9.8 Higher Order Derivatives

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مشتقات عليا

$y = f(x)$

1st derivative $y' = \frac{dy}{dx} = \frac{df}{dx} = f'(x)$

2nd derivative $y'' = \frac{d^2y}{dx^2} = \frac{d^2f}{dx^2} = f''(x)$

...

3rd " $y''' = f'''(x)$

4th derivative $y'''' = f''''(x) = y^{(4)}$ y super 4

⋮

nth derivative $y^{(n)} = f^{(n)}(x)$ y super n

f super n

$y^n = \underbrace{y \cdot y \cdot y \cdots y}_{n \text{ times}}$

$y^{(n)}$ n^{th} derivative of $y \equiv y$ super n

Exp Find y'' if $y = x^5 - 2x^4 + 3x^3 + 8$

$y' = 5x^4 - 8x^3 + 9x^2 + 0$

$y'' = 20x^3 - 24x^2 + 18x$

$y''' = 60x^2 - 48x + 18$

Find $y'''(1) \Rightarrow$ $y'''(1) = 60(1)^2 - 48(1) + 18$

$= 60 - 48 + 18$

$= 12 + 18$

$= 30$

Exp Find $f^{(4)}(-1)$ for $f(x) = 2x^4 - 3x^3 + 7x^2 - 5x$

$f'(x) = 8x^3 - 9x^2 + 14x - 5$

$f''(x) = 24x^2 - 18x + 14$

18

$f'(x) = \ldots$

$f''(x) = 48x^{-18}$

$f^{(4)}(x) = 48$

$f^{(4)}(-1) = 48$

Ex Find $f^{(10)}(2022)$ for $f(x) = 7x^8 - 6x^7 + 35$

$f' = 56x^7 - 42x^6$

$f'' = \ldots x^6 - \ldots x^5$

$f''' = \ldots x^5 - \ldots x^4$

$f'''' = \ldots x^4 - \ldots x^3$

$f^{(5)} = \ldots x^3 - \ldots x^2$

$f^{(6)} = \ldots x^2 - \ldots x^1$

$f^{(7)} = \ldots x - \ldots$

$f^{(8)} = \ldots$

$f^{(9)} = 0$

$f^{(10)}(x) = 0$

$f^{(10)}(2022) = 0$

Ex Find $y'(0)$ if $y = \sqrt[2]{(3x+1)^1}$

$= (3x+1)^{\frac{1}{2}}$

$y' = \frac{1}{2}(3x+1)^{\frac{1}{2}-1}(3)$ | $\frac{1}{2} - 1$, $\frac{1}{2} - \frac{2}{2}$, $\frac{-1}{2}$

$= \frac{3}{2}(3x+1)^{-\frac{1}{2}}$

$-\frac{1}{2} - 1$

$-\frac{1}{2} - \frac{2}{2}$

$\frac{-1-2}{}$

$y'' = \frac{3}{2}(-\frac{1}{2})(3x+1)^{-\frac{1}{2}-1}(3)$

$(3x+1)^{-\frac{3}{2}}$

$\frac{-1-2}{2}$

$\frac{-3}{2}$

$= \frac{-3}{4}(3)(3x+1)^{-\frac{3}{2}}$

$y'' = -\frac{9}{4} \cdot \frac{1}{(3x+1)^{\frac{3}{2}}}$

$y'' = -\frac{9}{4} \cdot \frac{1}{\sqrt{(3x+1)^3}}$

$y''(0) = -\frac{9}{4} \cdot \frac{1}{\sqrt{(3(0)+1)^3}}$

$= -\frac{9}{4} \cdot \frac{1}{\sqrt{1^3}}$

$= -\frac{9}{4}$

Exp Find ρ'' if $\rho = \frac{1}{q^3} + \sqrt[2]{q^3} - 2q^3$

Exp Find p if $p = q^3$ ...

$$p = q^{-3} + q^{\frac{3}{2}} - 2q^3$$

$$p' = -3q^{-4} + \frac{3}{2}q^{\frac{3}{2}-1} - 6q^2$$

$\frac{3}{2} - 1$

$\frac{3}{2} - \frac{2}{2}$

$\frac{3-2}{2}$

$\frac{1}{2}$

$$p' = -3q^{-4} + \frac{3}{2}q^{\frac{1}{2}} - 6q^2$$

$$p'' = 12q^{-5} + \frac{3}{2}\left(\frac{1}{2}\right)q^{\frac{1}{2}-1} - 12q$$

$$p'' = 12q^{-5} + \frac{3}{4}q^{-\frac{1}{2}} - 12q$$

$$p''' = -60q^{-6} + \frac{3}{4}\left(-\frac{1}{2}\right)q^{-\frac{1}{2}-1} - 12$$

$$p''' = -60q^{-6} - \frac{3}{8}q^{-\frac{3}{2}} - 12$$