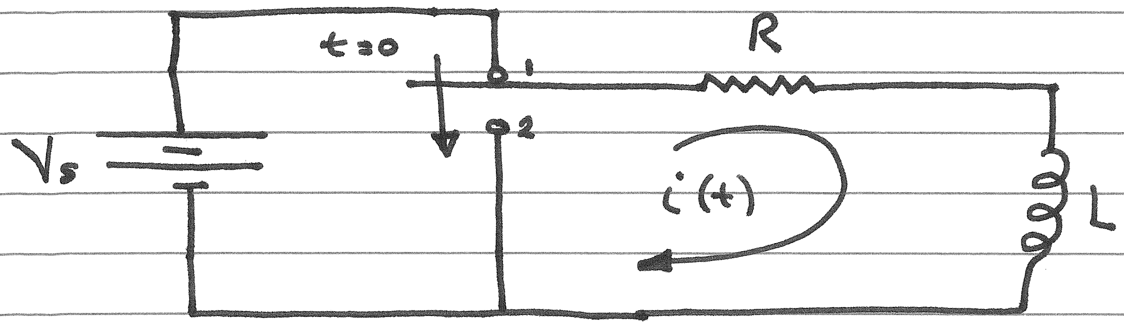
At $t = 0^-$, and $t = \infty$ (After the change)



First order Circuit

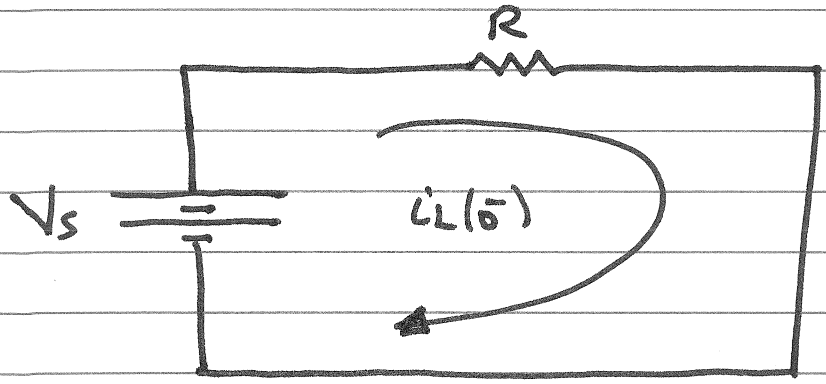
- A first-order Circuit can only contain one energy storage element (a Capacitor or an inductor) or a Combination of Capacitors or inductors that can be reduced to one Capacitor or inductor
- The Circuit will also Contain one or more resistances
- A first-order Circuit is characterized by a first-order differential equation

Natural Response of First-Order Circuit



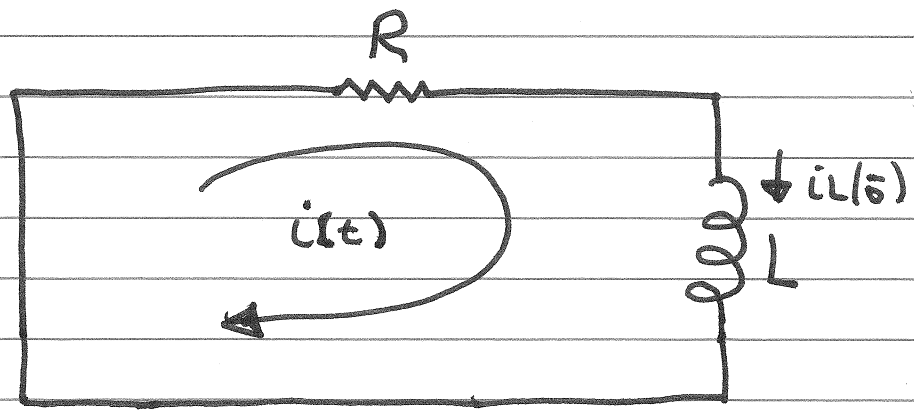
Find $i(t)$ for $t > 0$

1) For $t < 0$; $t = 0^-$

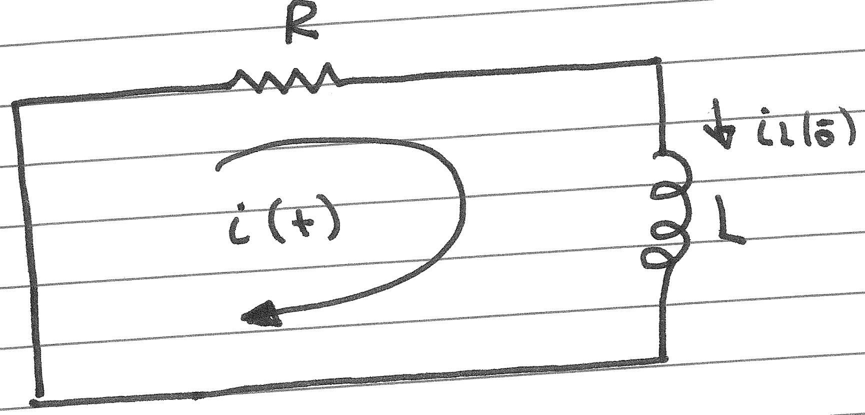


$$i_L(0^-) = \frac{V_s}{R}$$

2) For $t > 0$



For $t > 0$



KVL :

$$R\,i(t) + L\frac{di(t)}{dt} = 0$$

homogeneous first order differential equation

$$i(t) = A e^{st} \quad \text{for } t > 0$$

$$RAe^{st} + LAse^{st} = 0$$

$$Ae^{st}(R + LS) = 0$$

$$\therefore S = -\frac{R}{L}$$

To find A :

$$i(t) = A e^{st} \quad t > 0$$

$$i(0^+) = A$$

$$i(0^+) = i_L(0^+) = i_L(0^-)$$

$$\therefore A = i_L(0^-) = \frac{V_s}{R}$$

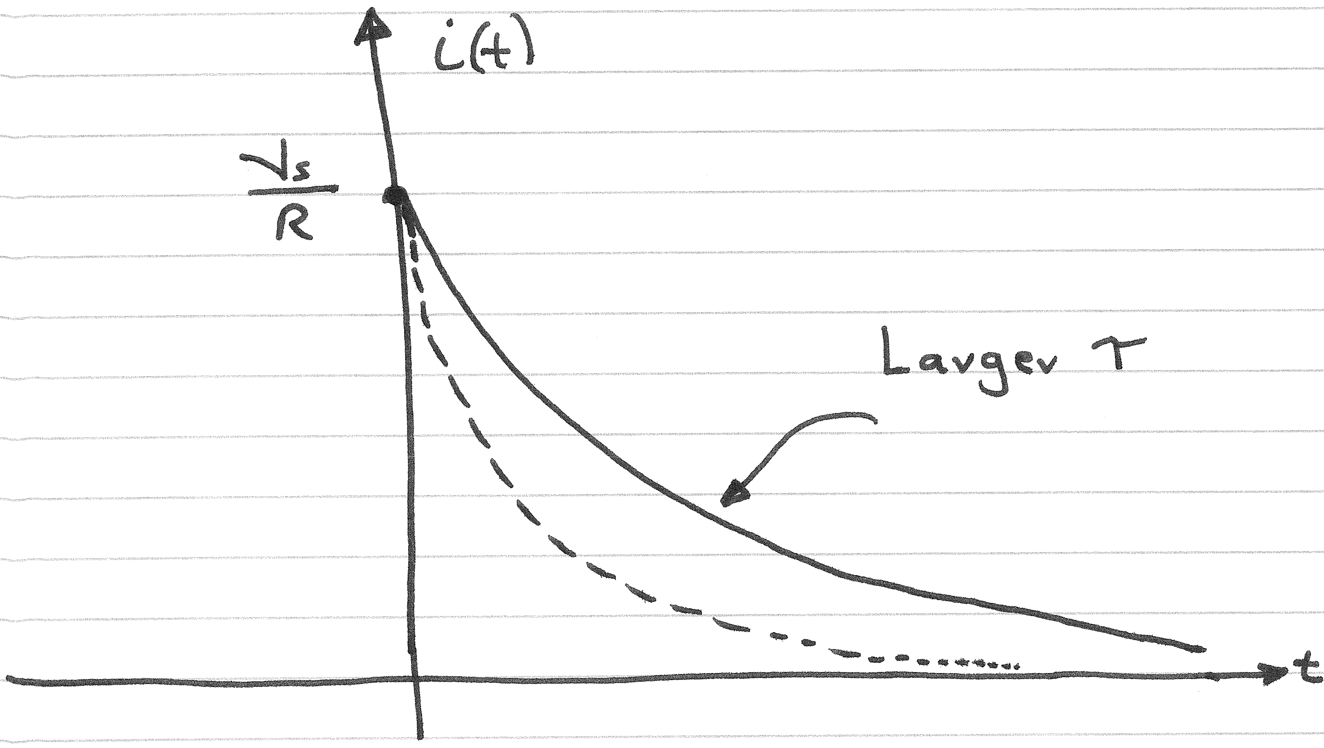
$$\therefore \quad i(t) = \frac{V_s}{R} e^{-\frac{R}{L}t} \qquad t > 0$$

let $\tau = \frac{L}{R}$

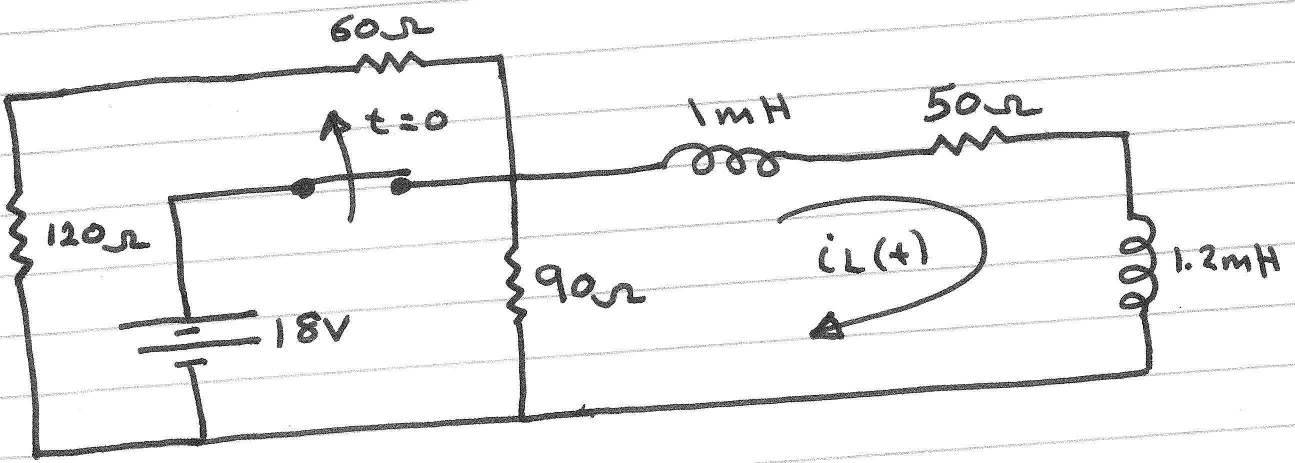
τ = Time Constant

$$\therefore \quad i(t) = \frac{V_s}{R} e^{-\frac{t}{\tau}} \qquad t > 0$$

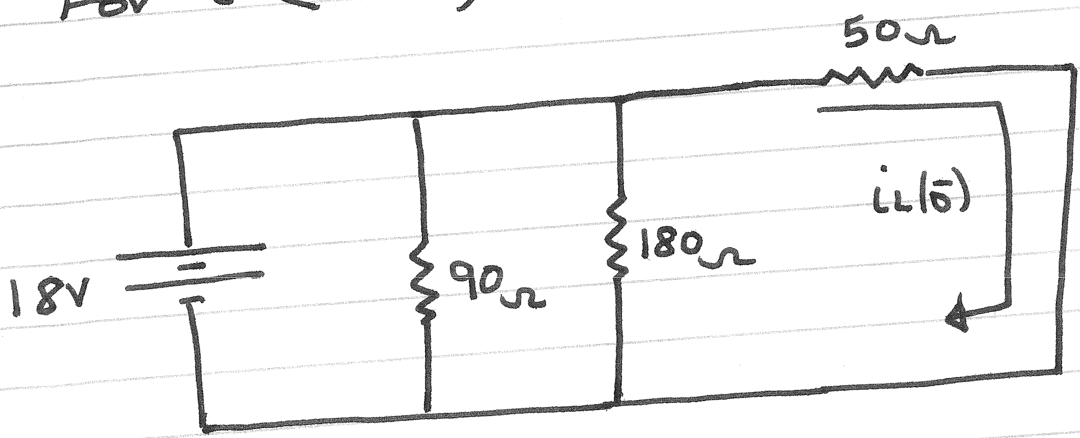
t	$i(t)$
0	$\frac{V_s}{R}$
τ	$0.37 \frac{V_s}{R}$
2τ	$0.14 \frac{V_s}{R}$
3τ	$0.05 \frac{V_s}{R}$
4τ	$0.018 \frac{V_s}{R}$
5τ	$0.0067 \frac{V_s}{R}$


i(t)
Vs
R
Larger τ
t

Find $i_L(t)$ for $t > 0$

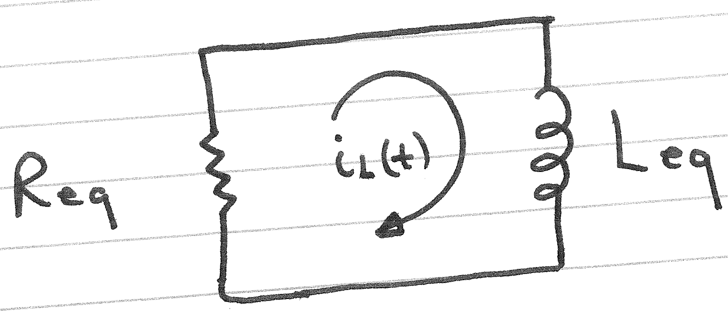


1) For $t < 0$; $t = 0^-$



$$i_L(0^-) = \frac{18V}{50\Omega} = 0.36\,A$$

2) For $t > 0$



$$R_{eq} = 90\Omega \| 180\Omega + 50\Omega = 110\Omega$$

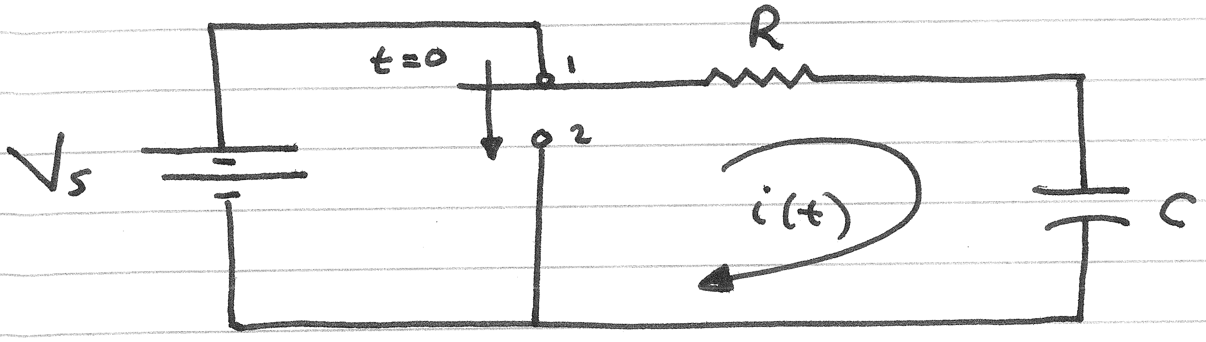
$$L_{eq} = 1mH + 1.2mH = 2.2mH$$

$$\tau = \frac{L_{eq}}{R_{eq}} = 20\mu s$$

$$\therefore i_L(t) = A e^{-t/\tau} \quad \text{for } t > 0$$

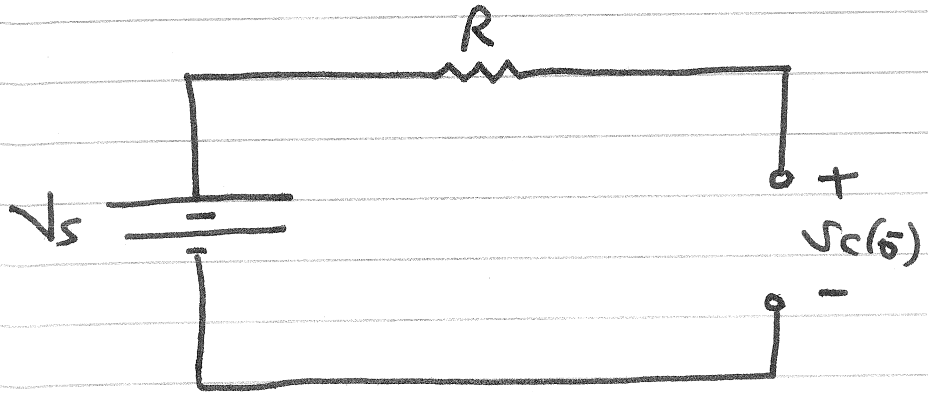
$$i_L(t) = 0.36 e^{-50,000t} \quad \text{for } t > 0$$

