IMAGE ENHANCEMENT - SPATIAL DOMAIN

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What is a Digital Image? (Review)

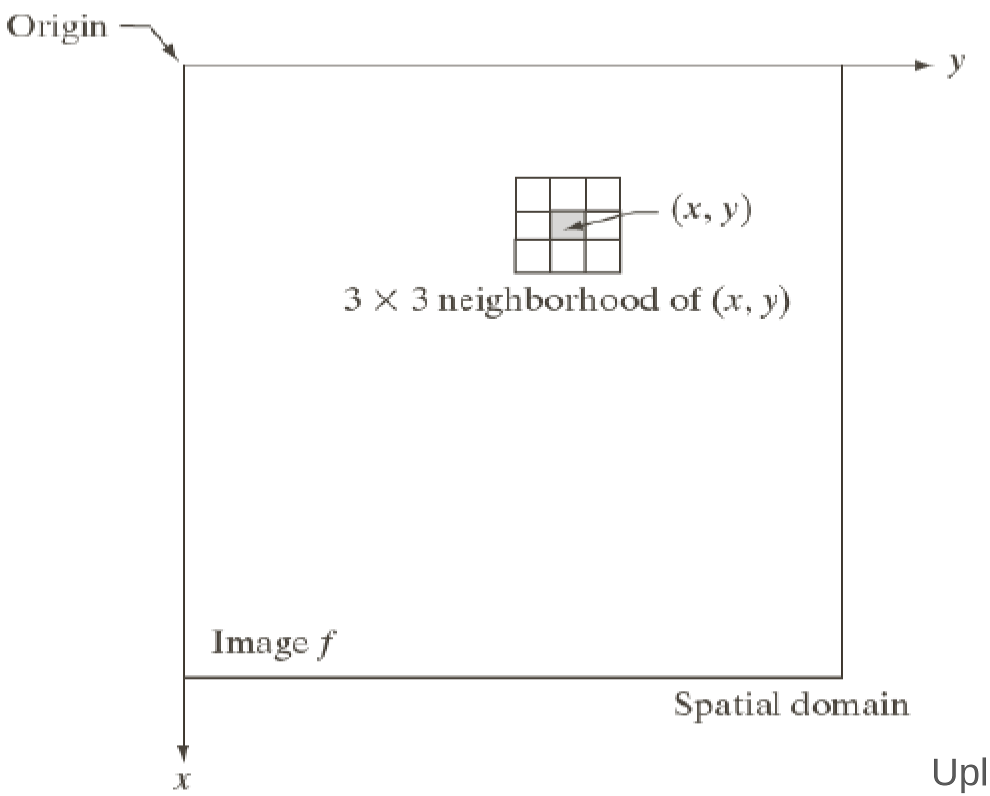
- Spatial domain of the image is the set of pixels composing the image
- Enhancement in the spatial domain involves direct operation on the pixel intensities
- This can be expressed mathematically as

$$g(x,y) = T[f(x,y)]$$

  - f(x,y) is the input image
  - g(x,y) is the output image
  - T[ ] is an operator defined over some neighborhood of (x,y)
- **Important**
  - Keep in mind that g(x,y) may take any value from the set of available gray levels only. Thus, when mapping we should 4 assign the mapped value to the closest level

Background

- Defining the neighborhood around (x,y)
  - Use a square/rectangular subimage that is centered at (x,y)
- Operation
  - Move the center of the subimage from pixel to pixel and apply the operator T at each location (x,y) to compute the output g(x,y)



Background

- The simplest form of the operator T is when the neighborhood size is 1x1 pixels. Accordingly, g(x,y) is only dependent on the value of f at (x,y)
- In this case, T is called the gray-level or intensity transformation function that can be represented as

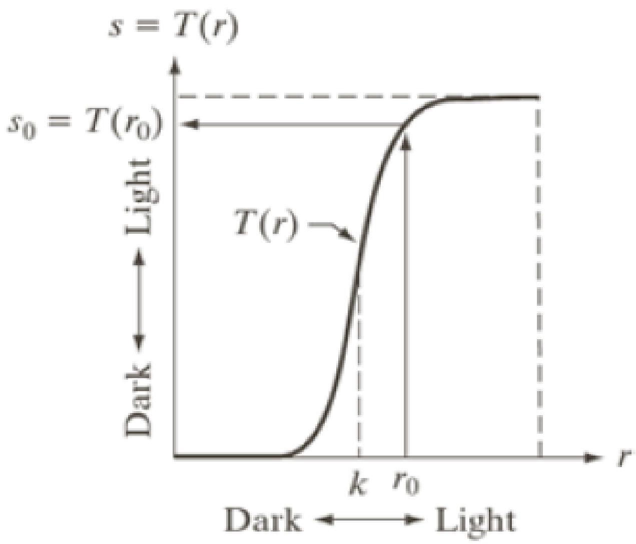
$$s = T(r)$$

  - s is a variable denoting g(x,y)
  - r is a variable denoting f(x,y)

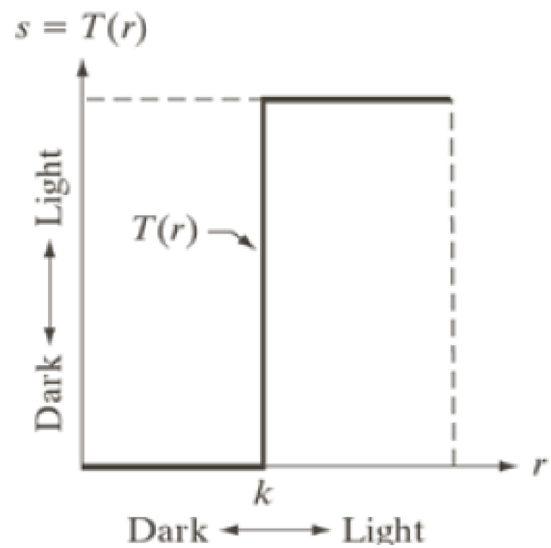
- This is kind of processing is referred as point processing

Background

- Intensity transformation function examples



T(r) performs contrast stretching by mapping levels less than k to narrow range while those above k are mapped to wider range



T(r) reduces the number of levels in the image to two

Fundamentals of Spatial Filtering

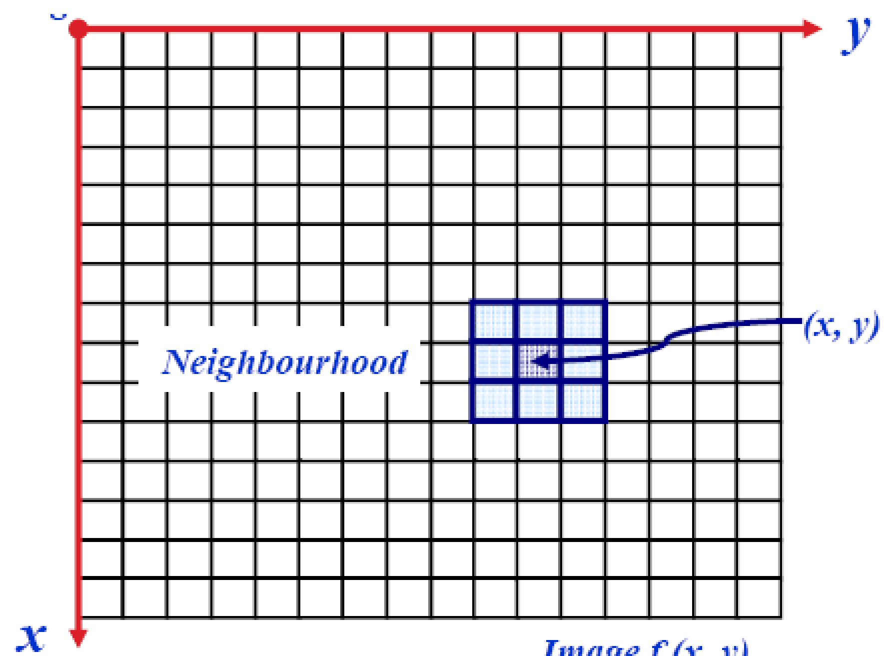
- Filtering is borrowed from the frequency domain processing and refers to the process of passing or rejecting certain frequency components
  - Highpass, lowpass, band-reject , and bandpass filters
- Filtering is achieved in the frequency domain by designing the proper filter (Chapter 4)
- Filtering can done in the spatial domain also by using filter masks (kernels, templates, or windows)
  - Unlike frequency domain filters, spatial filters can be nonlinear !

