Leen Sabri

Q3 $f(x) = \ln x \,,\ a = 1$ $\underline{TS}$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!}$$

$$= 0 + (x-1) - \frac{(x-1)^2}{2!} + \frac{2(x-1)^3}{3!}$$

$f(x) = \ln x \Rightarrow f(1) = \ln(1) = 0$	$P_0 = 0$
$f'(x) = \frac{1}{x} \Rightarrow f'(1) = 1$	$P_{(1)} = (x-1)$
$f''(x) = \frac{-1}{x^2} \Rightarrow f''(1) = -1$	$P_{(2)} = (x-1) - \frac{(x-1)^2}{2!}$
$f^{(3)}(x) = \frac{+1(2x)}{(x^2)^2} = \frac{2x}{x^4} = \frac{2}{x^3} \Rightarrow f^{(3)}(1) = 2$	$P_{(3)} = (x-1) - \frac{(x-1)^2}{2!} + \frac{2(x-1)^3}{3!}$

Q14 $f(x) = \frac{2+x}{1-x}$ $MS = TS$ at 0

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)x^3}{3!} + \dots$$

$$= 2 + 3x + \frac{6x^2}{2!} + \frac{18x^3}{3!} + \dots$$

$f(x) = \frac{2+x}{1-x}$ $\Rightarrow f(0) = 2$

$f'(x) = \frac{(1-x) - (2+x)(-1)}{(1-x)^2} = \frac{1-x+2+x}{(1-x)^2} = \frac{3}{(1-x)^2}$ $\Rightarrow f'(0) = 3$

$f''(x) = \frac{-3 * 2(1-x)(-1)}{(1-x)^4} = \frac{6(1-x)}{(1-x)^4} = \frac{6}{(1-x)^3}$ $\Rightarrow f''(0) = 6$

$f'''(x) = \frac{-6 * 3(1-x)^2(-1)}{(1-x)^6} = \frac{18}{(1-x)^4}$ $\Rightarrow f'''(0) = 18$

$\vdots$

$f^{(n)}(x) = \frac{3n!}{(1-x)^{(n+1)}}$ $\Rightarrow f^{(n)}(0) = 3n!$

نفس الاشي لكن مع سالب x وتم ايجادها اول الكتاب

Q20 $f(x) = \sinh x = \frac{e^x - e^{-x}}{2} = \frac{1}{2}[e^x - e^{-x}]$

$= \frac{1}{2}\left[1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots - \left(1 + (-x) + \frac{(-x)^2}{2!} + \frac{(-x)^3}{3!} + \frac{(-x)^4}{4!} + \dots\right]\right.$

$= \frac{1}{2}\left[\cancel{1} + x + \cancel{\frac{x^2}{2!}} + \frac{x^3}{3!} + \cancel{\frac{x^4}{4!}} + \dots \cancel{-1} + x - \cancel{\frac{x^2}{2!}} + \frac{x^3}{3!} - \cancel{\frac{x^4}{4!}} + \dots\right]$

$= \frac{1}{2}\left[2x + \frac{2x^3}{3!} + \frac{2x^5}{5!} + \dots\right]$

$= x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$

$$= \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$$

Q22 $f(x) = \frac{x^2}{x+1}$

$f(x) = \frac{x^2}{x+1}$ $\Rightarrow f(0) = 0$

$f'(x) = \frac{(x+1)(2x) - (x^2)}{(x+1)^2} = \frac{2x^2 + 2x - x^2}{(x+1)^2} = \frac{x^2+2x}{(x+1)^2} = \frac{x(x+2)}{(x+1)^2}$ $\Rightarrow f'(0) = 0$