RC Circuit



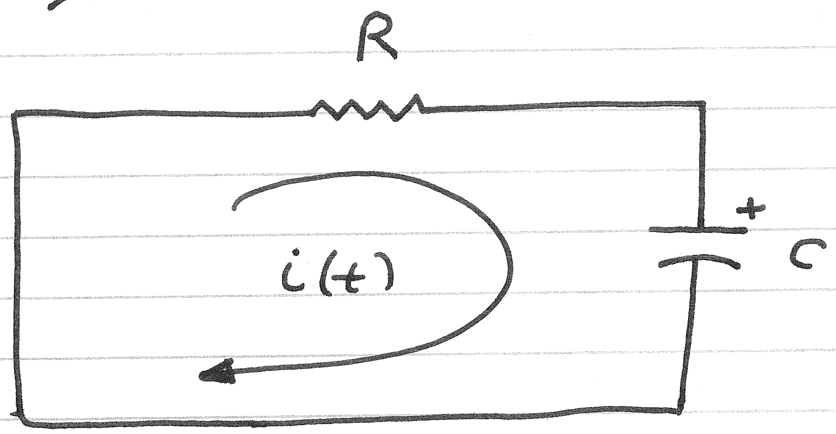
Find $i(t)$ for $t > 0$

1) For $t < 0$; $t = 0^-$

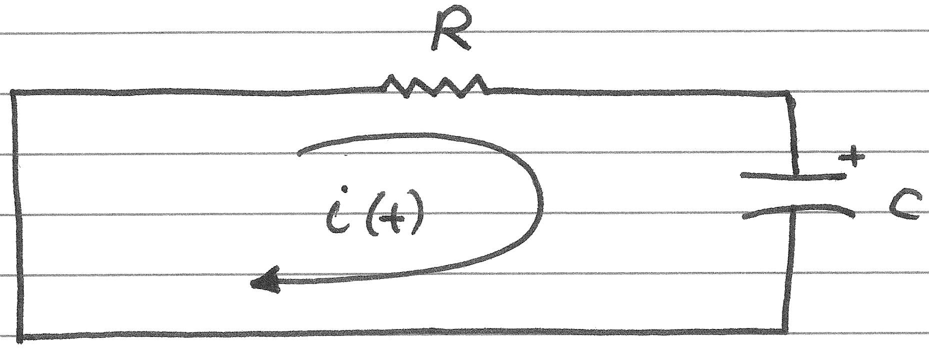


$V_c(0^-) = V_s$

2) For $t > 0$



For $t > 0$



KVL :

$$R i(t) + V_c(0^-) + \frac{1}{C}\int_{0^-}^{t} i(t)\, dt = 0 , \; t > 0$$

$$R \frac{di(t)}{dt} + \frac{1}{C} i(t) = 0$$

homogeneous First order differential equation

$$\therefore \; i(t) = A e^{st} \qquad t > 0$$

$$R A s e^{st} + \frac{1}{C} A e^{st} = 0$$

$$A e^{st} \left(Rs + \frac{1}{C} \right) = 0$$

$$\therefore \; s = -\frac{1}{RC}$$

$$\therefore \; i(t) = A e^{-\frac{1}{RC} t} \qquad t > 0$$

let $\tau = RC$ = time constant

$$\therefore \; i(t) = A e^{-t/\tau} \qquad t > 0$$

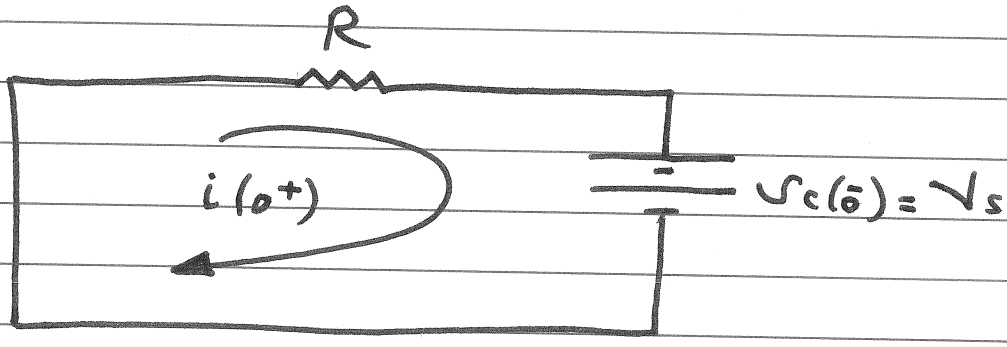
To find A

$$i(t) = A e^{-t/\tau} \qquad t > 0$$

$$i(0^+) = A$$

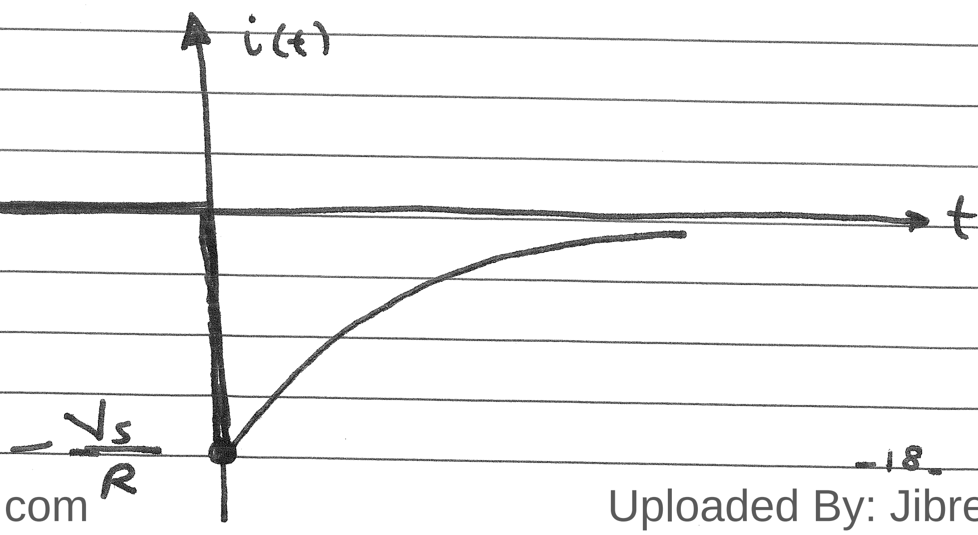
To find $i(0^+)$

at $t = 0^+$

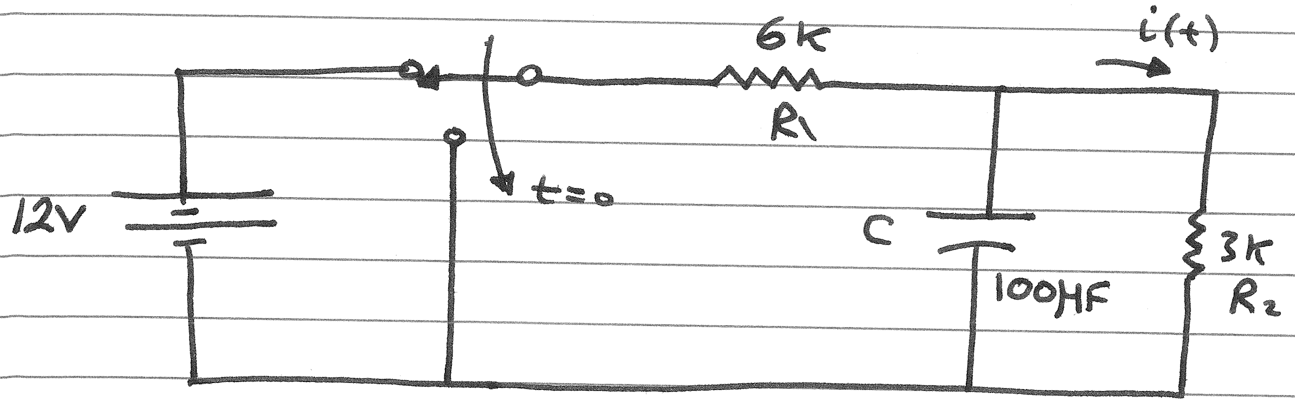


$$i(0^+) = -\frac{v_c(0^-)}{R} = -\frac{V_s}{R}$$

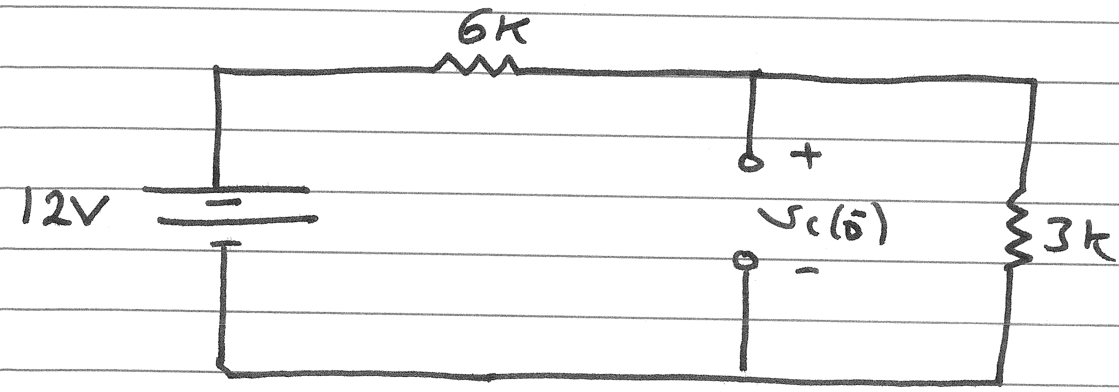
$$\therefore \; i(t) = -\frac{V}{R} e^{-t/\tau}$$



Calculate $v_c(t)$ and $i(t)$ for $t > 0$

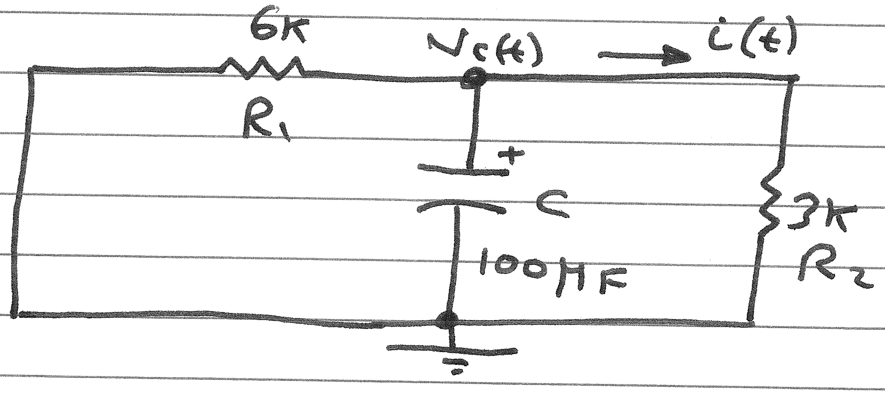


1) For $t < 0$; $t = 0^-$



$$v_c(0^-) = \frac{3k}{3k + 6k} 12V = 4V$$

2) For $t > 0$



KCL :

$$\frac{v_c(t)}{6k} + \frac{v_c(t)}{3k} + C\frac{dv_c(t)}{dt} = 0$$

$$\frac{dv_c(t)}{dt} + 5v_c(t) = 0$$

$$\therefore \; v_c(t) = A e^{-t/\tau} \qquad t > 0$$

$$\tau = R_{eq} C$$

$$R_{eq} = 6k \| 3k = 2k$$

$$C = 100 \mu F$$

$$\tau = R_{eq} C = 0.2 \text{ sec}$$

$$v_c(t) = A e^{-t/0.2} \qquad t > 0$$

<u>To find A</u>

$$v_c(0^+) = A = v_c(0^-) = 4V$$

$$\therefore \; v_c(t) = 4 e^{-5t} \qquad t > 0$$

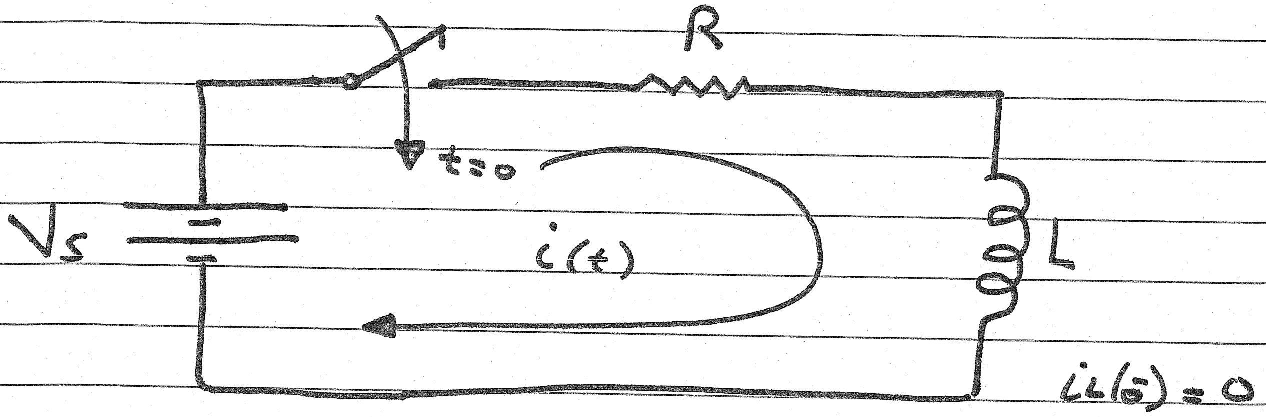
$$i(t) = \frac{v_c(t)}{R_2}$$

$$i(t) = \frac{4}{3} e^{-5t} \text{ mA} \qquad t > 0$$

The step Response of RC and RL Circuits

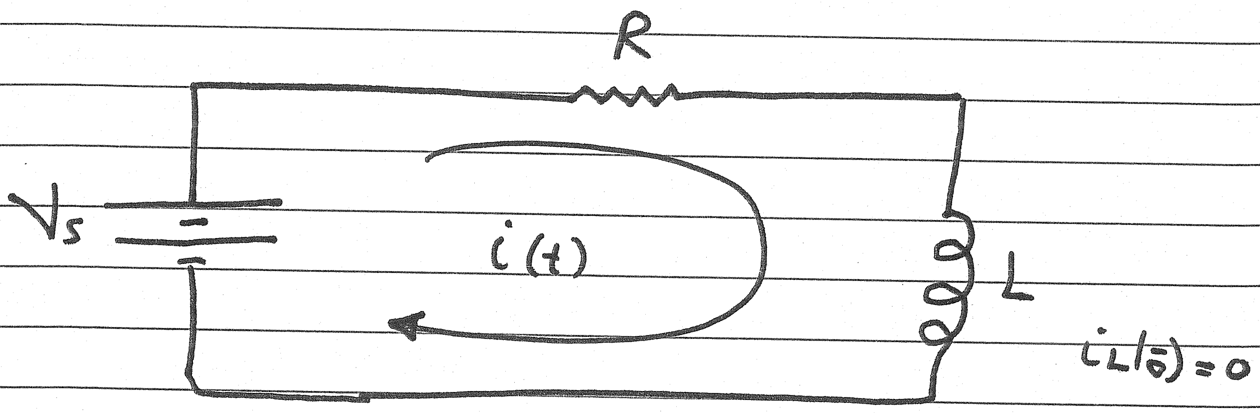
The response of a circuit to the sudden application of a constant voltage or current source is referred to as the step response of the circuit.