Spatial Filtering Mechanics

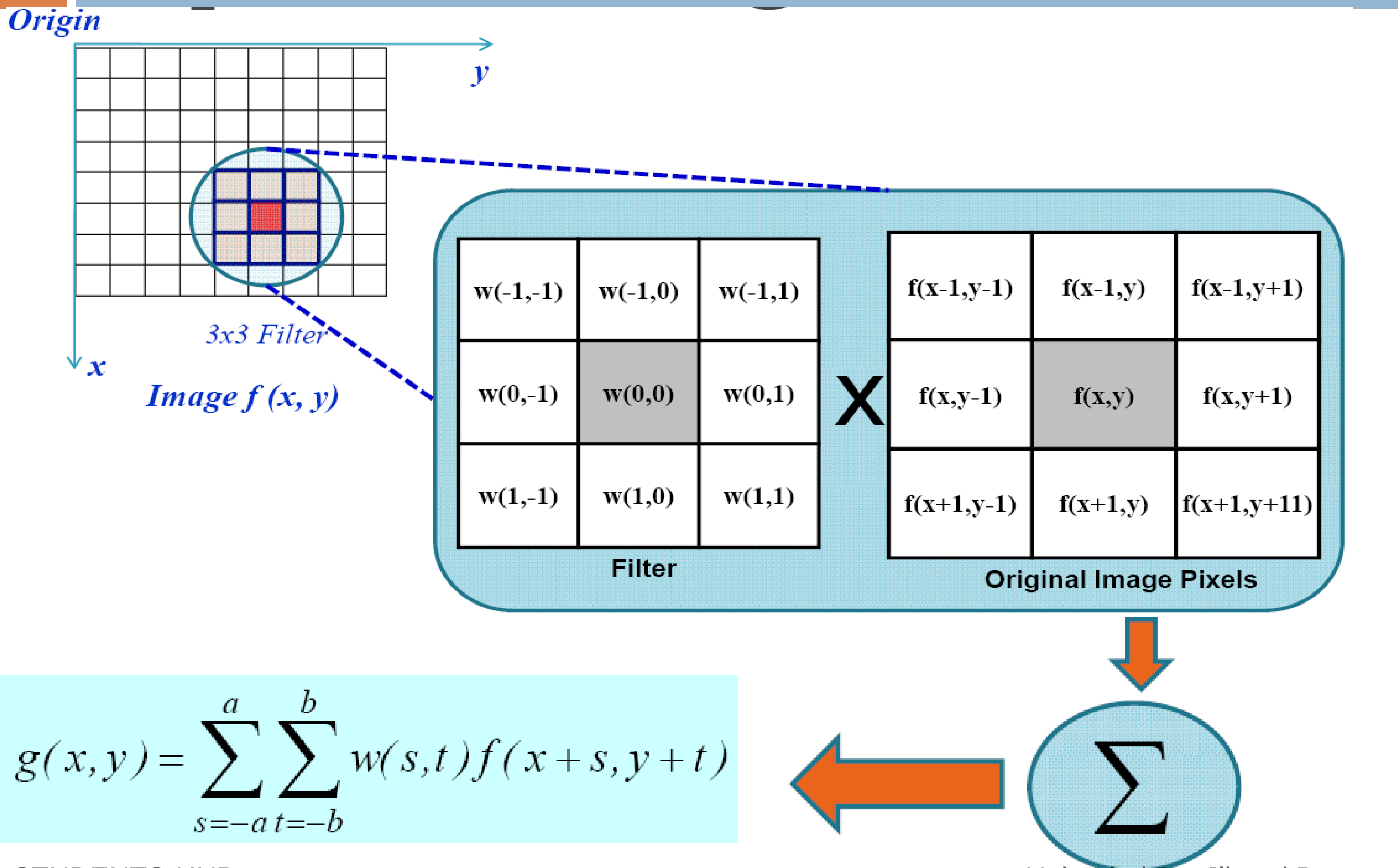
- A spatial filter is characterized by
  - A rectangular neighborhood of size mxn (usually m and n are odd)
  - A predefined operation that is specified by the mask values at each position.
- Spatial filtering Operation
  - The filter mask is centered at each pixel in the image and the output pixel value is computed based on the operation specified by the mask

w(-1,-1)	w(-1,0)	w(-1,1)
w(0,-1)	w(0,0)	w(0,1)
w(1,-1)	w(1,0)	w(1,1)

3x3 filter mask example



Spatial Filtering Mechanics



$$g(x,y)=\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)f(x+s,y+t)$$

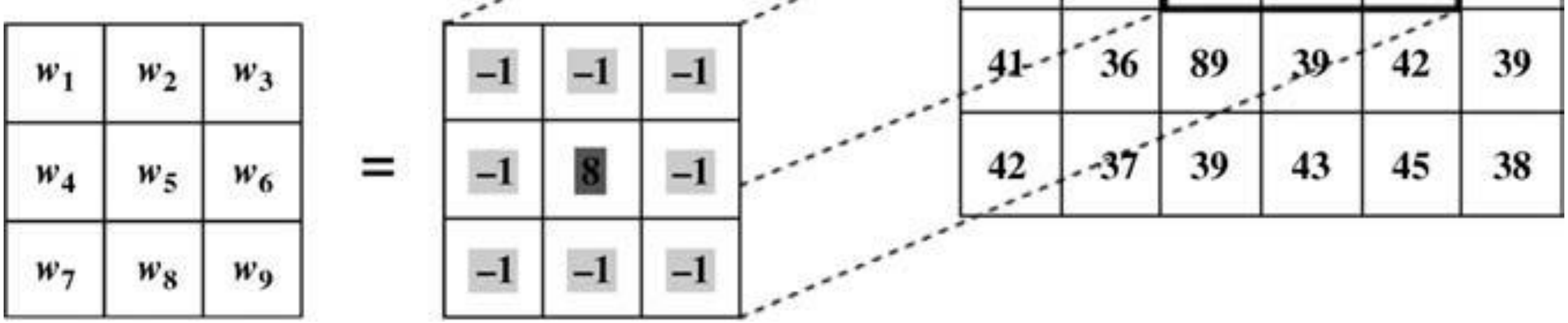
Spatial Filtering Mechanics

- **Spatial Filtering Process:** The spatial filtering on the whole image is given by:
  1. Move the mask over the image at each location.
  2. Compute sum of products between the mask coefficients and pixels inside subimage under the mask.
  3. Store the results at the corresponding pixels of the output image.
  4. Move the mask to the next location and go to step 2 until all pixel locations have been used.

Spatial Filtering Mechanics - Example

$$f_i = \sum_{k=1}^{9} w_k I_k(i)$$

$$=(-1x10) + (-1x11) + (-1x8) + (-1x40) + (8x35)$$
$$+(-1x42) + (-1x38) + (-1x36) + (-1x46) = 14$$



The mechanics of image filtering with an N*N=3*3 kernel filter

Spatial Correlation and Convolution

- Correlation is the process of moving a filter over the image and computing the sum of products at each location as explained in the previous slide

- The mechanics of convolution is similar to those of correlation, except that the filter mask is rotated by 180^o before sliding

Treatment of Pixels at Edges

- In the previous slides, we padded the image with zeros in both directions in order to compensate for unavailable values.
- Other approaches
  - Replicate edge pixels
  - Consider only available pixels that fall under the mask in the computation of the new values
  - Truncate the image
  - Allow pixels to wrapped around

Smoothing Spatial Filters

- A smoothing (averaging, blurring) filter replaces each pixel with the average value of all pixels under the mask

- Used for blurring and for noise reduction
  - Blurring is used in preprocessing steps, such as
    - Removal of small details from an image prior to object extraction
    - Bridging of small gaps in lines or curves
  - Noise reduction can be accomplished by blurring (noise as it is characterized with sharp transitions)

- Replacing the value of every pixel in an image by the average of the gray levels in the neighborhood will reduce the "sharp" transitions in gray levels.
  - sharp transitions
    - random noise in the image
    - edges of objects in the image

- Thus, smoothing can reduce noises (desirable) and blur edges (undesirable)

Smoothing Spatial Filters

- **Common smoothing masks**

1	1	1
1	1	1
1	1	1

X 1/9

Standard averaging mask

1	2	1
2	4	2
1	2	1

X 1/16

Weighted average mask

- General Form (weighted averaging filter):

$$g(x,y)=\frac{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)\, f(x+s,y+t)}{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)}$$

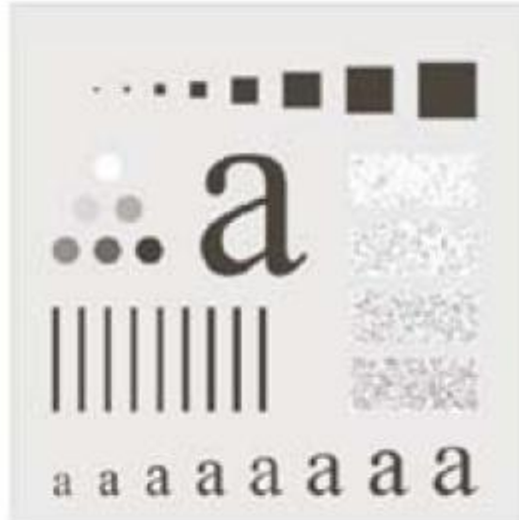
summation of all coefficient of the mask

Smoothing Spatial Filters

- Notes
  - The weighted average filter gives more weight to pixels near the center
  - The basic strategy behind weighting the center point the highest and then reducing the value of the coefficients as a function of increasing distance from the origin is simply an attempt to reduce blurring in the smoothing process.
  - It is hard to see the visual difference between the processing results of the two filters; however, the weighted average filter summation is 16, which make it more attractive for computers