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$$f''(x)=\frac{(x+1)^2(2x+2)-(x^2+2x)2(x+1)}{(x+1)^4}=\frac{(x+1)(2x+2)-(x^2+2x)2}{(x+1)^3}=\frac{2x^2+2x+2x+2-2x^2-4x}{(x+1)^3}=\frac{2}{(x+1)^3}\implies f''(0)=2$$

$$f^{(3)}(x)=\frac{-2*3(x+1)^2}{(x+1)^6}=\frac{-6}{(x+1)^4}\implies f'''(0)=-6$$

كيف أوجدها

$$f^{(n)}(x)=\frac{(-1)^n n!}{(x+1)^{n+1}}\quad \text{if } n>2 \qquad \implies f^{(n)}(0)=(-1)^n(n!)$$

$$f(x)=\sum_{n=2}^{\infty}(-1)^n x^n$$

Q27 $f(x)=\frac{1}{x^2}\ , a=1$

$f(x)=\frac{1}{x^2} \implies f(1)=1 = 1$

$f'(x)=\frac{-2x}{x^4}=\frac{-2}{x^3} \implies f'(1)=-2 = -2$

$f''(x)=\frac{2(3x^2)}{x^6}=\frac{6}{x^4}=\frac{3!}{x^4} \implies f''(1)=6=3!$

$f'''(x)=\frac{-6(4x^3)}{x^8}=\frac{-24}{x^5}=\frac{-(4!)}{x^5} \implies f'''(1)=-24=-4!$

$f^{(n)}(x)=\frac{(-1)^n(n+1)!}{x^{n+2}} \implies f^{(n)}(1)=(-1)^n(n+1)!$

$$f(x)=1-2(x-1)+3(x-1)^2-4(x-1)^3+\dots=\sum_{n=0}^{\infty}(-1)^n(n-1)(x-1)^n$$

Q32 $f(x)=\sqrt{x+1}\ , a=0$

$f(x)=\sqrt{x+1} \implies f(0)=1$

$f'(x)=\frac{1}{2\sqrt{x+1}}=\frac{1}{2}(x+1)^{-1/2} \implies f'(0)=\frac{1}{2}$

$f''(x)=\frac{1}{2}*\frac{-1}{2}(x+1)^{-3/2}=\frac{-1}{4}(x+1)^{-3/2} \implies f''(0)=\frac{-1}{4}$

$f'''(x)=\frac{-1}{4}*\frac{-3}{2}(x+1)^{-5/2}=\frac{3}{8}(x+1)^{-5/2} \implies f'''(0)=\frac{3}{8}$

$f^{(4)}(x)=\frac{3}{8}*\frac{-5}{2}(x+1)^{-7/2}=\frac{-15}{16}(x+1)^{-7/2} \implies f^{(4)}(0)=\frac{-15}{16}$

$\vdots$

$$f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n=f(a)+f'(a)(x-a)+\frac{f''(a)(x-a)^2}{2!}+\frac{f'''(a)(x-a)^3}{3!}$$
$$=f(0)+f'(0)(x-0)+\frac{f''(0)(x-0)^2}{2!}+\frac{f'''(0)(x-0)^3}{3!}+\frac{f^{(4)}(0)(x-0)^4}{4!}$$
$$=1+\frac{1}{2}(x)-\left(\frac{1}{4}\frac{x^2}{2}\right)+\left(\frac{3}{8}\frac{(x)^3}{6}\right)-\left(\frac{15}{16}\frac{x^3}{24}\right)$$
$$=1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3-\frac{5}{128}x^4+\dots$$

Q37

عوضن بدنا ال $a=1$

$$\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n=f(a)+f'(a)(x-a)+\frac{f''(a)(x-a)^2}{2!}+\frac{f'''(a)(x-a)^3}{3!}$$

$f(a)=e^x,\ f'(a)=e,\ f''(a)=e\ \dots\ \therefore f^{(n)}(a)=e^a\ \forall n=0,1,2,3,\dots$

$$=e^a+e^a(x-a)+\frac{{}^a(x-a)^2}{2!}+\dots$$
$$={}^a\left[1+(x-a)+\frac{(x-a)^2}{2!}+\dots\right]$$