The step response of an RL Circuit



Find $i(t)$ for $t > 0$

For $t > 0$



KVL :

$$V_s = R\,i(t) + L\,\frac{di(t)}{dt} \qquad t > 0$$

non homogenous First order differential equation

$\therefore$

$$i(t) = i_n(t) + i_f(t)$$

$i_n(t) \equiv$ natural response

$i_f(t) \equiv$ forced response

<u>To find $i_f(t)$</u>

let $i_f(t) = K$

$$V_s = R\,i(t) + L\,\frac{di(t)}{dt}$$

$$V_s = RK + L(0)$$

$$V_s = RK$$

$$\therefore K = \frac{V_s}{R} = i_f(t)$$

now

$$i(t) = i_n(t) + i_f(t) \qquad t > 0$$

$$i(t) = A e^{-t/\tau} + K \qquad t > 0$$

$$\tau = \frac{L}{R}$$

$$i(t) = A e^{-t/\tau} + \frac{V_s}{R} \qquad t > 0$$

To find A

$$i(t) = \frac{V_s}{R} + A e^{-t/\tau} \qquad t > 0$$

$$i(0^+) = \frac{V_s}{R} + A$$

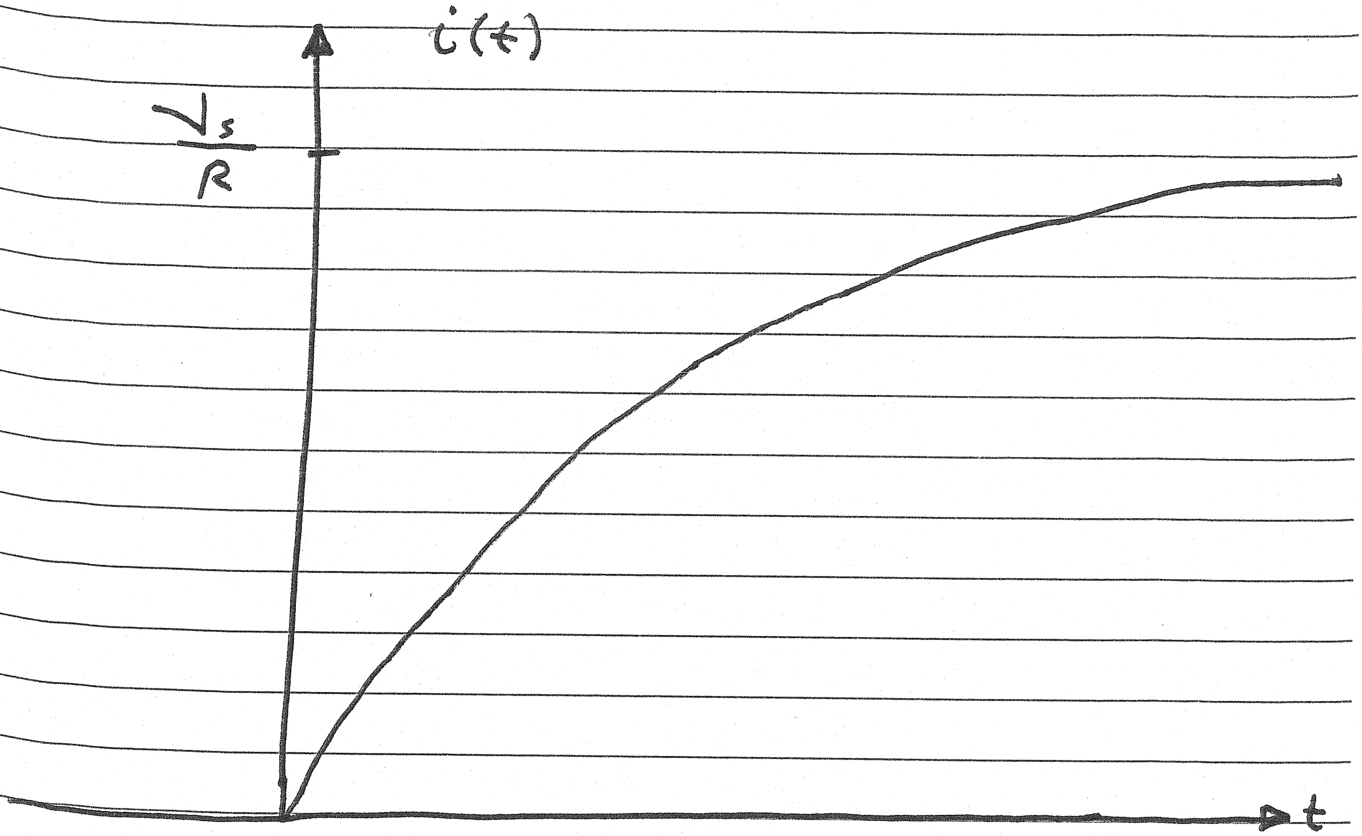
But $i(0^+) = i_L(0^+) = i_L(0^-) = 0$

$$\therefore A = -\frac{V_s}{R}$$

$$\therefore i(t) = \frac{V_s}{R} - \frac{V_s}{R} e^{-t/\tau} \qquad t \geq 0$$

$$i(t) = \frac{V_s}{R}\left(1 - e^{-t/\tau}\right) \qquad t \geq 0$$

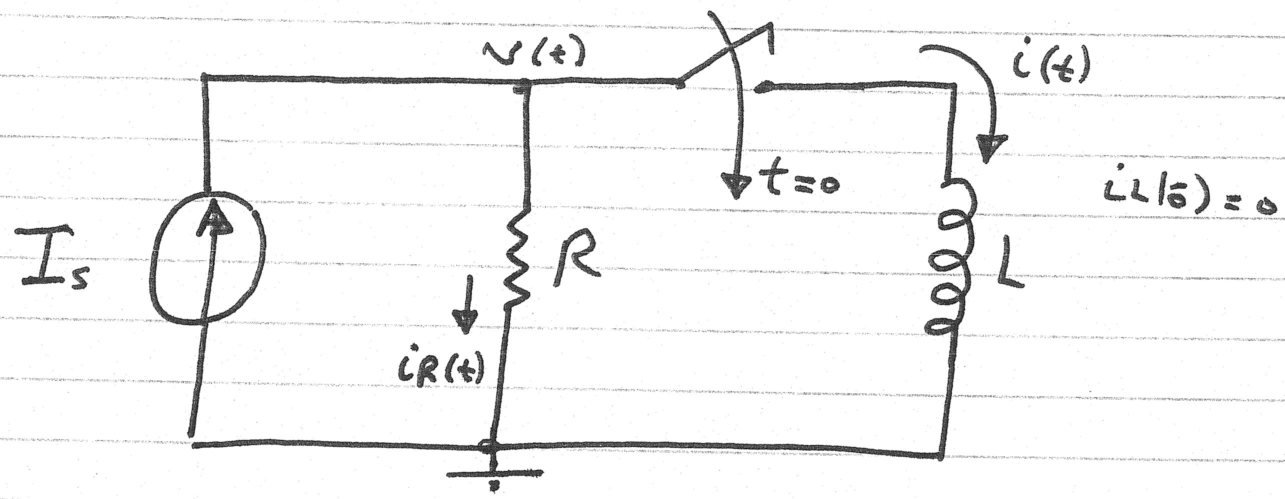
t	$i(t)$
0	0
τ	$0.63 \frac{V_s}{R}$
2τ	$0.86 \frac{V_s}{R}$
3τ	$0.95 \frac{V_s}{R}$
4τ	$0.98 \frac{V_s}{R}$
5τ	$0.99326 \frac{V_s}{R}$



$$i(t) = \frac{V_s}{R}\left(1 - e^{-t/\tau}\right)$$

$$\tau = \frac{L}{R}$$

Find $i(t)$ for $t > 0$



For $t > 0$

KCL :

$$I_s = i_R(t) + i(t) \qquad t > 0$$

$$I_s = \frac{v(t)}{R} + i(t) \qquad t > 0$$

$$v(t) = v_R(t) = v_L(t) = L\frac{di(t)}{dt}$$

$$I_s = \frac{L}{R}\frac{di(t)}{dt} + i(t) \qquad t > 0$$

$$\therefore\ i(t) = i_n(t) + i_f(t)$$

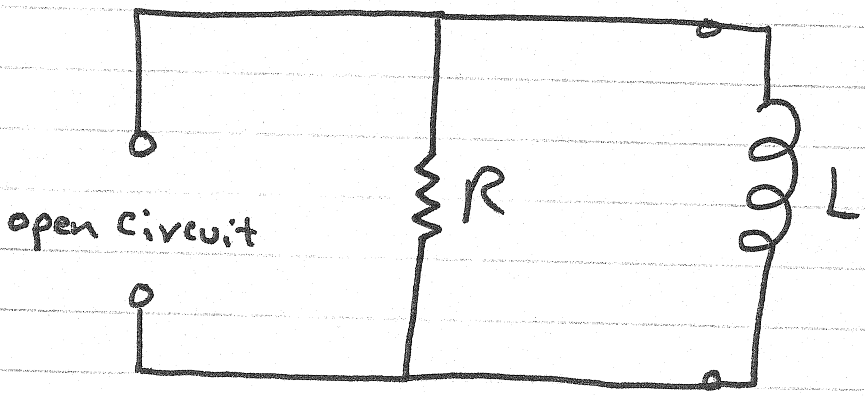
let $i_f(t) = K$

$$I_s = \frac{L}{R} \cdot 0 + K$$

$$\therefore\ K = I_s = i_f(t)$$

$$\tau = \frac{L}{R_{eq}}$$

R_{eq} = The Thevenin resistance seen by the inductor



$\therefore R_{eq} = R$

$i(t) = K + A e^{-t/\tau}$; $t > 0$

$i(t) = I_s + A e^{-t/\tau}$; $t > 0$

<u>To find A</u>

$i(0^+) = i_L(0^+) = i_L(0^-) = 0$

$i(0^+) = I_s + A = i_L(0^-) = 0$

$\therefore A = -I_s$

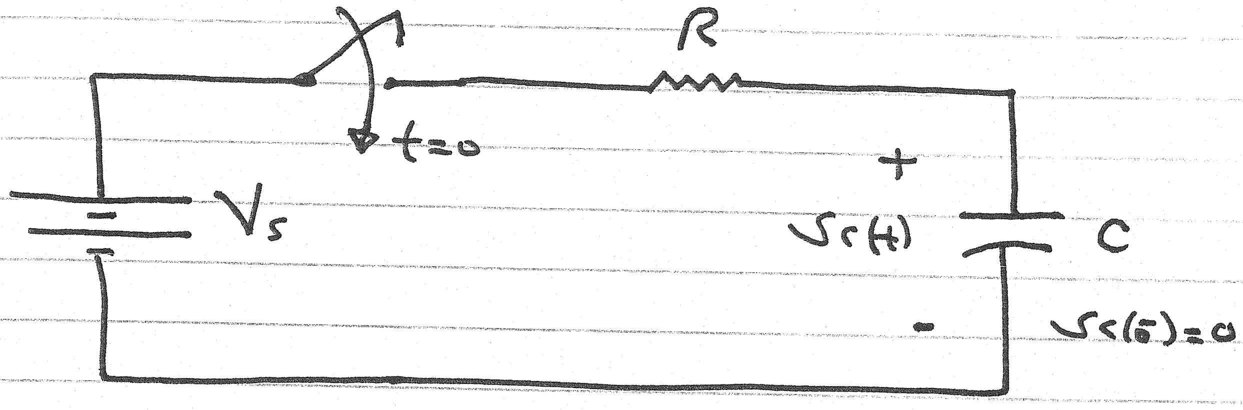
$$\therefore\; i(t) = i_n(t) + i_f(t) \quad ; t > 0$$

$$i(t) = A e^{-t/\tau} + I_s \quad ; t > 0$$

$$i(t) = -I_s e^{-t/\tau} + I_s \quad ; t > 0$$

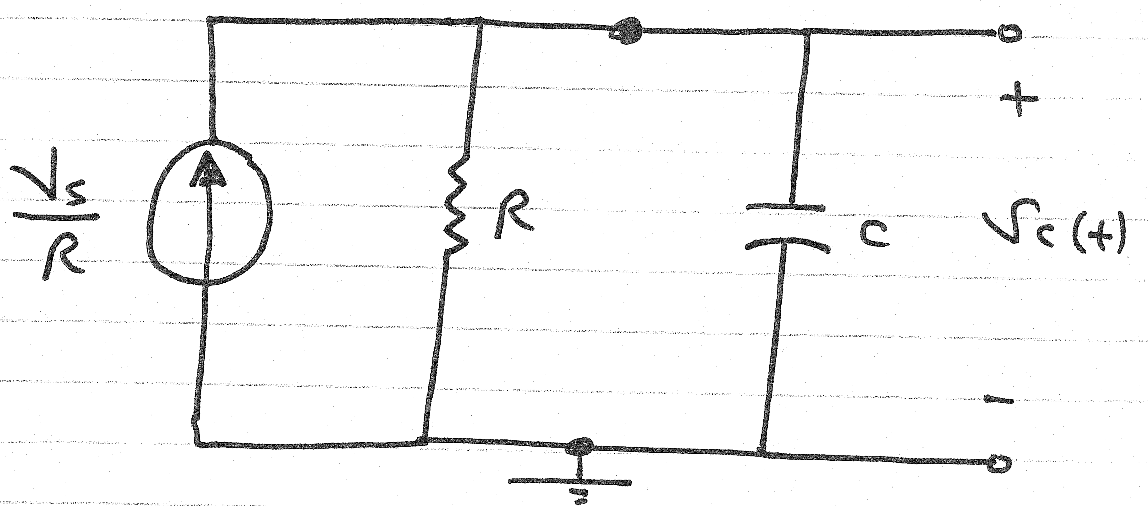
$$\therefore\; i(t) = I_s \left(1 - e^{-t/\tau}\right) \quad t \geq 0$$

The step response of an RC Circuit



Find $v_c(t)$ for $t > 0$

For $t > 0$



KCL :

$$\frac{V_s}{R} = i_R(t) + i_c(t) \qquad t > 0$$

$$\frac{V_s}{R} = \frac{v_c(t)}{R} + C\,\frac{dv_c(t)}{dt} \qquad t > 0$$

$$\frac{V_s}{R} = \frac{V_c(t)}{R} + C\frac{dV_c(t)}{dt}$$

First order nonhomogenous differential equation

$\therefore$

$$V_c(t) = V_{cn}(t) + V_{cf}(t) \qquad t > 0$$

$$V_c(t) = Ae^{-t/\tau} + K$$

<u>To find K</u>

$$\frac{V_s}{R} = \frac{K}{R} + 0$$

$$\therefore K = V_s$$

$$\therefore V_c(t) = Ae^{-t/\tau} + V_s \qquad t > 0$$

$$\tau = R_{eq}C$$

$$R_{eq} = R$$

<u>To find A</u>

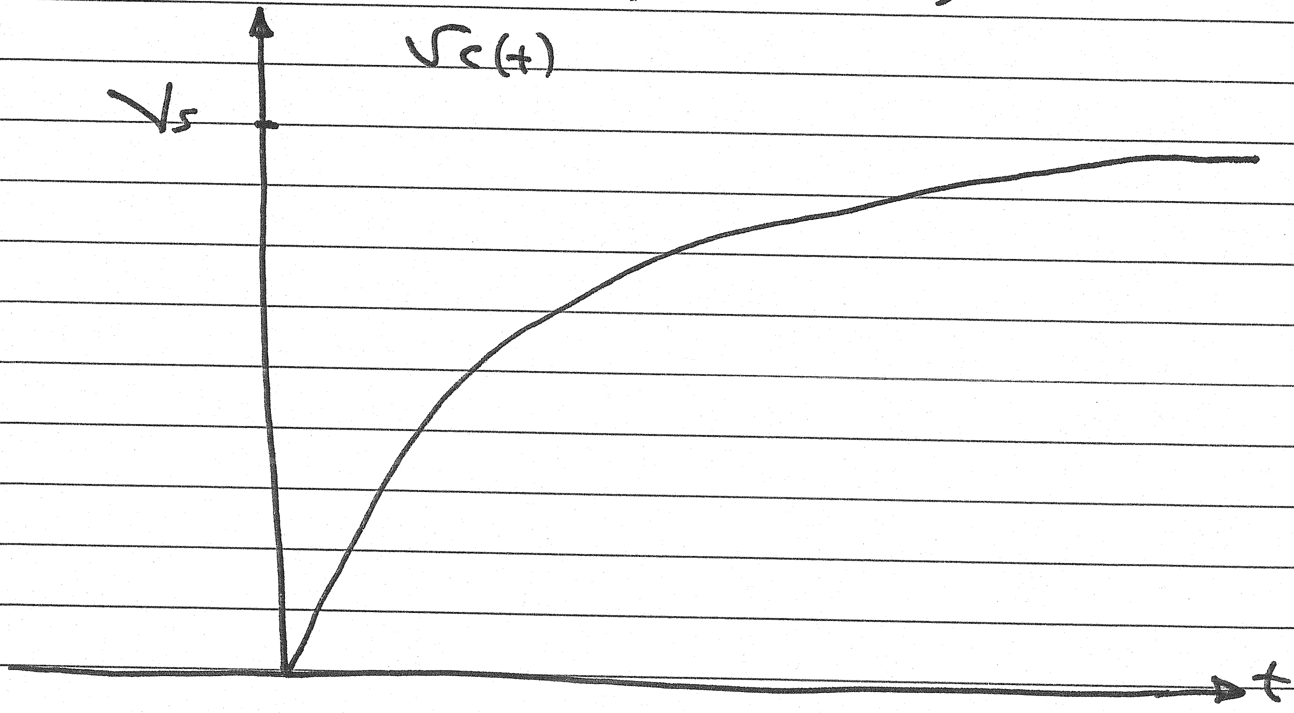
$$V_c(0^+) = V_c(0^-) = 0$$

$$v_c(0^+) = K + A = v_c(0^-) = 0$$

$$\therefore A = -K = -V_s$$

$$\therefore v_c(t) = V_s - V_s e^{-t/\tau} \qquad t \geq 0$$

$$v_c(t) = V_s \left(1 - e^{-t/\tau}\right) \qquad t \geq 0$$



When all independent sources are Constant, the response of the First order circuit has the form