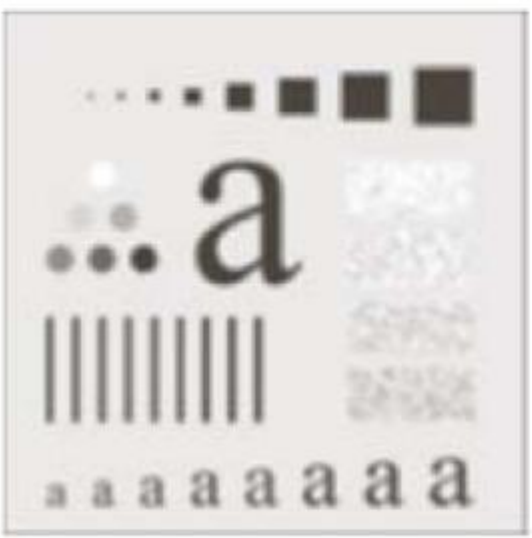
Smoothing Spatial Filters - Example

Original

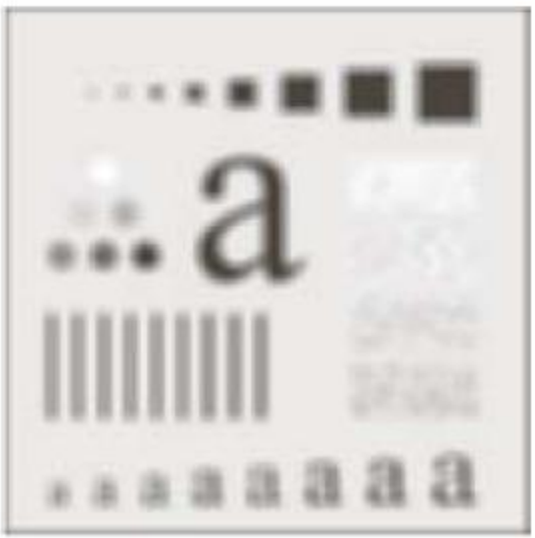


Smoothing by 3x3 Mask

Smoothing by 5x5 Mask



Smoothing by 9x9 Mask

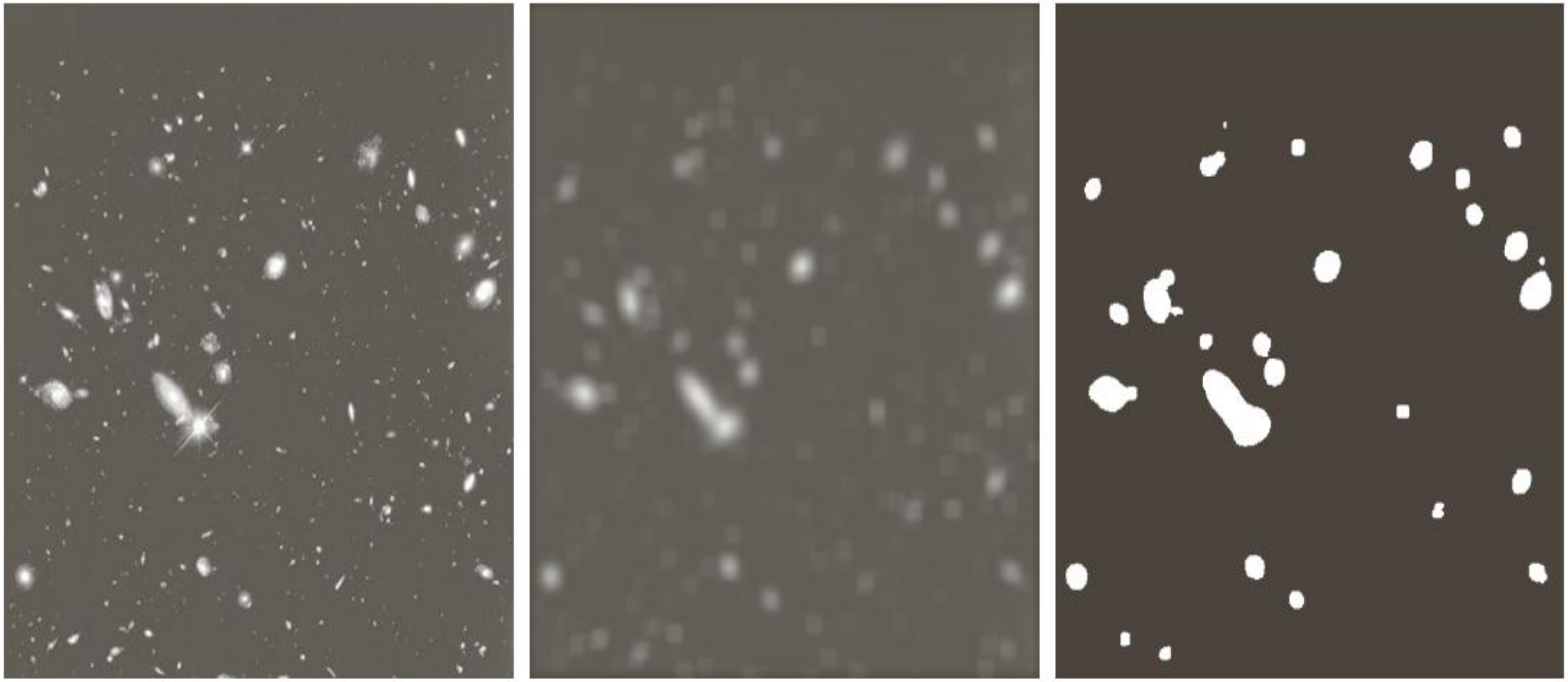


Smoothing by 15x15 Mask

Smoothing by 35x35 Mask

Smoothing Spatial Filters - Example

- Smoothing highlights gross details. Could be useful in providing better segmentation results