$$r(t) = r_n(t) + K \qquad t>0$$

$$r(t) = A e^{-t/\tau} + K \qquad t>0$$

$$r(\infty) = K$$

$$\therefore\ r(t) = A e^{-t/\tau} + r(\infty) \qquad t>0$$

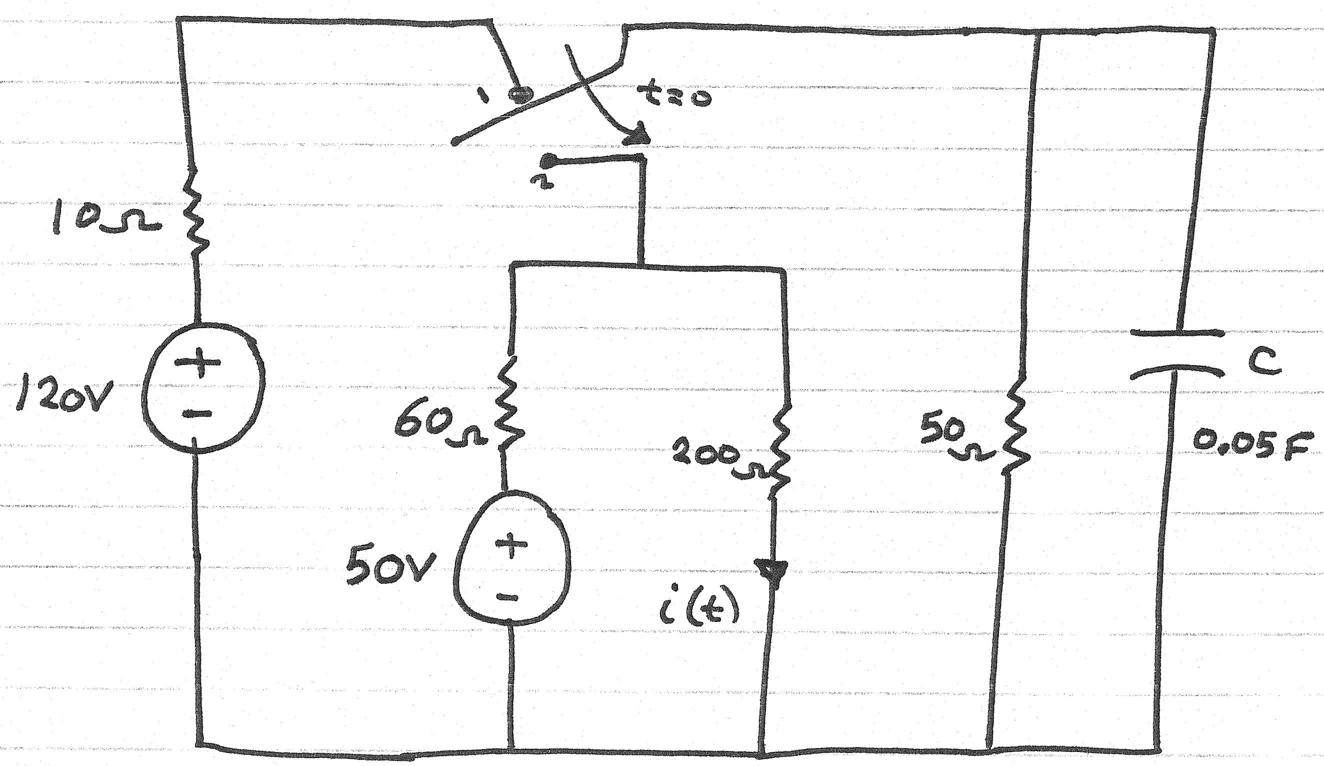
$$r(0^+) = A + r(\infty)$$

$$\therefore\ A = r(0^+) - r(\infty)$$

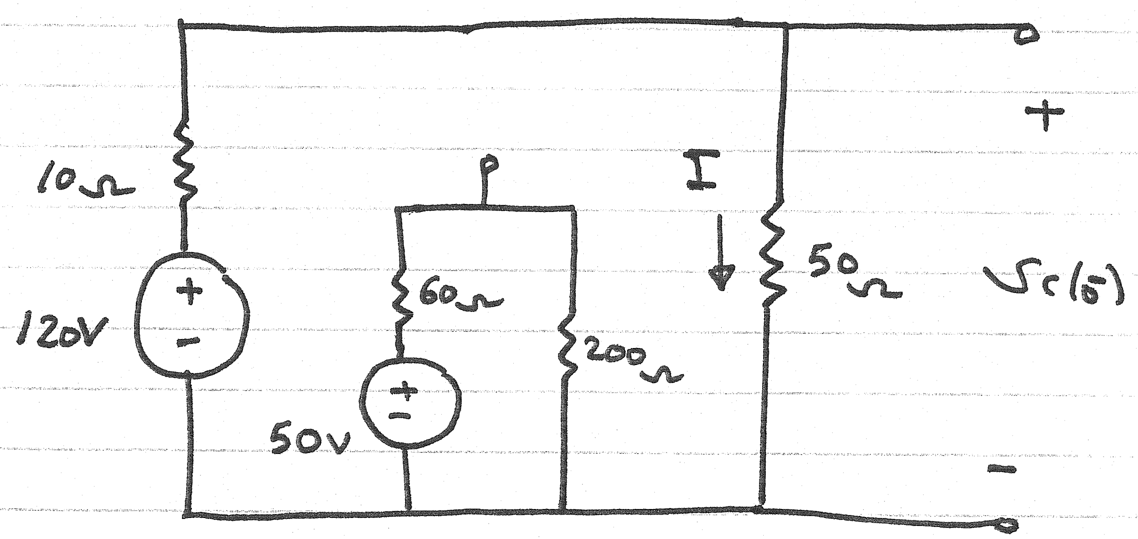
$$\therefore\ r(t) = r(\infty) + \left[r(0^+) - r(\infty)\right] e^{-t/\tau} \qquad t>0$$

$$\tau = \frac{L}{R_{eq}} \quad \text{or} \quad \tau = R_{eq}\, C$$

Find $i(t)$ for $t>0$



For $t<0$; $t=0^-$

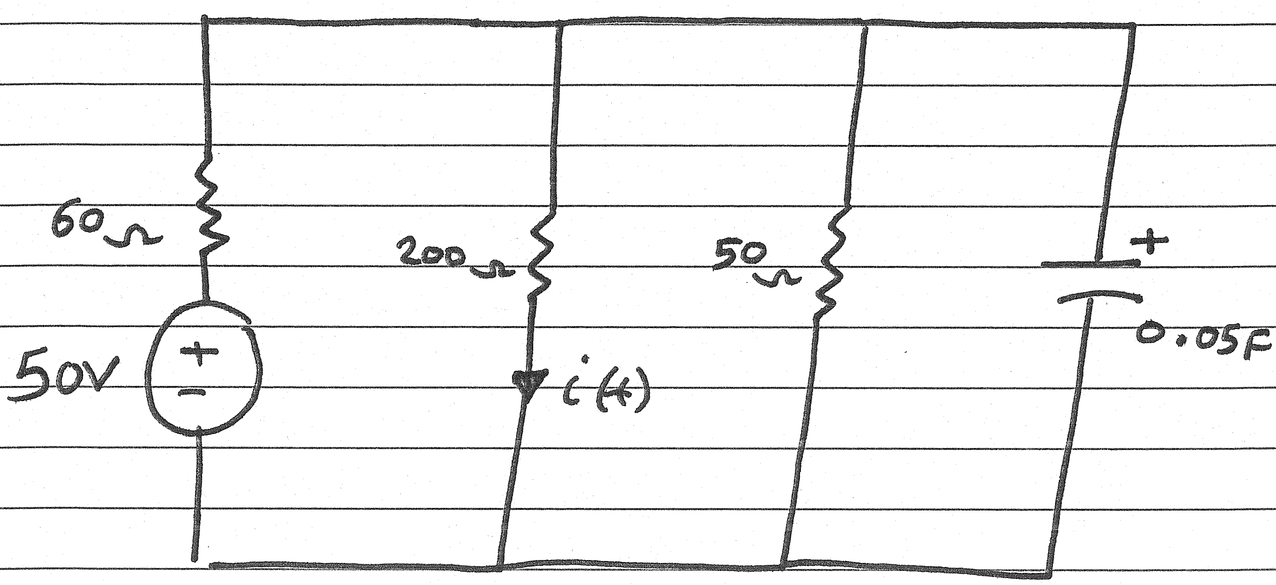


KVL : $-120+10I+50I=0$

$\therefore \quad I = 2A$

$v_c(0^-) = 50 I = 100V$

<u>For $t > 0$</u>

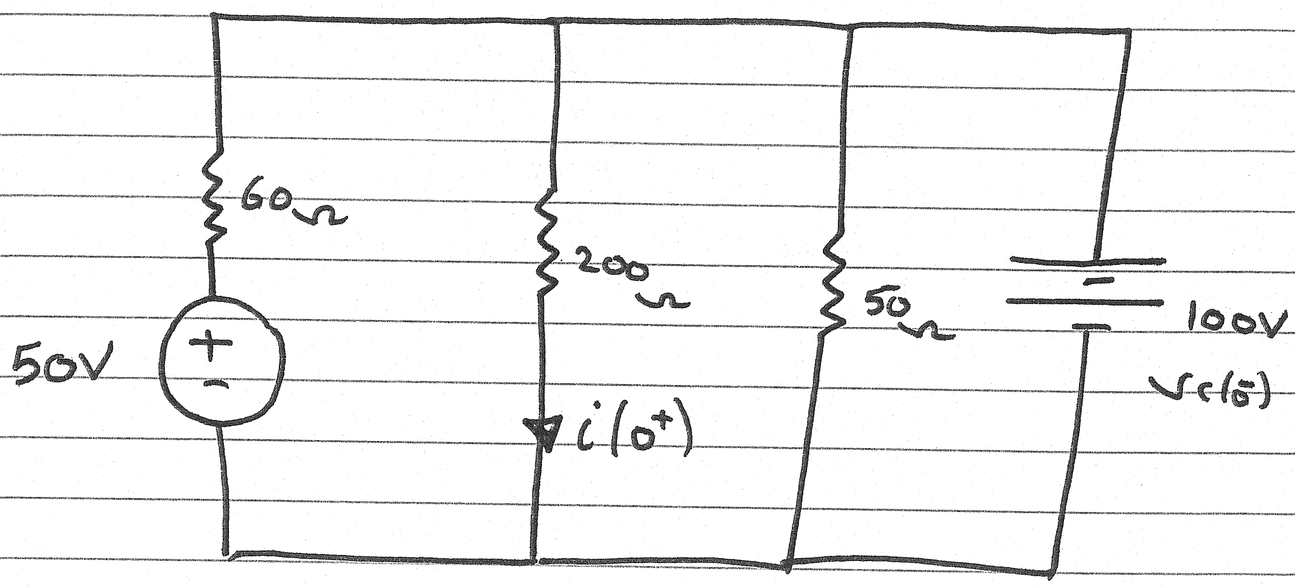


$$v(t) = v(\infty) + [v(0^+) - v(\infty)] e^{-t/\tau} \quad t > 0$$

$$i(t) = i(\infty) + [i(0^+) - i(\infty)] e^{-t/\tau} \quad t > 0$$

To find $i(0^+)$

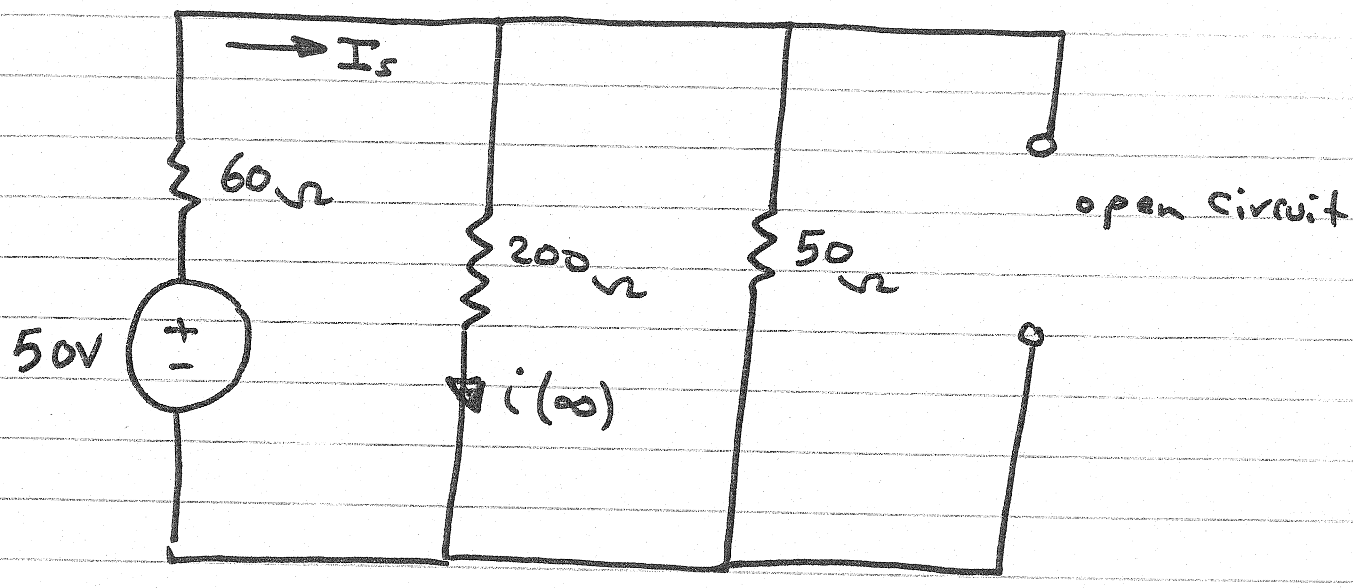
at $t = 0^+$



$$i(0^+) = \frac{v_c(0^-)}{200} = \frac{100}{200} = 0.5\,A$$

To find $i(\infty)$

at $t = \infty$



$$i(\infty) = \frac{50}{50+200} I_s$$

$$I_s = \frac{50}{50\Omega \| 200\Omega + 60\Omega} = 0.5A$$

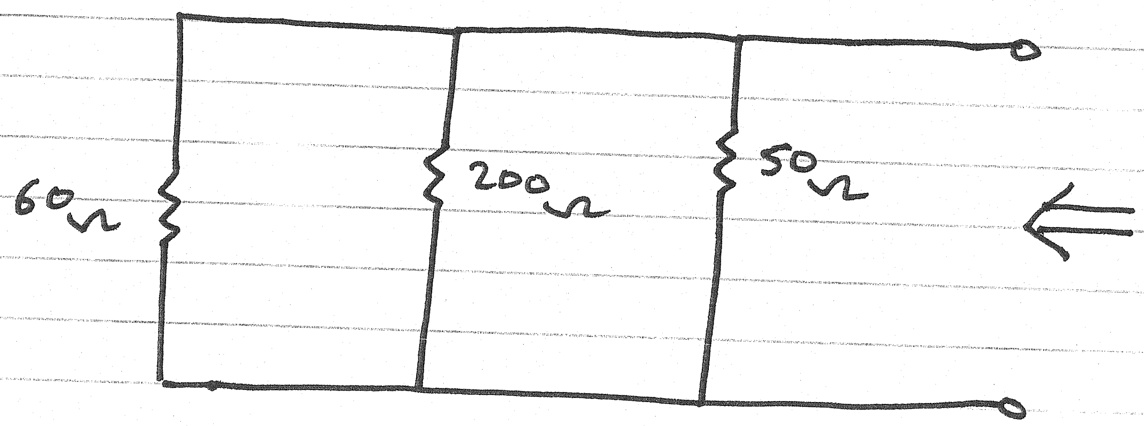
$\therefore\ i(\infty) = 0.1A$

To find τ

for $t > 0$

$$\tau = R_{eq} C$$

$R_{eq} = R_{TH}$ seen by the Capacitor



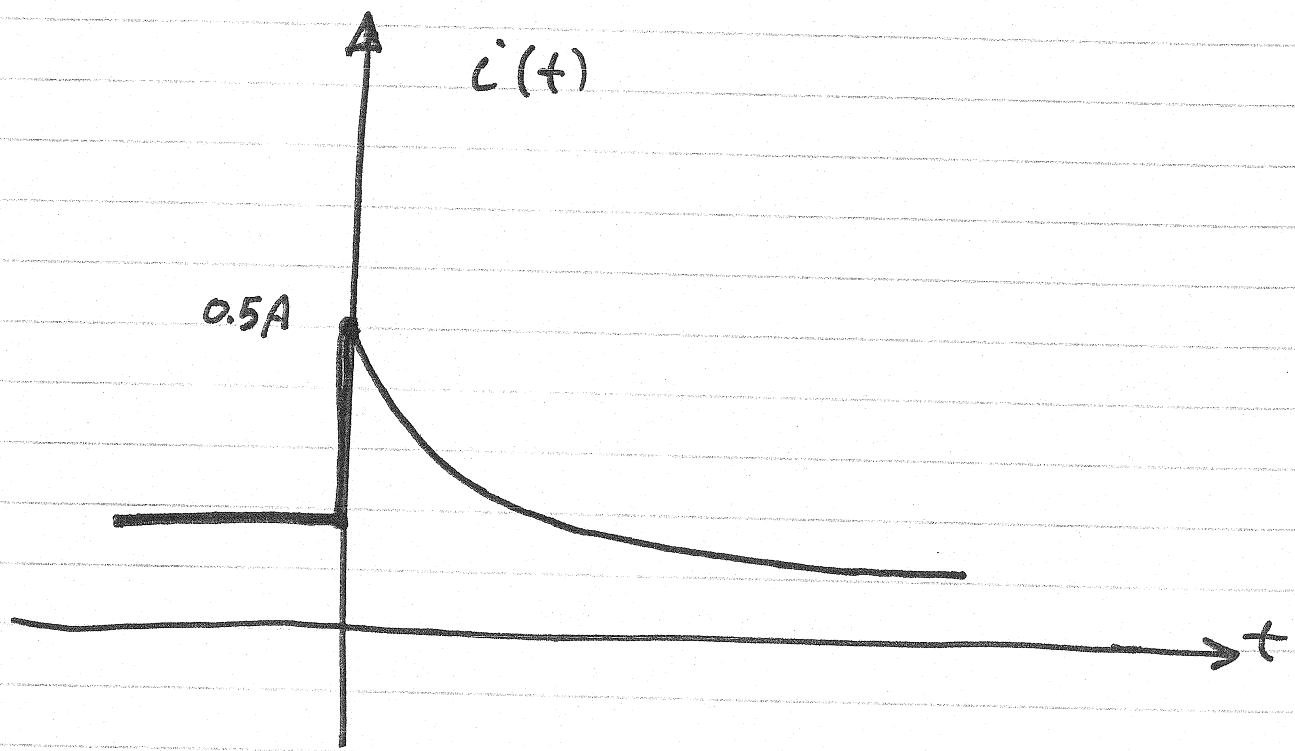
$$R_{TH} = 60\,\Omega \parallel 200\,\Omega \parallel 50\,\Omega$$

$$R_{TH} = 24\,\Omega$$

$$\tau = (0.05)(24) = 1.2 \text{ sec}$$

$$\therefore\ i(t) = \left(0.1 + 0.4\,e^{-t/1.2}\right) A \ ; t > 0$$

$$i(t) = \left(0.1 + 0.4\, e^{-t/1.2}\right) \text{ A} \quad ; t > 0$$



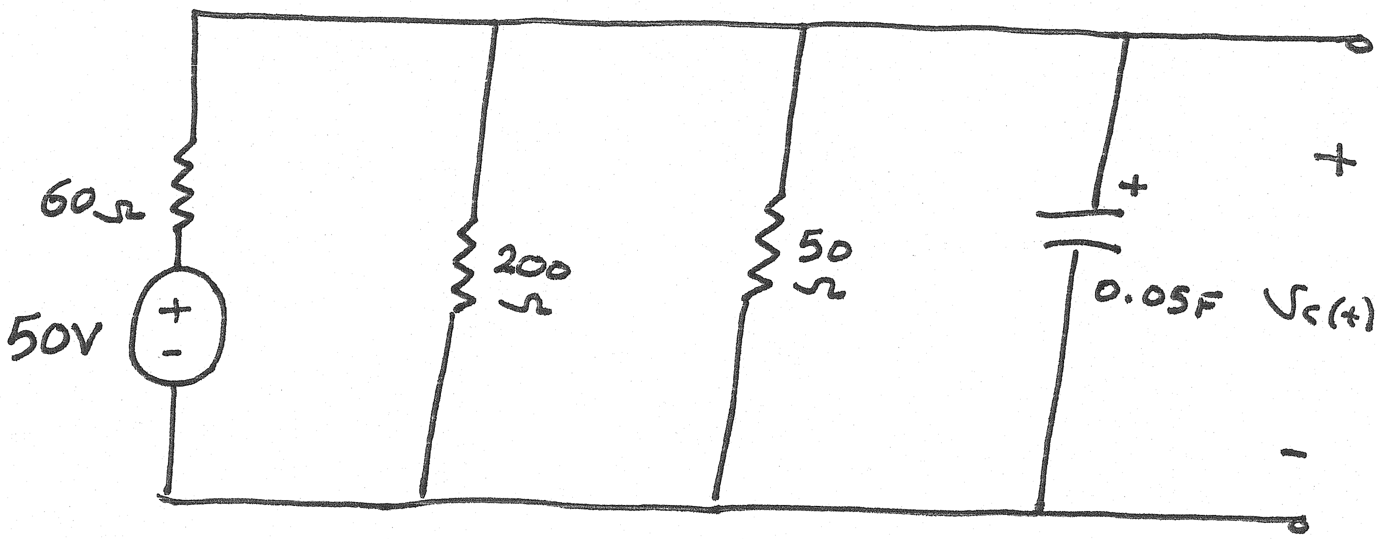
$i(0^-) = 0.192 \text{ A}$

$i(0^+) = 0.5 \text{ A}$

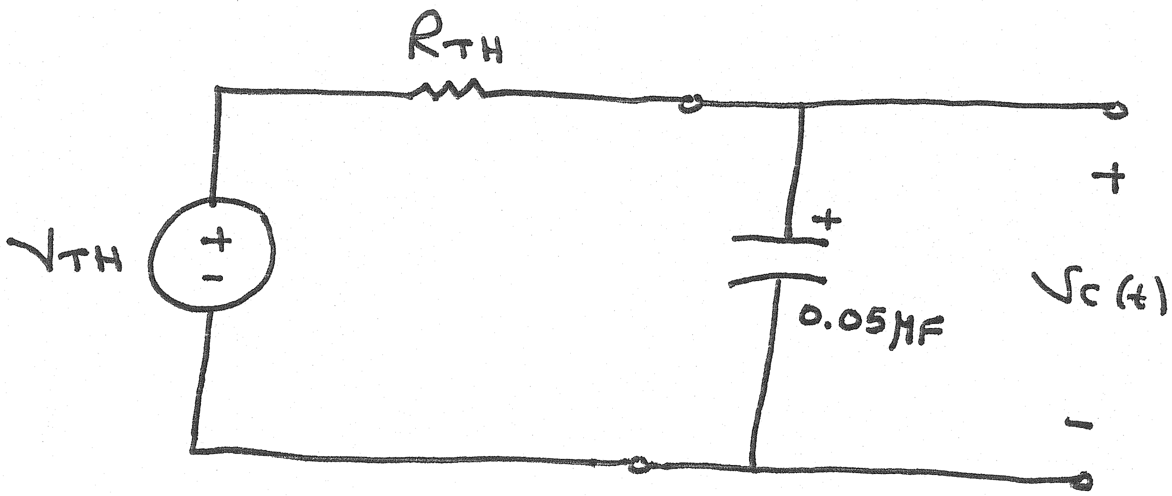
$i(\infty) = 0.1 \text{ A}$

To find $v_c(t)$ for $t>0$

For $t>0$

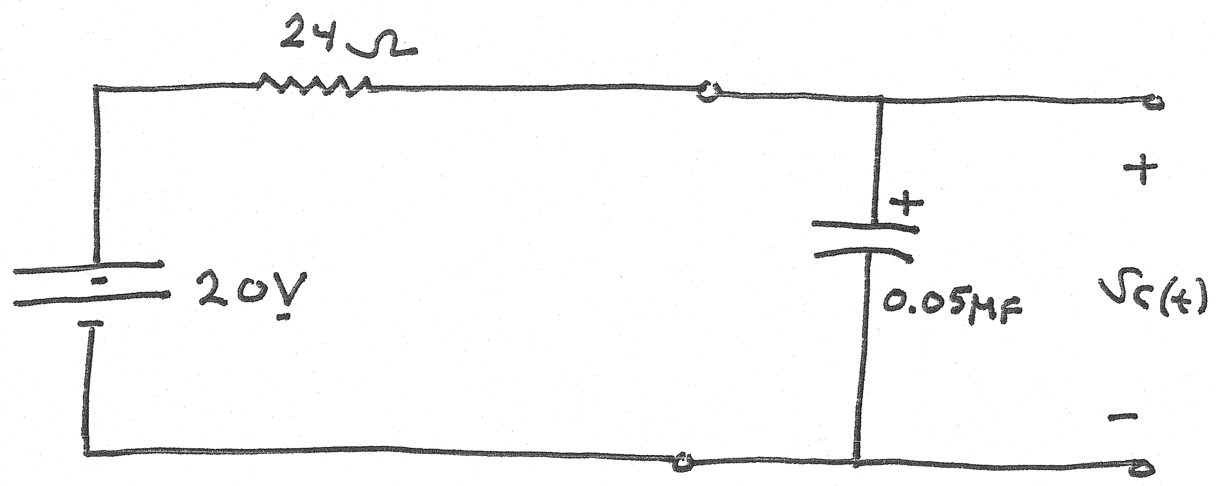


The Circuit can be simplified to



$$R_{TH} = 60\Omega \,||\, 200\Omega \,||\, 50\Omega = 24\,\Omega$$

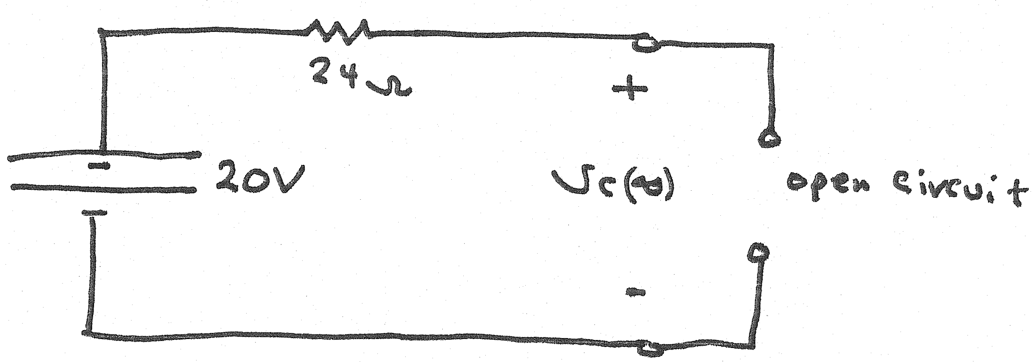
$$V_{TH} = \frac{50\Omega \,||\, 200\Omega}{50\Omega \,||\, 200\Omega + 60\Omega} \cdot 50 = 20V$$



$$v_c(t) = v_c(\infty) + \left[v_c(0^+) - v_c(\infty)\right] e^{-t/\tau} \quad ; \; t>0$$

1) $v_c(0^+) = v_c(0^-) = 100\,V$

2) To find $v_c(\infty)$



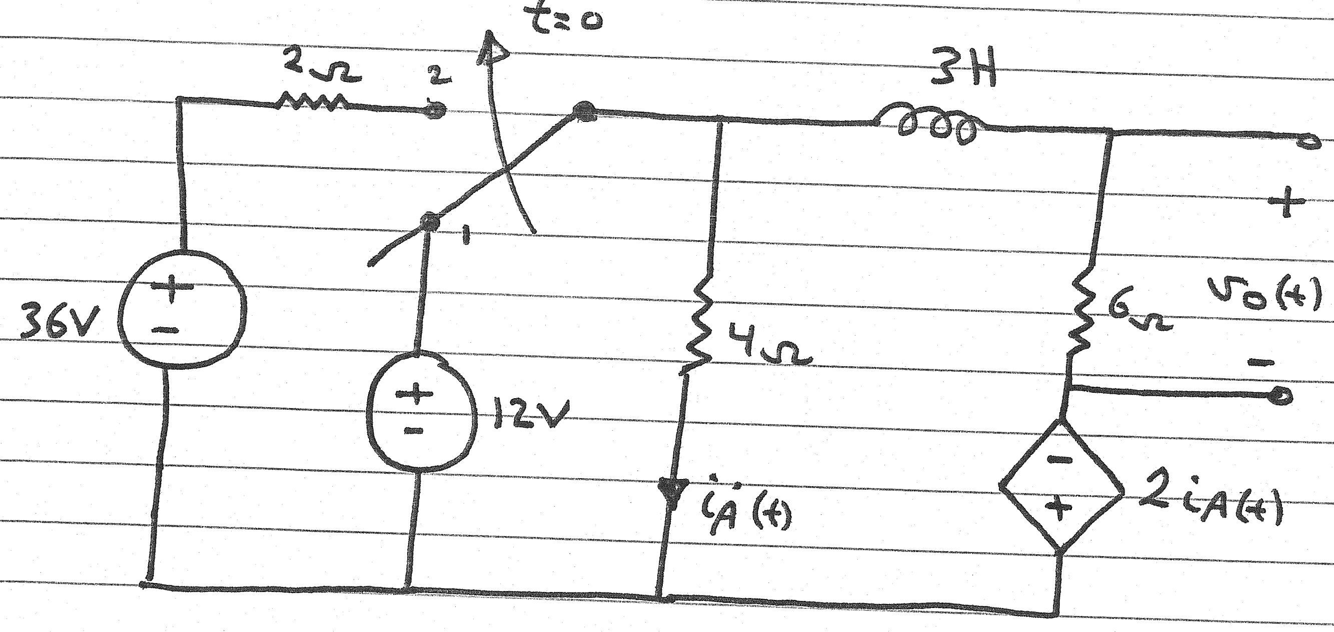
$\therefore \; v_c(\infty) = 20V$

3) $\tau = R_{TH}\, C = 1.2 \text{ sec}$

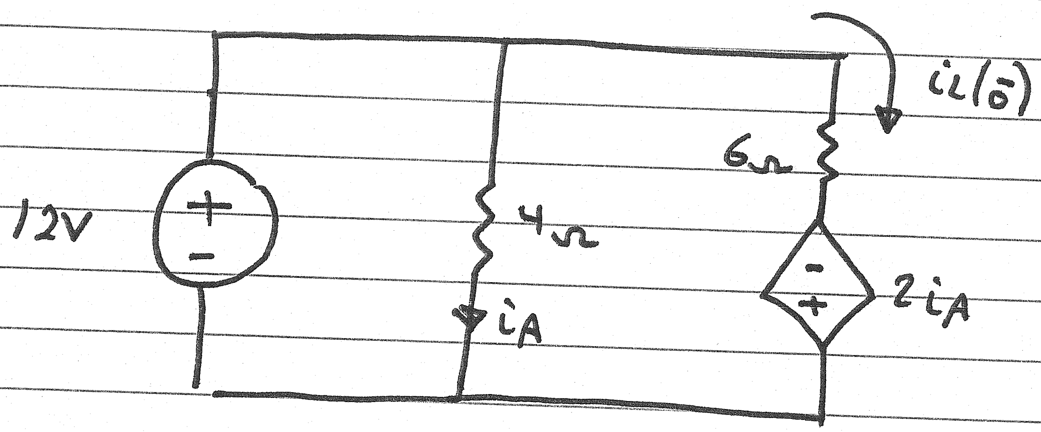
$$v_c(t) = 20 + (100 - 20)\, e^{-t/1.2} \; V \; , \; t \geq 0$$