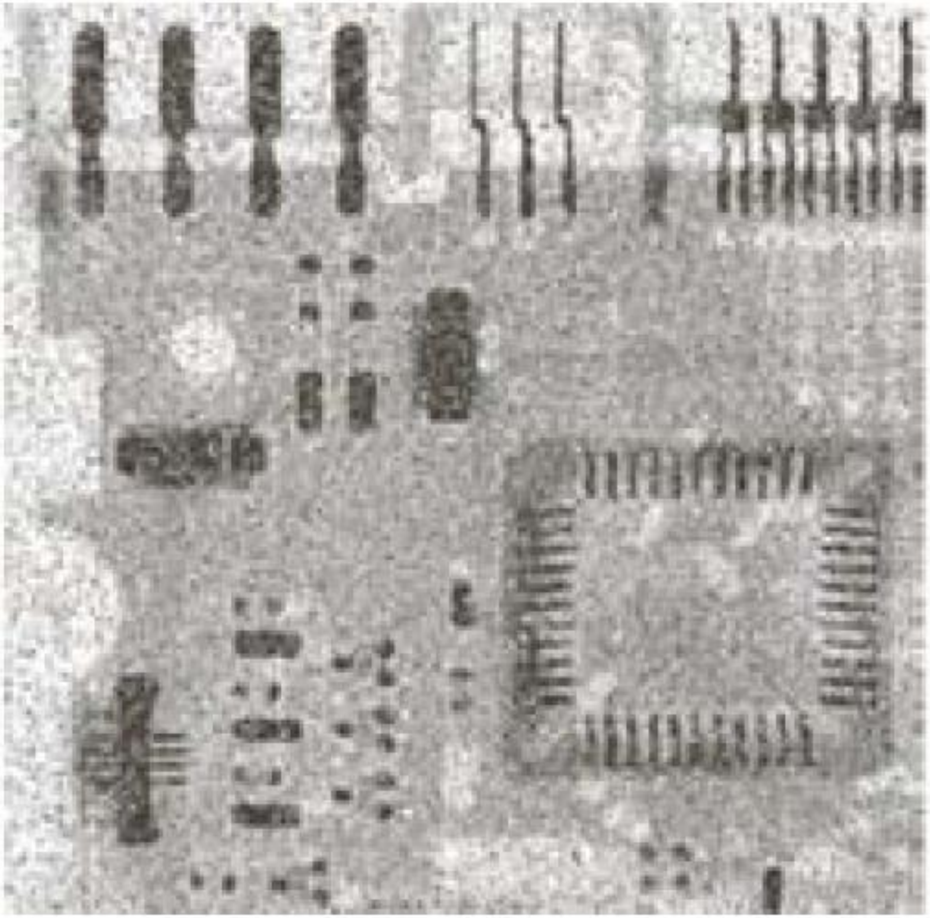


a b c

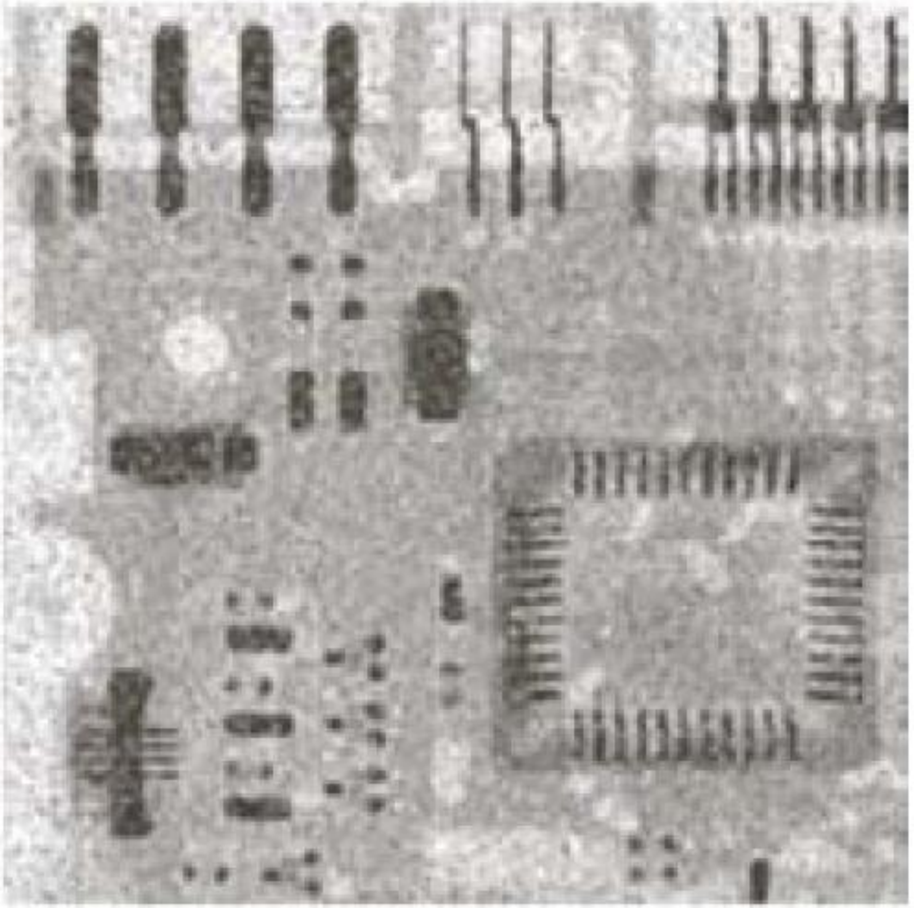
FIGURE 3.34 (a) Image of size 528 × 485 pixels from the Hubble Space Telescope. (b) Image filtered with a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

Smoothing Spatial Filters - Example

- Noise Reduction



Original



Smoothed

Sharpening Spatial Filters

- In addition to noise removal, the other two main uses of image filtering are for (a) feature extraction and (b) feature enhancement.
- The principle objective of sharpening is to highlight transitions in intensity which usually correspond to edges in images; thus sharpening is the opposite of smoothing
- If we examine the smoothing operation we can think of it as integration
- Thus to perform sharpening in the spatial domain, it is intuitive to use differentiation
- The strength of the response of a derivative operator is proportional to the degree of discontinuity of the image at the point at which the operator is applied.
  - Thus, image differentiation
    - enhances edges and other discontinuities (noise)
    - deemphasizes area with slowly varying gray-level values.

Sharpening Spatial Filters

- We are concerned about the behavior of 1st and 2nd derivatives in the following areas
  - Constant intensity
  - Onset and end of discontinuities (ramps and steps)
  - Intensity ramps
- Properties of 1st derivative
  - Zero in areas of constant intensity
  - Nonzero at the onset of a step and intensity ramp
  - Nonzero along intensity ramp
- Properties 2nd derivative
  - Zero in areas of constant intensity
  - Nonzero at the onset and end of a step and intensity ramp
  - Zero along intensity ramp

Sharpening Spatial Filters

- Derivatives can be approximated as differences
- 1st derivative at x

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

- 2nd derivative at x

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

Sharpening Spatial Filters

Investigation of derivatives behavior

Intensity profile

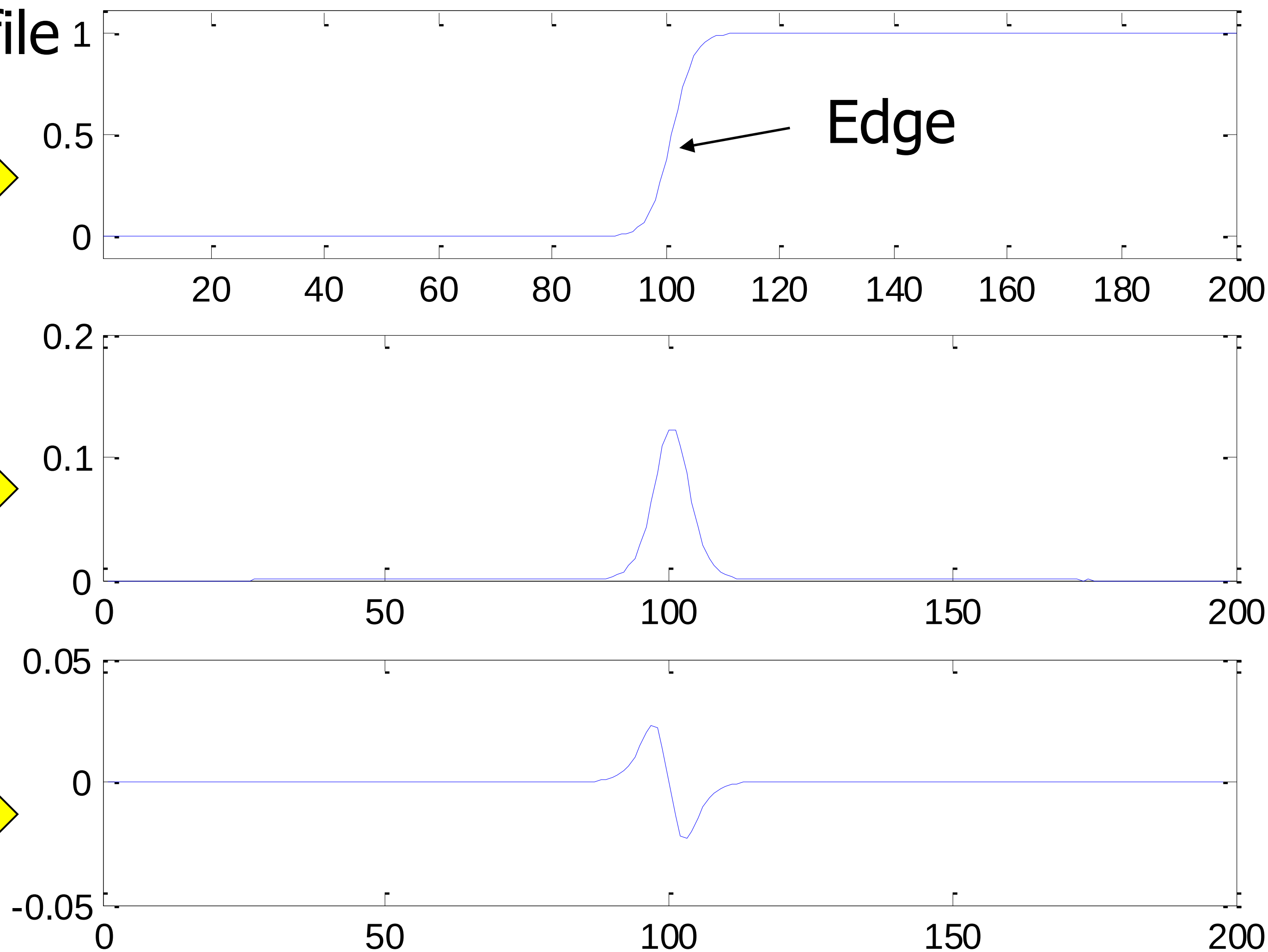
$p(x)$

1st derivative

$$\frac{dp}{dx}$$

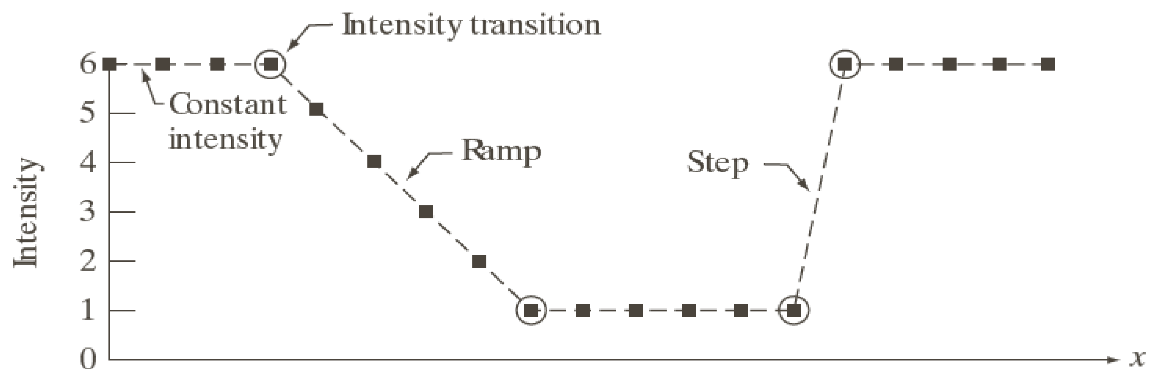
2nd derivative

$$\frac{d^2p}{dx^2}$$

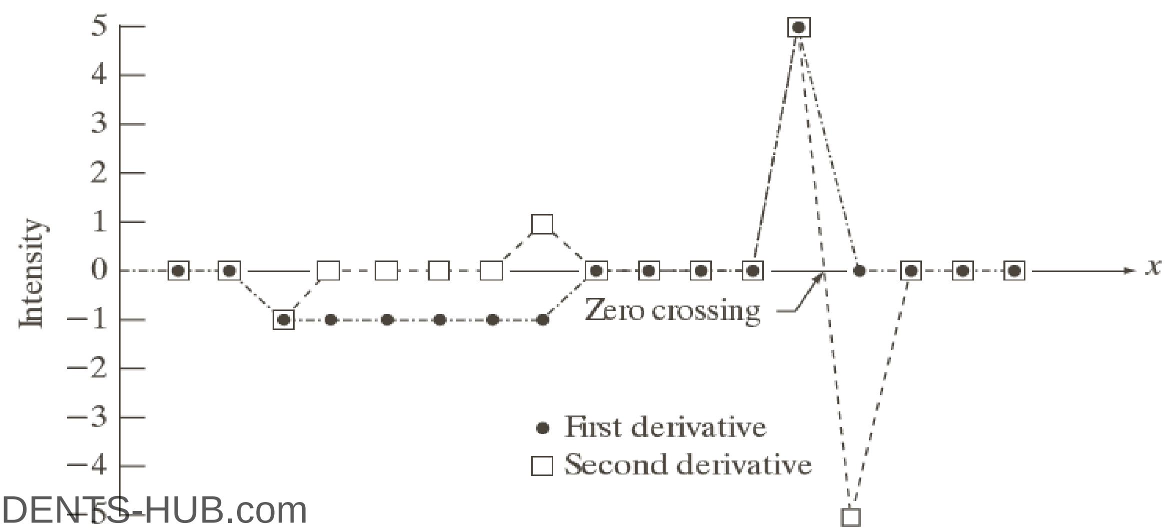


Sharpening Spatial Filters

- Investigation of derivatives behavior



Scan line	6	6	6	6	5	4	3	2	1	1	1	1	1	1	6	6	6	6	6
1st derivative		0	0	−1	−1	−1	−1	−1	0	0	0	0	0	5	0	0	0	0	
2nd derivative		0	0	−1	0	0	0	0	1	0	0	0	0	5	−5	0	0	0	



a
b
c

FIGURE 3.36 Illustration of the first and second derivatives of a 1-D digital function representing a section of a horizontal intensity profile from an image. In (a) and (c) data points are joined by dashed lines as a visualization aid.

Sharpening Spatial Filters

- **Notes**
  - Examining the 1st and 2nd derivatives plots shows that all of their properties are satisfied
  - 1st derivative produce thicker edges than 2nd derivatives
  - 2nd derivative produce double edge separated by a zero crossing
  - 2nd derivative is commonly used in sharpening since it has simpler implementation and finer edges

Sharpening Using 2nd Derivative

- when we consider an image function of two variables, f(x,y), at which time we will dealing with partial derivatives along the two spatial axes.
- The second derivative (Laplacian) in 2-D is defined as

$$\nabla^2 f = \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}$$

- If we define

$$\frac{\partial^2 f}{\partial x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$$

- Then

$$\nabla^2 f = [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)]$$

Sharpening Using 2nd Derivative

- The Laplacian can be implemented as a filter mask

0	1	0
1	-4	1
0	1	0

- Or

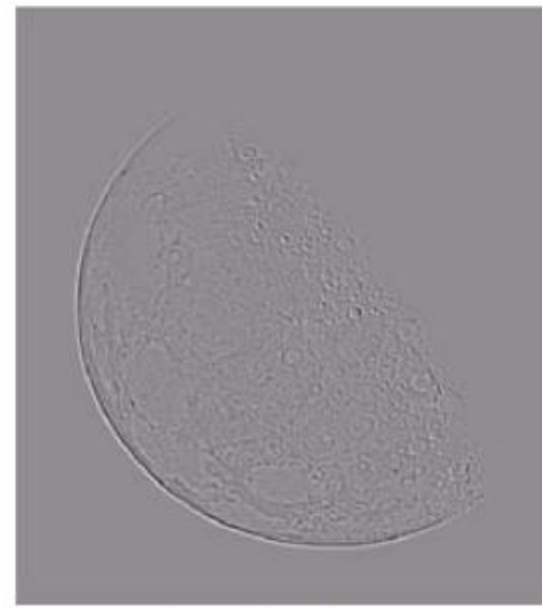
1	1	1
1	-8	1
1	1	1

Sharpening Using 2nd Derivative

- Computing the Laplacian doesn't produce a sharpened image. However, grayish edge lines and discontinuities superimposed on a dark background
  - It is common practice to scale the Laplacian image to [0,255] for better display

Original Image

Laplacian Filtered Image



Laplacian Filtered Image Scaled for Display

Sharpening Using 2nd Derivative

- Alternatively, to obtain a sharpened image g(x,y), subtract the Laplacian image from the original image

$$g(x,y) = f(x,y) - \nabla^2 f(x,y)$$

Original Image

-

Laplacian Filtered Image

=

Sharpened Image

Sharpening Using 2nd Derivative

- The two steps required to achieve sharpening can be combined into a single filtering operation

$$
\begin{aligned}
g(x,y) = f(x,y) - [&f(x+1,y) + f(x-1,y) \\
&+ f(x,y+1) + f(x,y-1) + 4f(x,y)] \\
= 5f(x,y) - [&f(x+1,y) + f(x-1,y) \\
&+ f(x,y+1) + f(x,y-1)]
\end{aligned}
$$

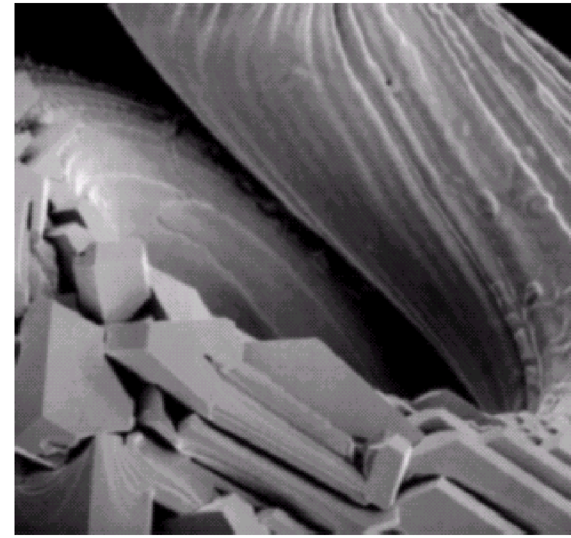
0	-1	0
-1	5	-1
0	-1	0

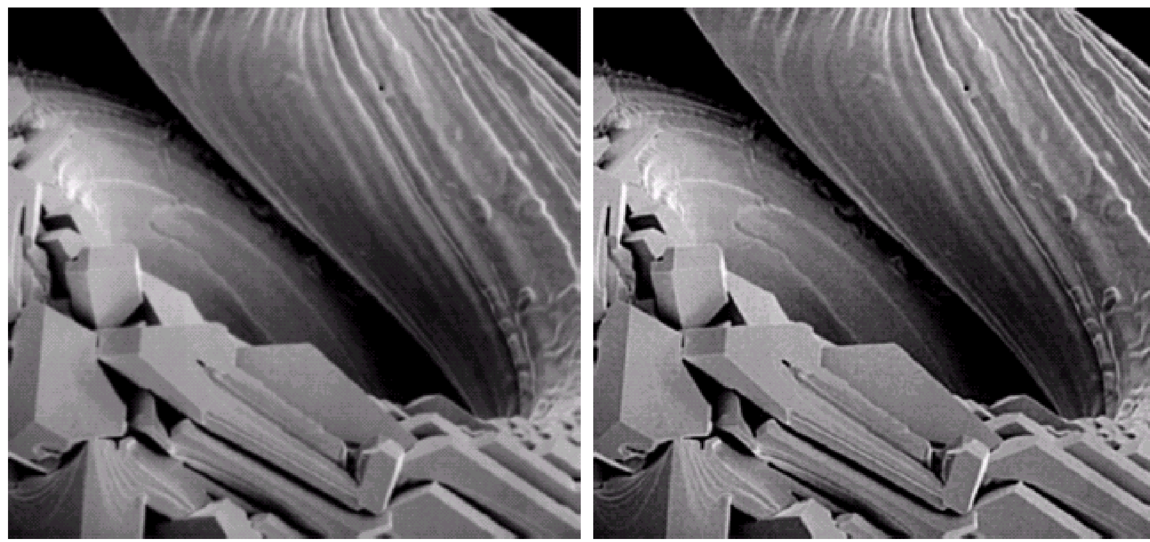
Sharpening Using 2nd Derivative

- Example

0	–1	0
–1	5	–1
0	–1	0

–1	–1	–1
–1	9	–1
–1	–1	–1





a b c
d e

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Unsharp Masking and High-Boost Filtering

- A process that has been used for many years by the printing process and publishing industry to sharpen images.