$$v_c(t) = 20 + 80\, e^{-t/1.2} \; V \; , \; t \geq 0$$

Find $v_o(t)$ for $t > 0$



For $t < 0$; $t = 0^-$



$$i_A = \frac{12}{4} = 3A$$

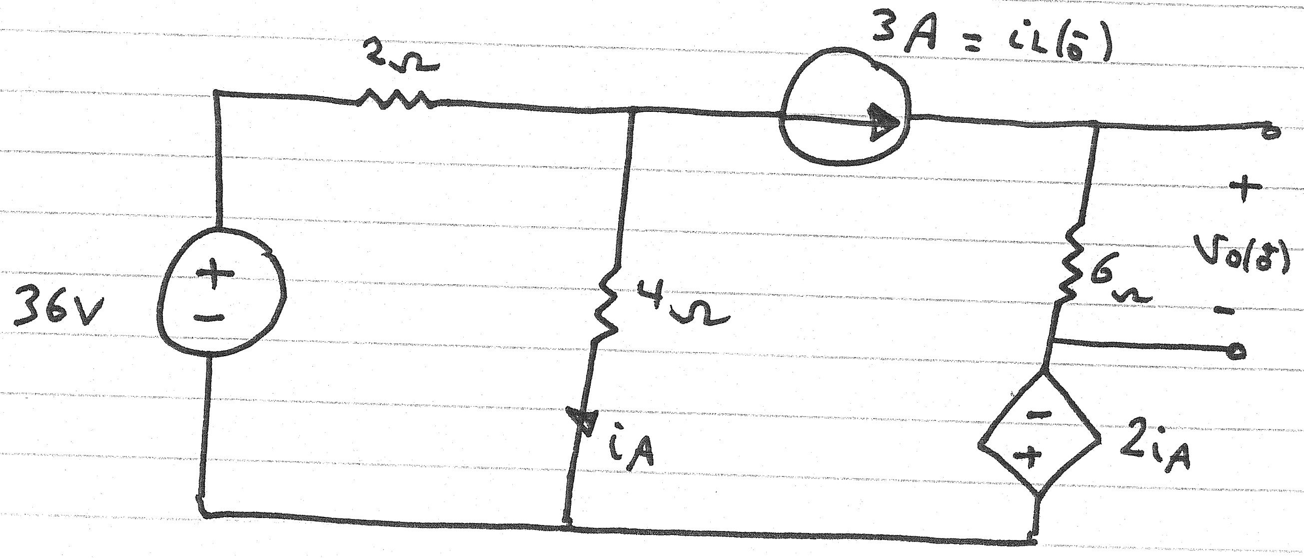
KVL :

$$-12 + 6\,i_L(0^-) - 2i_A = 0$$

$$\therefore i_L(0^-) = 3A$$

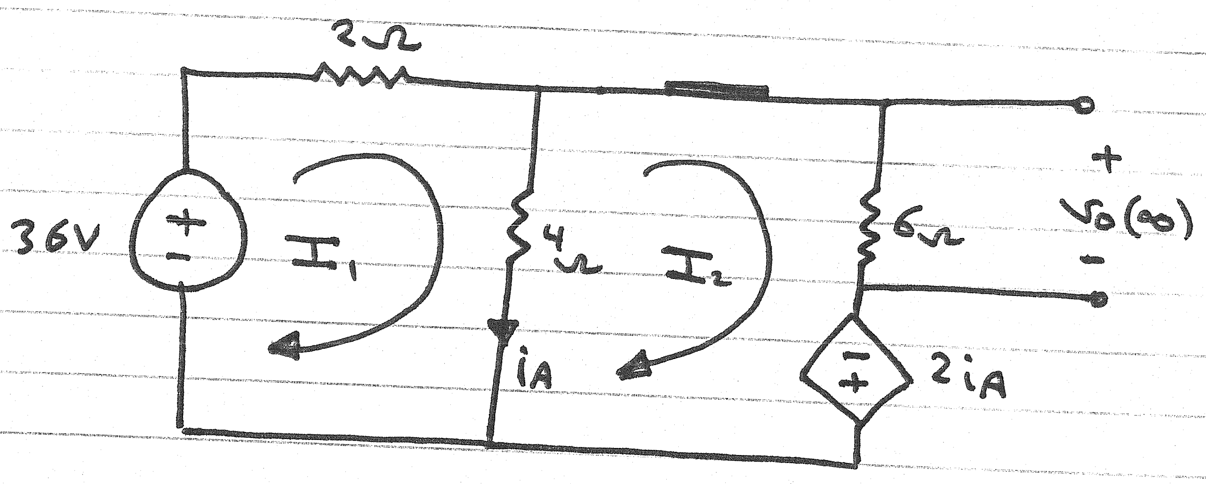
$$v_o(t) = v_o(\infty) + \left[v_o(0^+) - v_o(\infty)\right] e^{-t/\tau} \qquad t > 0$$

To find $v_o(0^+)$



$$v_o(0^+) = (3A)(6\Omega) = 18V$$

To find $v_o(\infty)$



KVL for mesh ①:

$$36 = 6I_1 - 4I_2$$

KVL for mesh ②:

$$2i_A = -4I_1 + 10I_2$$

$$i_A = I_1 - I_2$$

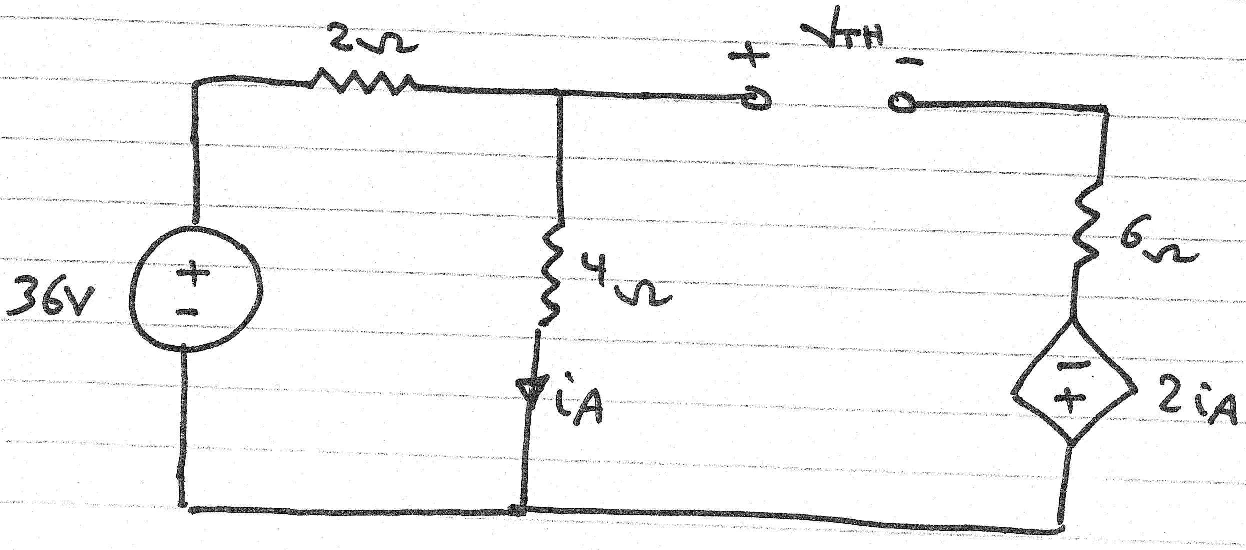
Solving for I_2; we get $I_2 = \frac{36}{8}$ A

$$v_o(\infty) = (6\Omega)\left(\frac{36}{8}A\right) = 27V$$

To find $\tau = \frac{L}{R_{TH}}$

<u>To find R_{TH}</u>

$$R_{TH} = \frac{V_{TH}}{I_{SC}}$$

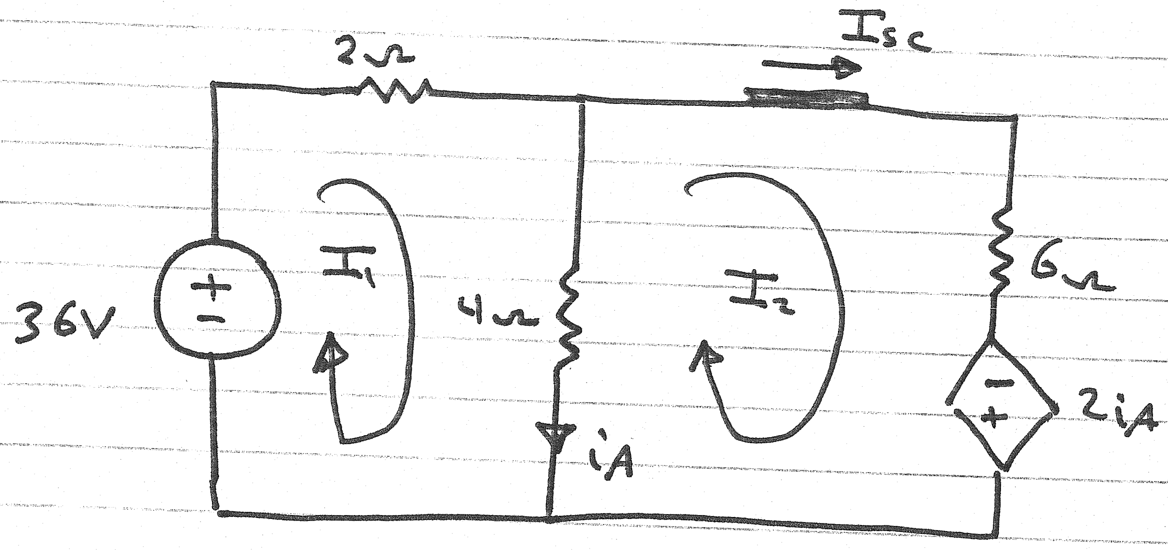


$$V_{TH} = 4\Omega\, i_A + 2 i_A = 6 i_A$$

$$i_A = \frac{36}{6} = 6A$$

$\therefore V_{TH} = 36V$

To find I_{sc}



KVL for mesh 1 :

$$36 = 6I_1 - 4I_2$$

KVL for mesh 2 :

$$2i_A = -4I_1 + 10I_2$$

$$i_A = I_2 - I_1$$

Solving for $I_2 = I_{sc} = \frac{36}{8}$ A

$$\therefore R_{TH} = \frac{V_{TH}}{I_{sc}} = 8\,\Omega$$

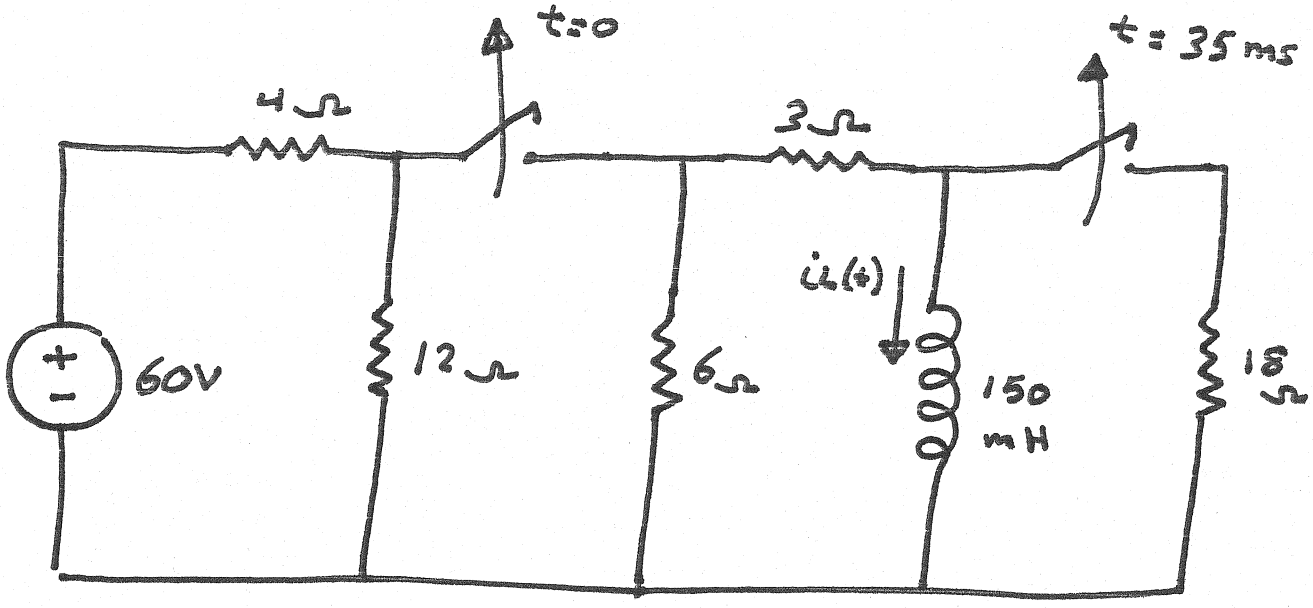
$$\therefore \tau = \frac{L}{R_{TH}} = \frac{3}{8} \text{ sec}$$

$$\therefore v_o(t) = v_o(\infty) + [v_o(0^+) - v_o(\infty)]\, e^{-t/\tau} \quad t>0$$

$$\therefore v_o(t) = 27 + [18 - 27]\, e^{-t/\tau} \quad t>0$$

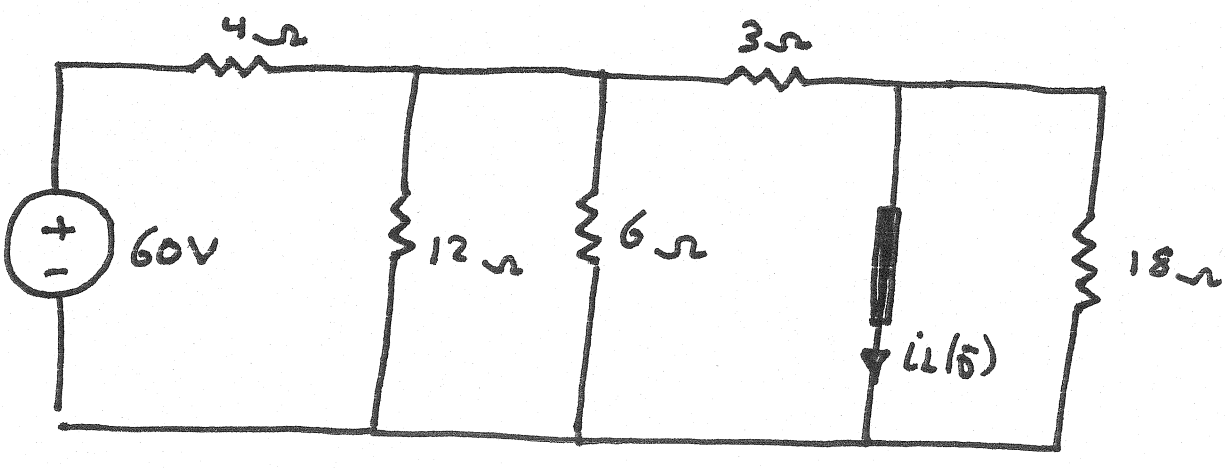
$$\therefore v_o(t) = (27 - 9e^{-\frac{8}{3}t}) \text{ V} \quad \text{for } t>0$$

Sequential Switching



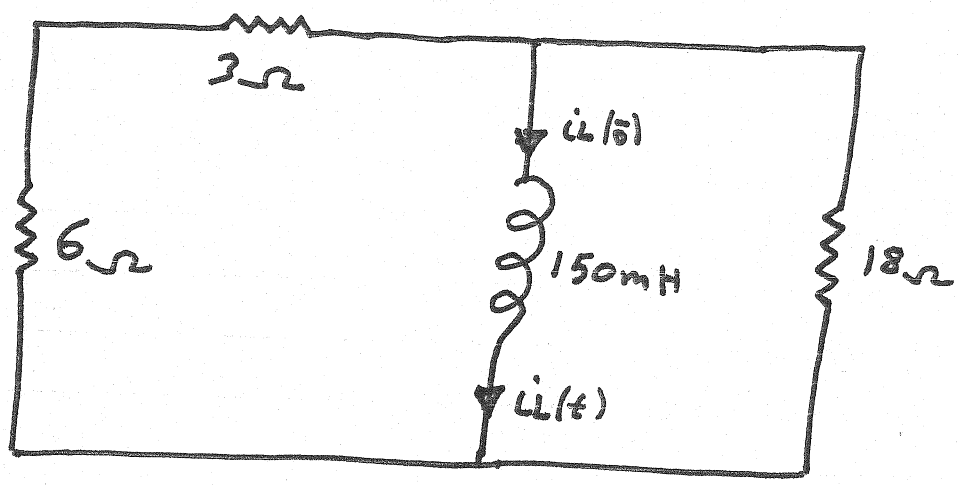
Find $i_L(t)$ for $t > 0$

1) For $t < 0$; $t = 0^-$



$i_L(0^-) = 6A$

2) For $35ms \geq t \geq 0$



Source-free circuit

$\therefore \; i_L(t) = A e^{-t/\tau_1} \quad , \quad t \geq 0$

$$\tau_1 = \frac{L}{R_{TH_1}}$$

$$R_{TH_1} = 18\Omega \parallel (6\Omega + 3\Omega) = 6\Omega$$

$$\therefore \; \tau_1 = \frac{150mH}{6\Omega} = 25 \text{ ms}$$

<u>To find A</u>

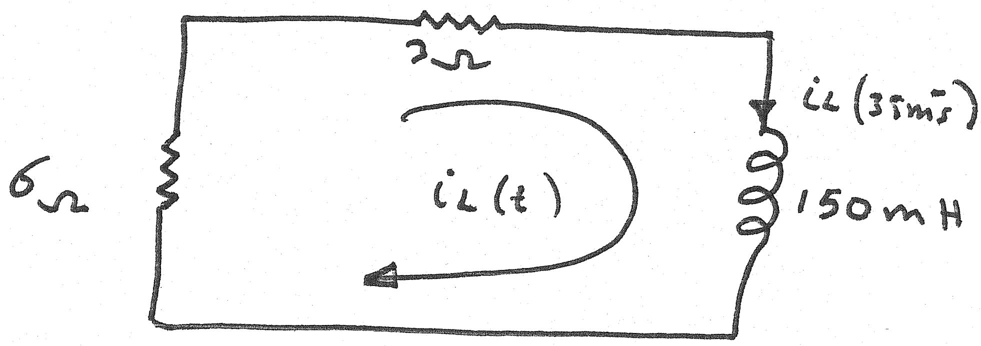
$$i(0^+) = A = i_L(0^-) = 6\,\underline{A}$$