- Steps:
  - Blur the original image
  - Subtract the blurred from the original (mask)
  - Add the mask to the original

Unsharp Masking and High-Boost Filtering

- If we use Laplacian filter to create sharpen image $f_s(x,y)$ with addition of original image then:

$$f_{hb}(x,y)=\begin{cases} Af(x,y)-\nabla^2 f(x,y) \\ Af(x,y)+\nabla^2 f(x,y) \end{cases}$$

if the center coefficient of the Laplacian mask is negative

if the center coefficient of the Laplacian mask is positive

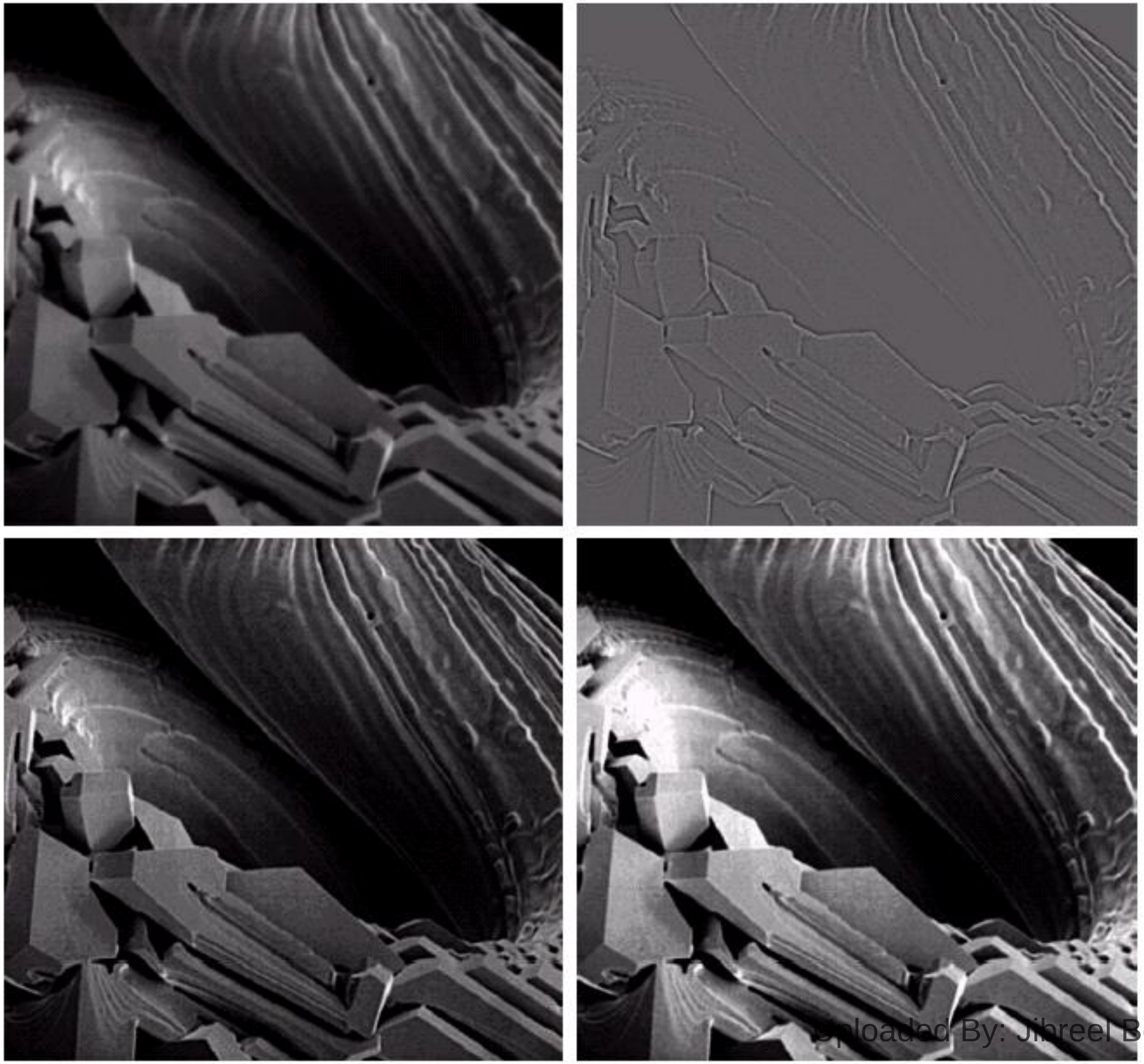
Unsharp Masking and High-Boost Filtering

0	−1	0
−1	$A+4$	−1
0	−1	0

−1	−1	−1
−1	$A+8$	−1
−1	−1	−1

- If $A \geq 1$, High-Boost
- if $A = 1$, it becomes "standard" Laplacian sharpening

Unsharp Masking and High-Boost Filtering



a b
c d

FIGURE 3.43
(a) Same as Fig. 3.41(c), but darker.
(a) Laplacian of (a) computed with the mask in Fig. 3.42(b) using $A = 0$.
(c) Laplacian enhanced image using the mask in Fig. 3.42(b) with $A = 1$. (d) Same as (c), but using $A = 1.7$.

Sharpening Using 1st Derivative

- The first derivative in image processing is implemented as a gradient which is defined as a vector

$$\nabla \mathrm{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

- The gradient points in the direction of greatest rate of change at location (x,y)
- The magnitude of the gradient is defined as

$$\nabla f = mag(\nabla \mathrm{f}) = [G_x^2 + G_y^2]^{1/2}$$

$$= \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{1/2}$$

Or, approximately

$$\nabla f \approx |G_x| + |G_y|$$

Sharpening Using 1st Derivative - Gradient Mask

- simplest approximation, 2x2

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

$$G_x = (z_8 - z_5) \quad \text{and} \quad G_y = (z_6 - z_5)$$

$$\nabla f = [G_x^2 + G_y^2]^{1/2} = [(z_8 - z_5)^2 + (z_6 - z_5)^2]^{1/2}$$

$$\nabla f \approx |z_8 - z_5| + |z_6 - z_5|$$

Sharpening Using 1st Derivative

- **Computation of the gradient using Roberts cross-gradient operators**

z1	z2	z3
z4	z5	z6
z7	z8	z9

-1	0
0	1

Horizontal Operator

0	-1
1	0

Vertical Operator

$$G_x = (z_9 - z_5) \quad \text{and} \quad G_y = (z_8 - z_6)$$

$$\nabla f = [G_x^2 + G_y^2]^{1/2} = [(z_9 - z_5)^2 + (z_8 - z_6)^2]^{1/2}$$

$$\nabla f \approx |z_9 - z_5| + |z_8 - z_6|$$

Sharpening Using 1st Derivative

- **Computation of the gradient using Sobel operators**

z1	z2	z3
z4	z5	z6
z7	z8	z9

Pixel z5 and its neighbours

-1	-2	-1
0	0	0
1	2	1

Mask to Compute Gx

-1	0	1
-2	0	2
-1	0	1

Mask to Compute Gy

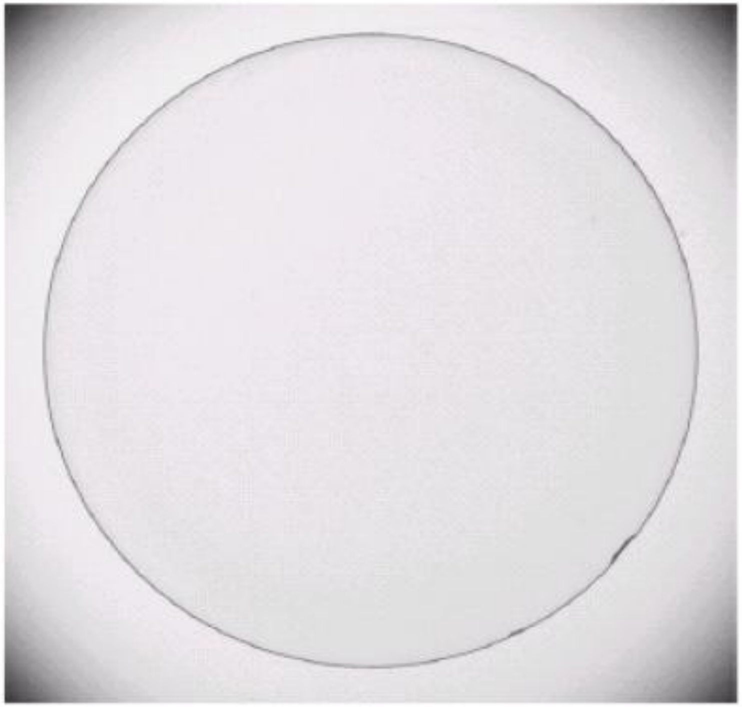
$$G_x = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)$$

$$G_y = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)$$

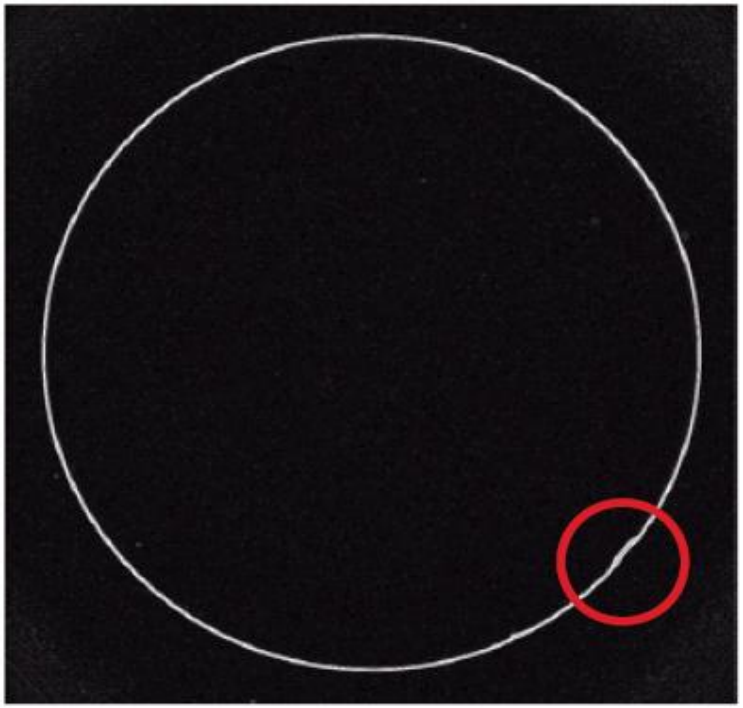
$$\nabla f \approx |G_x| + |G_y|$$

Sharpening Using 1st Derivative

- **Example**
  - Gradient is widely used in industrial inspection as it produce thicker edges in the result, which make it easier for machines to detect artifacts