$\therefore \; i_L(t) = 6 e^{-40t} \, \underline{A} \quad , \quad 0 \leq t \leq 35ms$

at $t = 35ms$

$i_L(35ms) = 1.48\ \underline{A}$

2) For $t > 35ms$



Source free Circuit

$\therefore\ i_L(t) = A\, e^{\frac{-(t-35ms)}{\tau_2}}$

$\tau_2 = \frac{L}{R_{TH2}}$

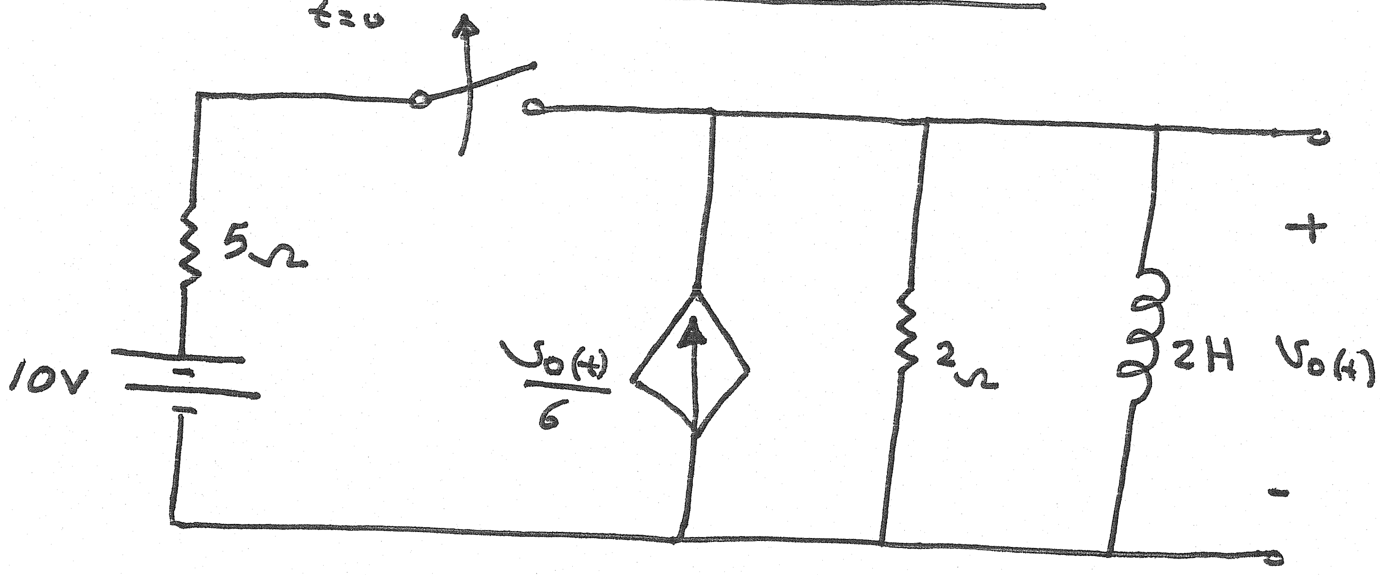
$R_{TH2} = 6\Omega + 3\Omega = 9\Omega$

$\therefore\ \tau_2 = 16.67\ ms$

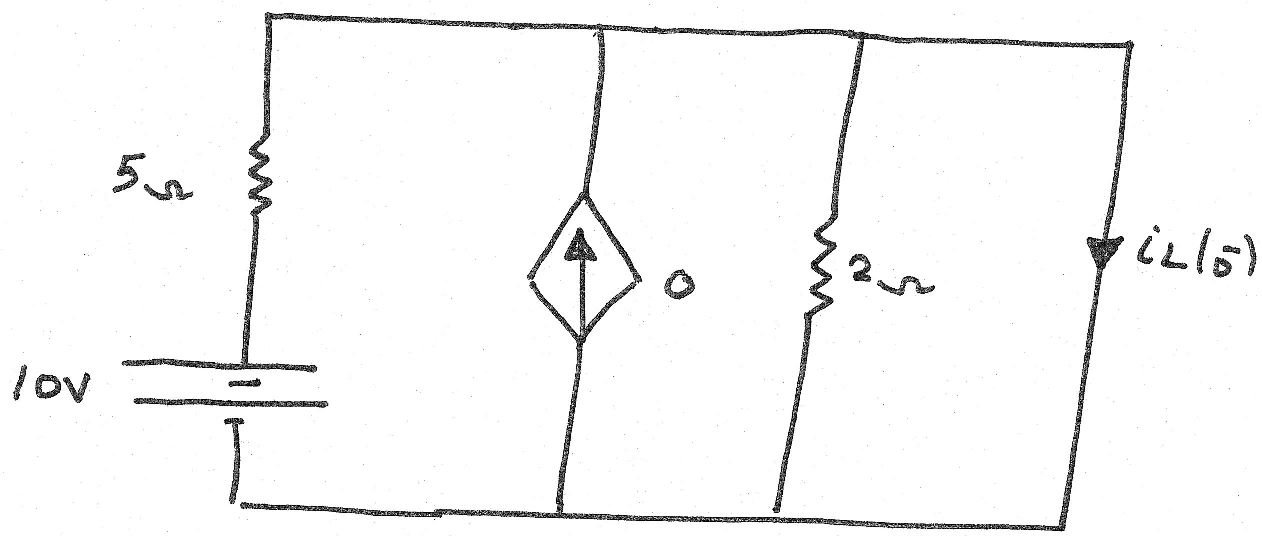
$\therefore\ i_L(t) = 1.48\, e^{\frac{-(t-35ms)}{16.67ms}}$

$i_L(t) = 1.48\, e^{-60(t-0.035)}\ \underline{A}\ ;\ t > 35ms$

Circuit with dependent sources

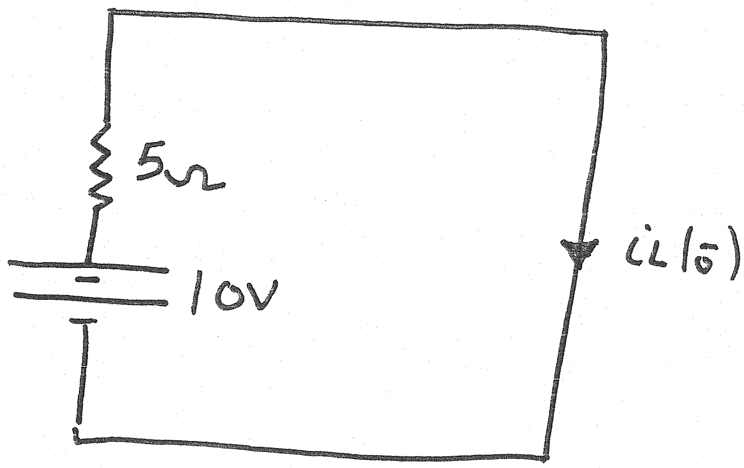


1) for $t < 0$; $t = 0^-$



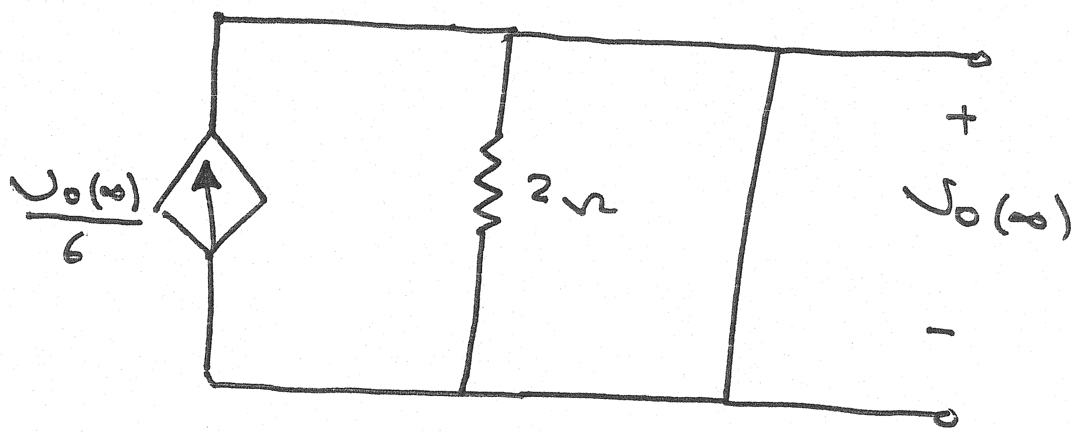
$\frac{v_o}{6} = 0 \longrightarrow$ open Circuit

$2\Omega \parallel 0 = 0 \longrightarrow 2\Omega \longrightarrow$ open Circuit



$\therefore \; i_L(0^-) = 2\,A$

2) At $t = \infty$

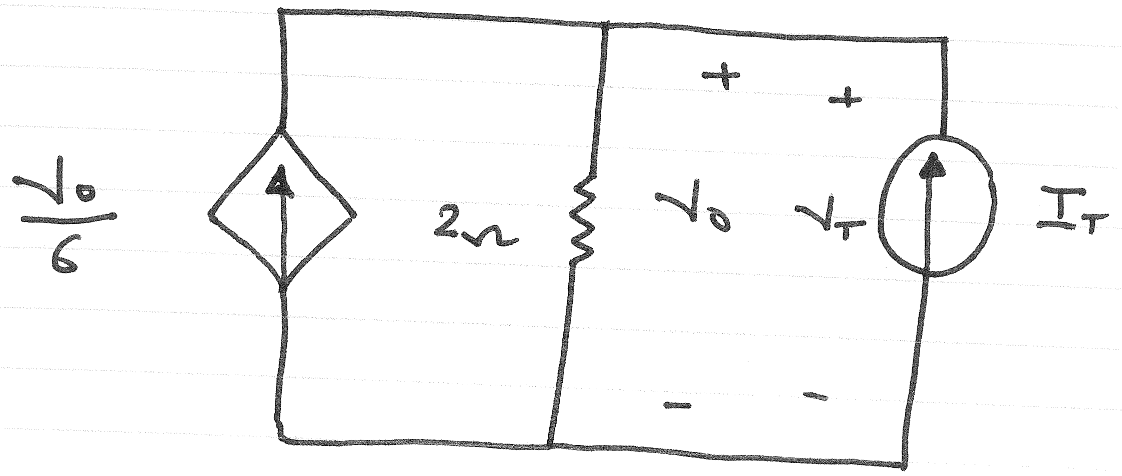


$\therefore \; v_o(\infty) = 0$

3) To find τ

$$\tau = \frac{L}{R_{TH}}$$

$$R_{TH} = \frac{V_T}{I_T}$$



KCL :

$$\frac{V_o}{6} + I_T = \frac{V_o}{2}$$

$$V_o = V_T$$

$$\therefore \frac{V_T}{I_T} = 3\,\Omega = R_{TH}$$

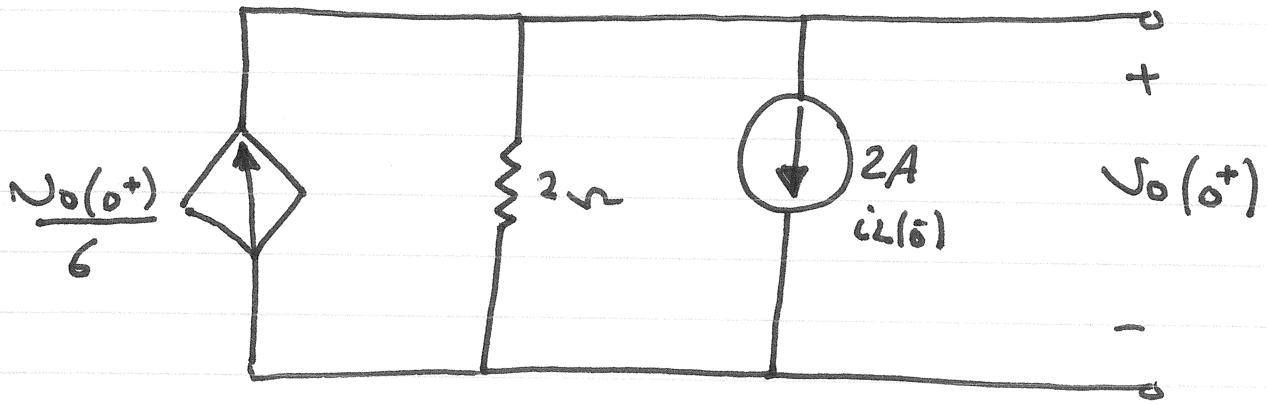
$$\therefore \tau = \frac{L}{R_{TH}} = \frac{2}{3}\text{ sec}$$

$$V_o(t) = V_o(\infty) + [V_o(0^+) - V_o(\infty)]\,e^{-t/\tau} \quad ; t>0$$

$$\therefore V_o(t) = -6e^{-1.5t}\ A \quad ; \quad t>0$$

-50-

4) at $t = 0^+$



KCL :

$$\frac{v_o(0^+)}{6} = 2 + \frac{v_o(0^+)}{2}$$

$$\therefore v_o(0^+) = -6\,\underline{V}$$

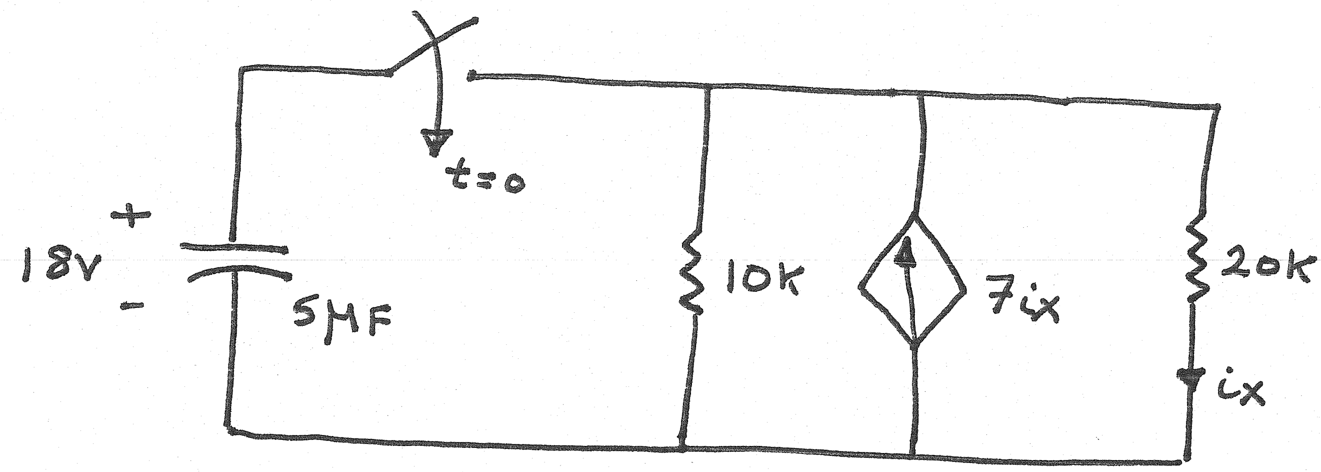
Now

$$v_o(t) = v_o(\infty) + [v_o(0^+) - v_o(\infty)]\, e^{-t/\tau}, \quad t > 0$$

$$\therefore v_o(t) = -6\, e^{-t/\tau} \quad \underline{A} \;\; ; \;\; t > 0$$

$$\therefore v_o(t) = -6\, e^{-1.5t} \quad A \;\; ; \;\; t > 0$$

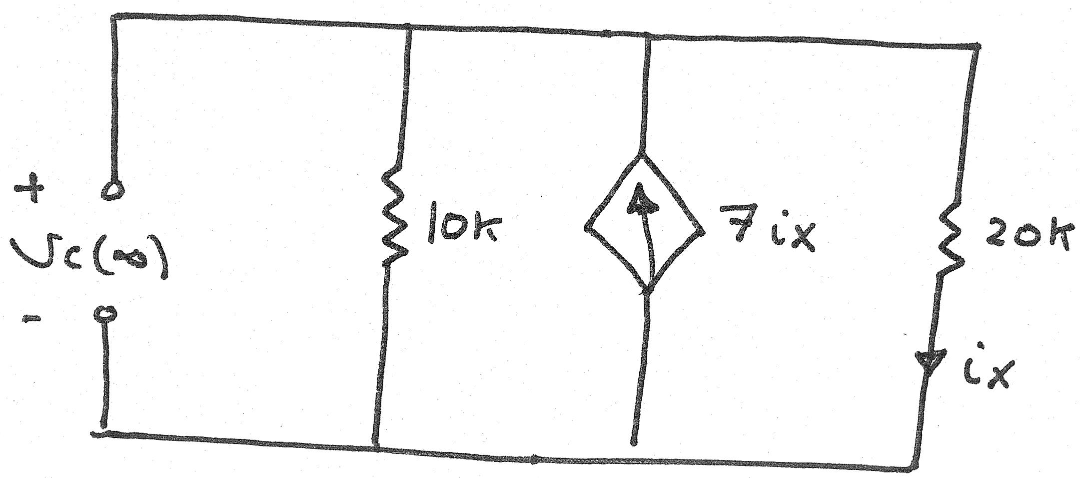
Unbounded Response



Find $v_c(t)$ for $t > 0$

1) $v_c(0^+) = v_c(0^-) = 18\,\underline{V}$

2) To find $v_c(\infty)$



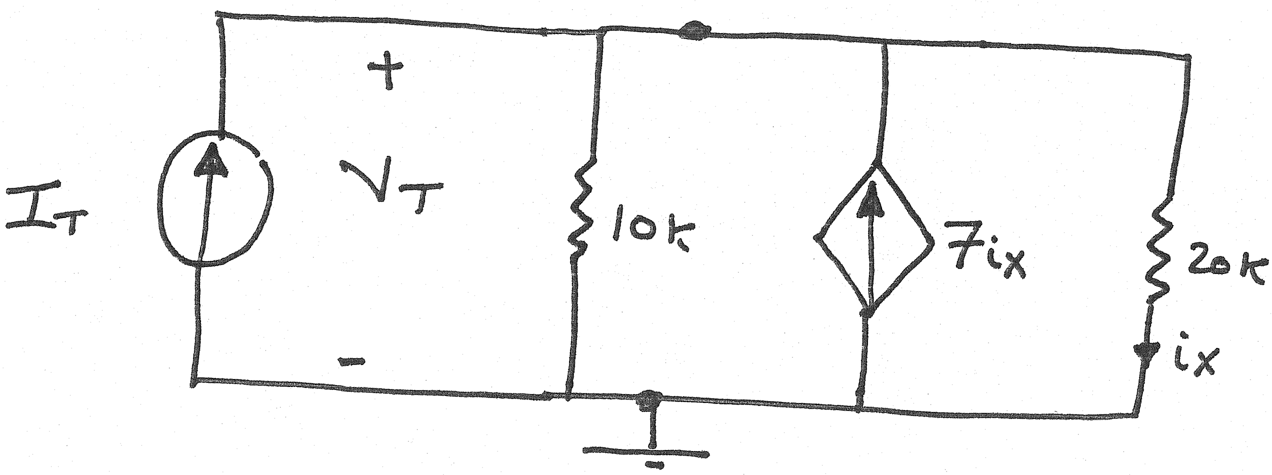
Since the Circuit is dead

$\therefore v_c(\infty) = 0$

3) To find τ

$$\tau = R_{TH} C$$

$$R_{TH} = \frac{V_T}{I_T}$$



KCL :

$$I_T + 7 i_x = \frac{V_T}{10k} + \frac{V_T}{20k}$$

$$i_x = \frac{V_T}{20k}$$

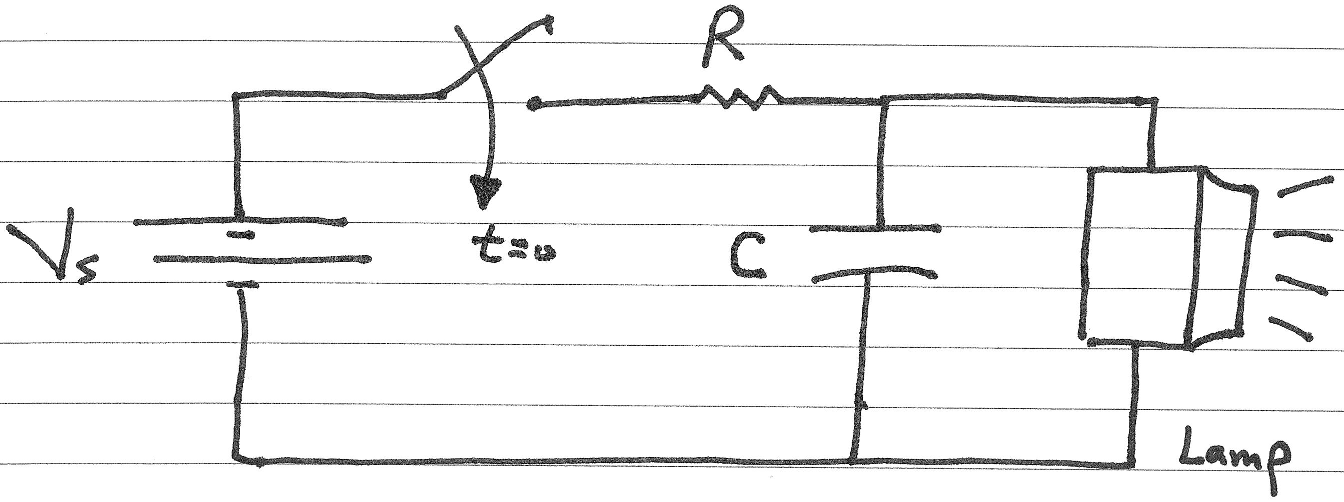
$$\therefore \frac{V_T}{I_T} = -5k = R_{TH} \quad *$$

$$\therefore \tau = R_{TH} C = -25 ms$$

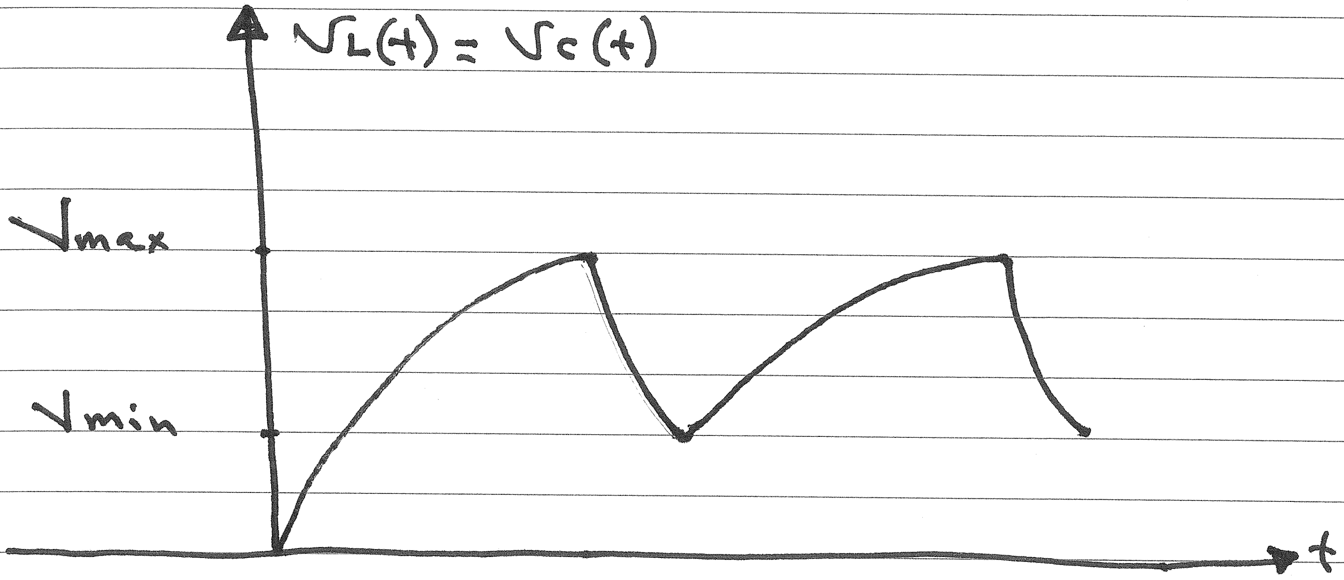
$$\therefore V_c(t) = 18 e^{+40t} \text{ V} \quad \text{for } t \geq 0$$

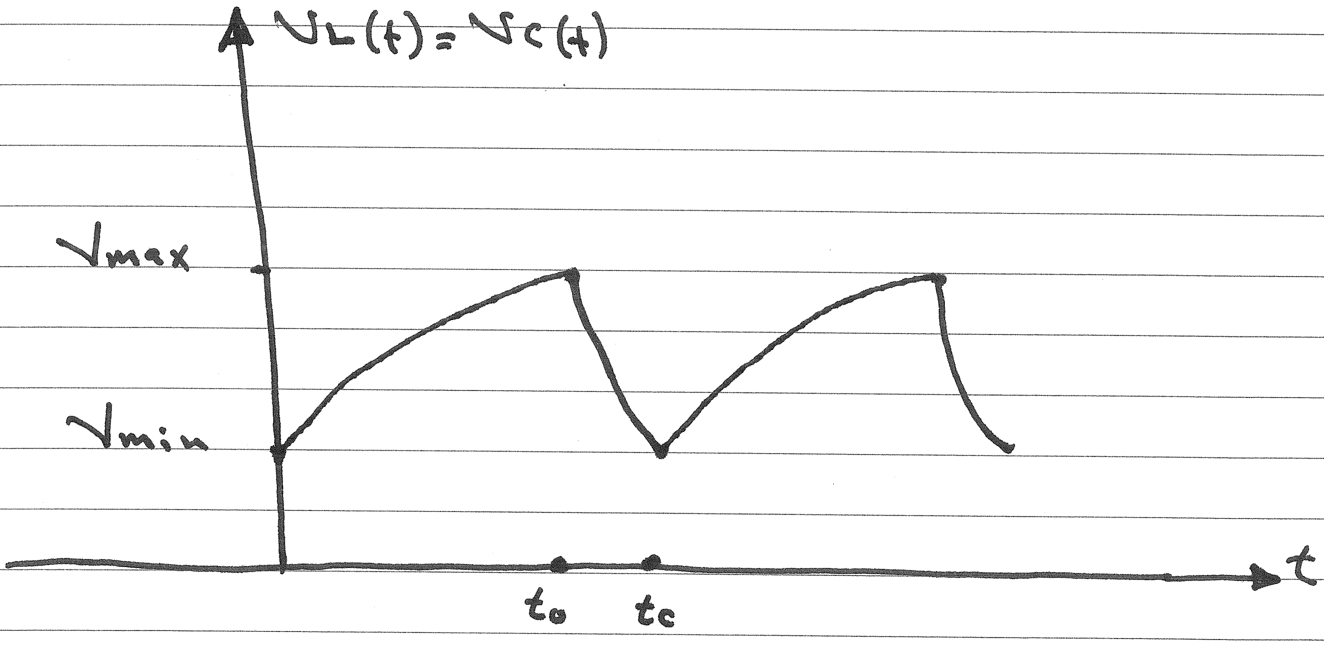
Practical Perspective

A Flashing Light Circuit



- The Lamp starts to Conduct whenever the Lamp voltage reaches V_{max}
- During the time the Lamp Conducts, it Can be modeled as R_L
- The Lamp will Continue to Conduct untill the Lamp voltage drops to V_{min}
- During the time the lamp is not Conducting, it can be modeled as open Circuit.





1) In the interval $t_o > t > 0$

$$V_L(t) = V_s + (V_{min} - V_s)\, e^{-t/\tau_1}$$

$\tau_1 = RC$, $R_L = \infty$

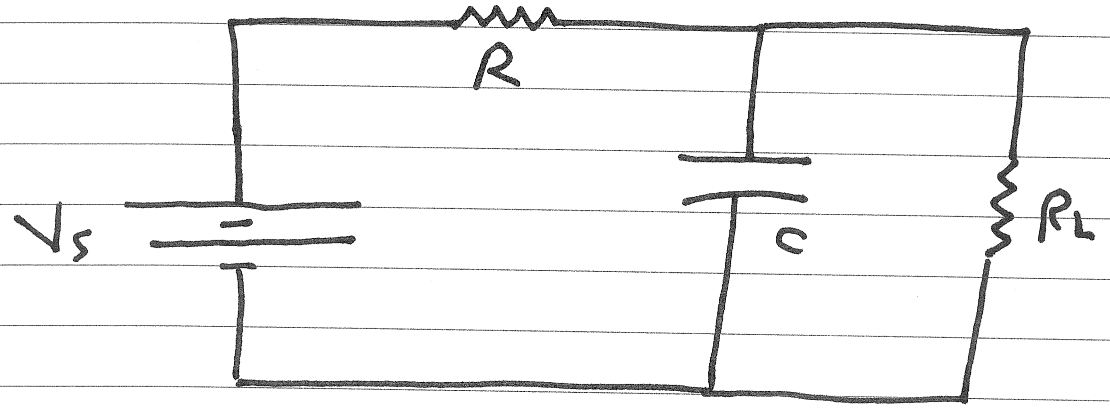
$$v_L(t) = V_s + (V_{min} - V_s)\, e^{-t/\tau_1}$$

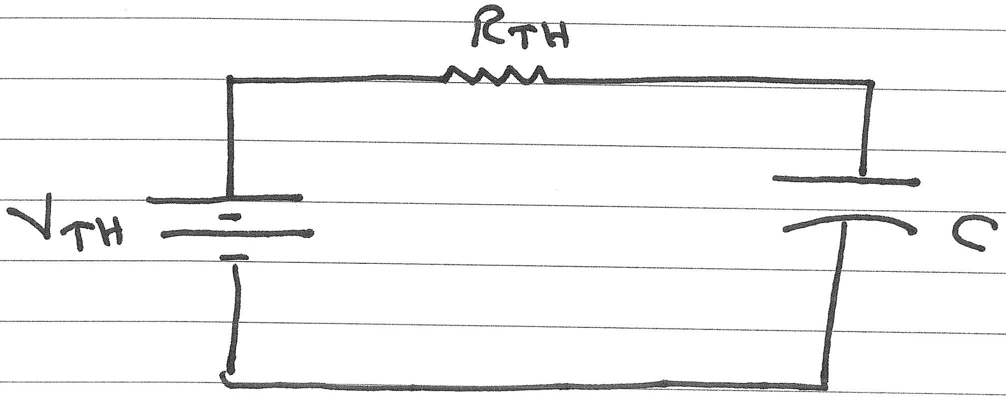
at $t = t_o$; $v_L(t_o) = V_{max}$

$$V_{max} = V_s + (V_{min} - V_s)\, e^{-t/RC}$$

$$\therefore\ t_o = RC \ln \frac{V_{min} - V_s}{V_{max} - V_s}$$

2) In the interval $t_c > t > t_o$





$$R_{TH} = R \parallel R_L$$

$$V_{TH} = \frac{R_L}{R_L + R_{TH}} V_s$$

$$V_L(t) = V_{TH} + (V_{max} - V_{TH})\, e^{\frac{-(t-t_0)}{\tau_2}}$$

$$\tau_2 = R_{TH}C = \frac{R R_L}{R_L + R} C$$

at $t = t_c$; $V_L(t_c) = V_{min}$

$$V_{min} = V_{TH} + (V_{max} - V_{TH})\, e^{\frac{-(t_c-t_0)}{\tau_2}}$$

$$\therefore \quad t_c - t_0 = \frac{R R_L}{R + R_L} C \ln \frac{V_{max} - V_{TH}}{V_{min} - V_{TH}}$$