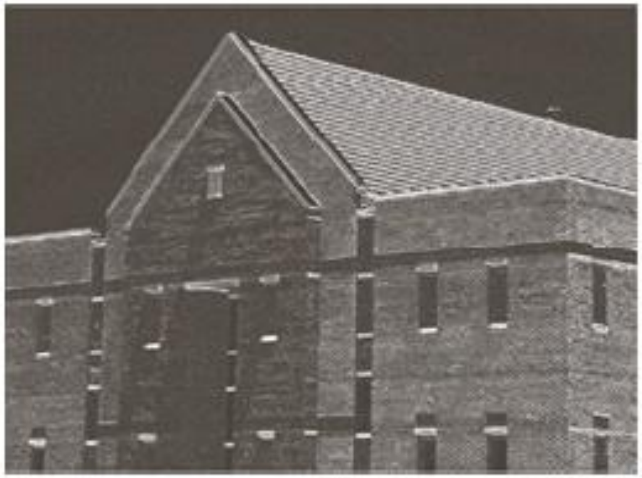
Optical Image for a contact lens



Gradient Image

Sharpening Using 1st Derivative

Image

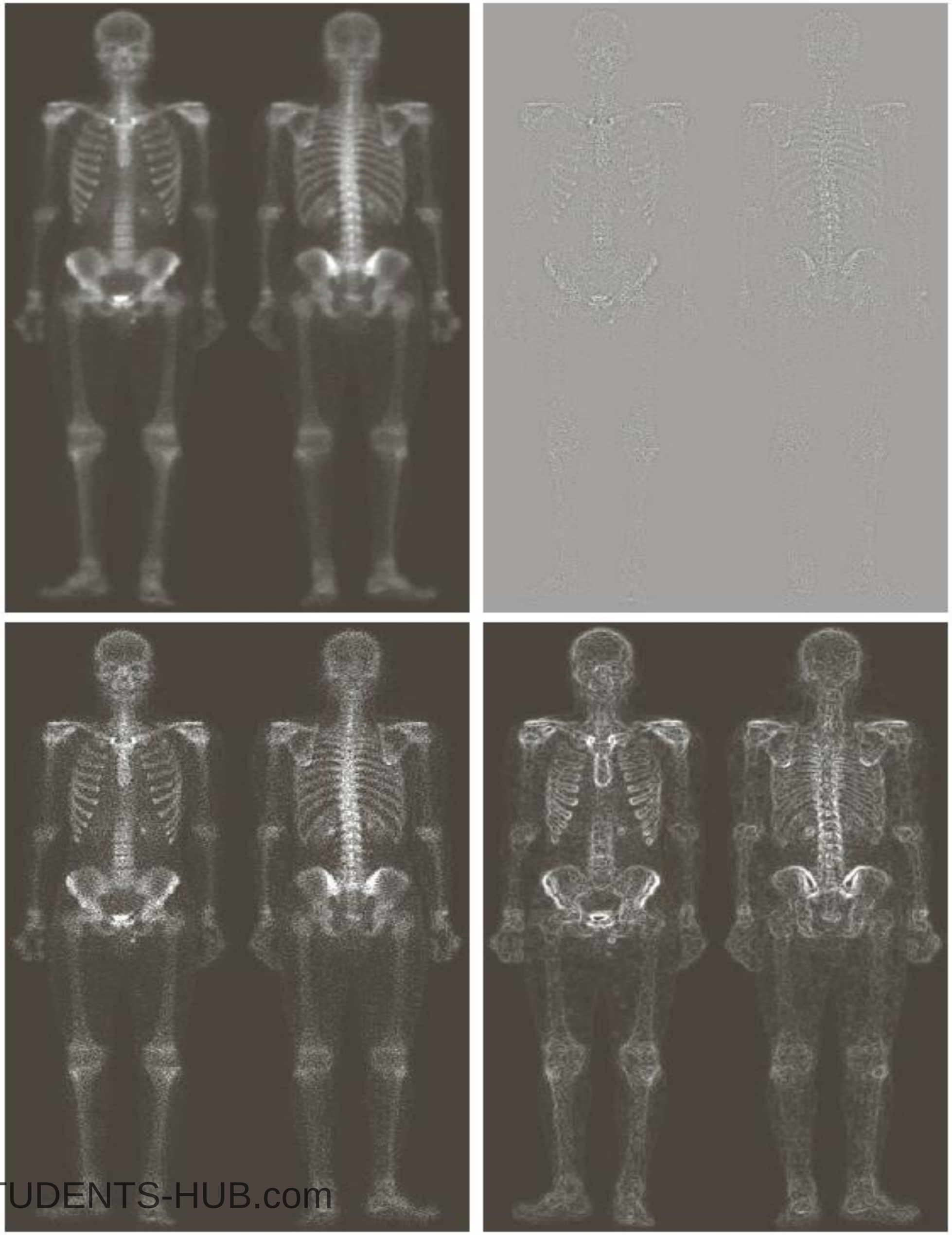


Gx computed using
Sobel Operator

Gy computed using
Sobel Operator

|Gx| + |Gy|

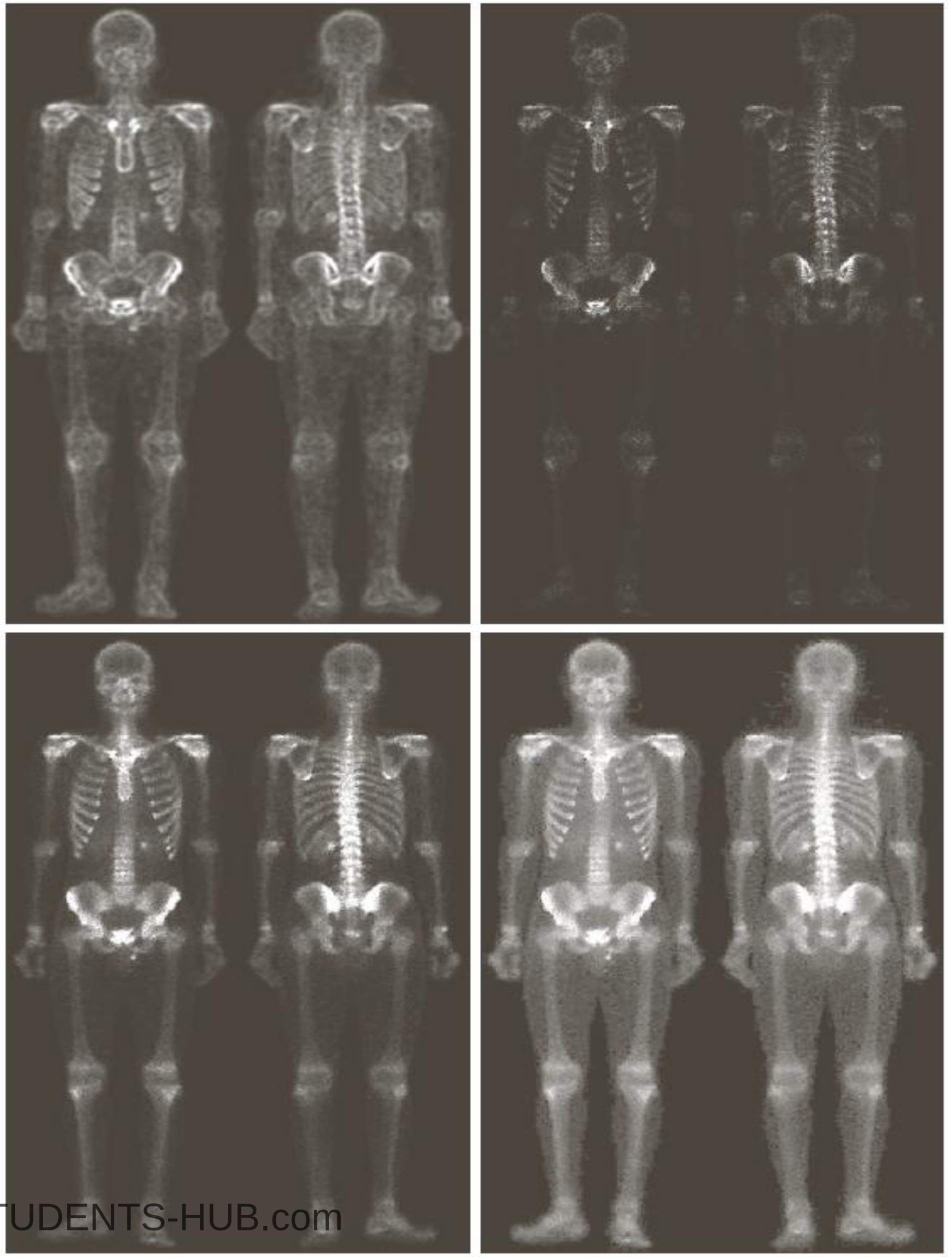
Combining Spatial Enhancements



a	b
c	d

FIGURE 3.43
(a) Image of whole body bone scan.
(b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b).
(d) Sobel gradient of (a).

Combining Spatial Enhancements



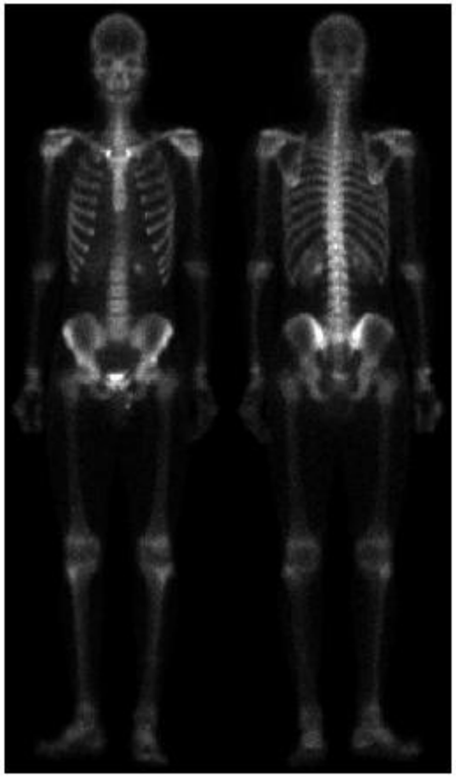
e	f
g	h

FIGURE 3.43 *(Continued)* (e) Sobel image smoothed with a 5×5 averaging filter. (f) Mask image formed by the product of (c) and (e). (g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)

Combining Spatial Enhancements

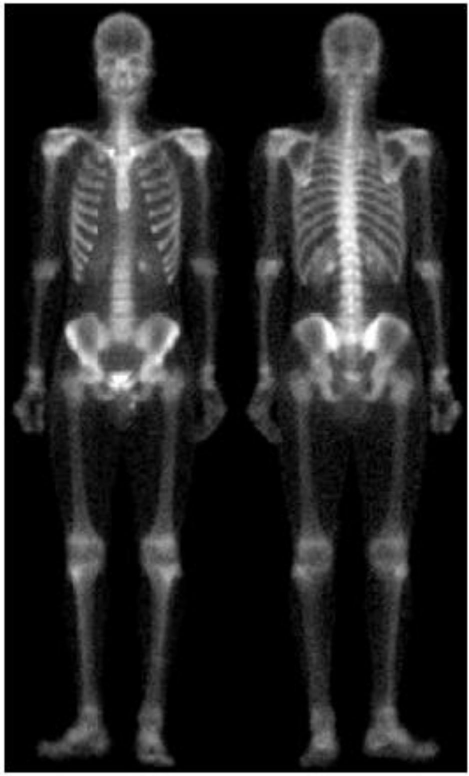
- Compare to enhancement by single method

Processed by a combination of methods



Sharpened by Laplacian

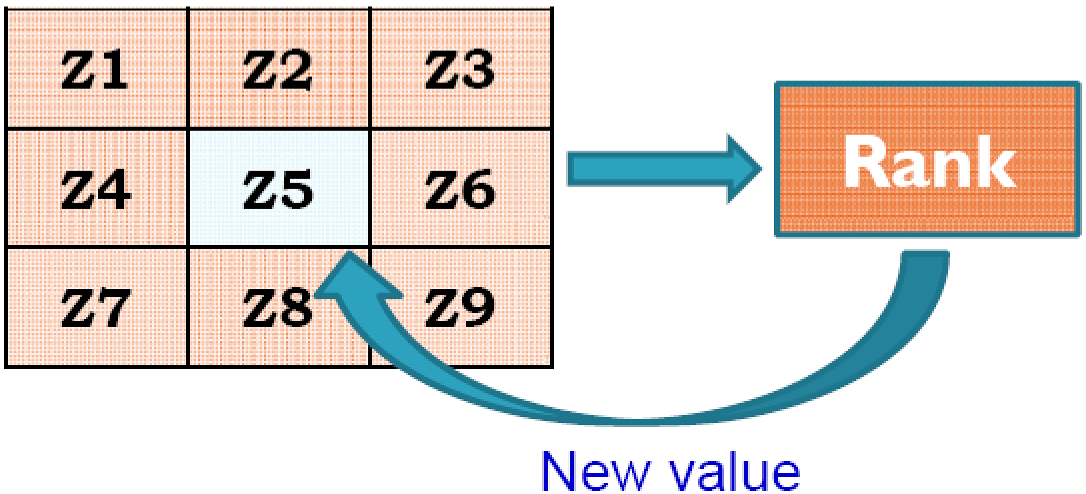
Histogram Equalization



Power-Law with gamma = 0.5

Nonlinear (Order-statistic) Spatial Filters

- Order-statistic filters are nonlinear filters whose response is based on ordering the pixels under the mask and then replacing the centre pixel with the value determined by the ranking result

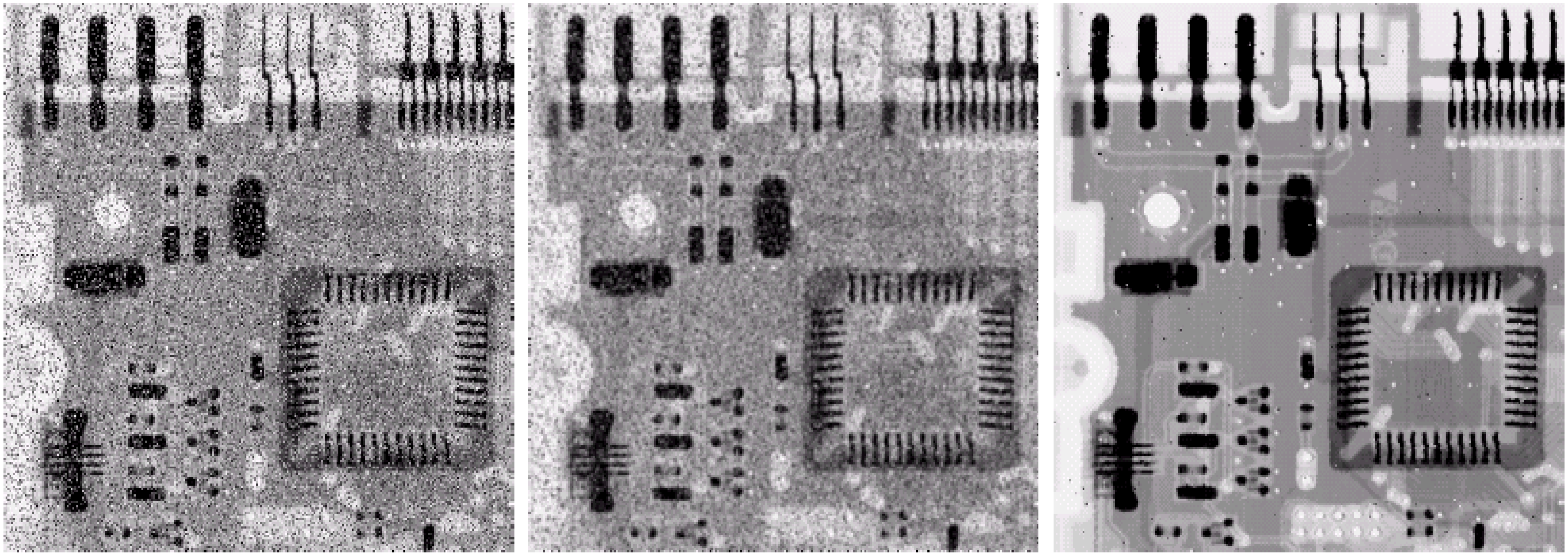


- Example
  - median filter : $R = \text{median}\{z_k \mid k = 1,2,\ldots,n \times n\}$
  - max filter : $R = \max\{z_k \mid k = 1,2,\ldots,n \times n\}$
  - min filter : $R = \min\{z_k \mid k = 1,2,\ldots,n \times n\}$
    - note: n x n is the size of the mask

Median Filters

- Replaces the value of a pixel by the median of the gray levels in the neighborhood of that pixel
  - The original value of the pixel is included in the computation of the median)
- Quite popular because for certain types of random noise (impulse noise ⇨ salt and pepper noise) , they provide excellent noise-reduction capabilities, with considering less blurring than linear smoothing filters of similar size.
- Forces the points with distinct gray levels to be more like their neighbors.
- Isolated clusters of pixels that are light or dark with respect to their neighbors, and whose area is less than $n^2/2$ (one-half the filter area), are eliminated by an n x n median filter.
- Eliminated = forced to have the value equal the median intensity of the neighbors.
- Larger clusters are affected considerably less

Median Filters - Example



a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)