Fundamentals Physics

Tenth Edition

Halliday

Chapter 9_1

Center of Mass and Linear Momentum

9-1 Center of Mass (2 of 12)

- The motion of rotating objects can be complicated (imagine flipping a baseball bat into the air)
- But there is a special point on the object for which the motion is simple
- The **center of mass** of the bat traces out a parabola, just as a tossed ball does
- All other points rotate around this point

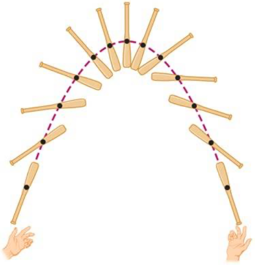


Figure 9-1

9-1 Center of Mass (3 of 12)

- The **center of mass** (com) of a system of particles:

  The center of mass of a system of particles is the point that moves as though: (1) all of the system's mass were concentrated there and (2) all external forces were applied there.

- For two particles separated by a distance *d*, where the origin is chosen at the position of particle 1:

$$x_{com} = \frac{m_2}{m_1 + m_2} d.$$

Equation (9-1)

9-1 Center of Mass (4 of 12)

- For two particles, for an arbitrary choice of origin:

$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}.$$

Equation (9-2)

9-1 Center of Mass (5 of 12)

- The center of mass is in the same location regardless of the coordinate system used
- It is a property of the particles, not the coordinates

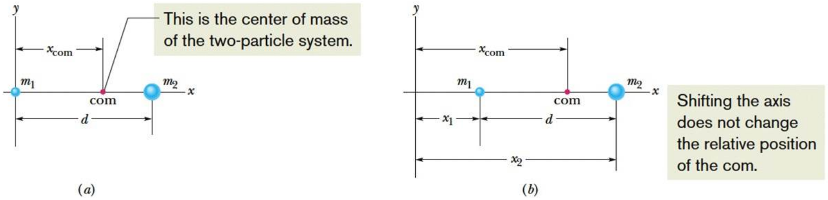


Figure 9-2

9-1 Center of Mass (6 of 12)

- For many particles, we can generalize the equation, where $M = m_1 + m_2 + \ldots + m_n$:

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots + m_nx_n}{M} = \frac{1}{M}\sum_{i=1}^{n} m_ix_i.$$

Equation (9-4)

9-1 Center of Mass (7 of 12)

- In three dimensions, we find the center of mass along each axis separately:

$$x_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i, \quad y_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i, \quad z_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i.$$

Equation (9-5)

9-1 Center of Mass (8 of 12)

- More concisely, we can write in terms of vectors:

$$\vec{r}_{\text{com}} = \frac{1}{M} \sum_{i=1}^{n} m_i \vec{r}_i, \qquad \textbf{Equation (9-8)}$$

- For solid bodies, we take the limit of an infinite sum of infinitely small particles → **integration**!
- Coordinate-by-coordinate, we write:

$$x_{\text{com}} = \frac{1}{M} \int x\, dm, \quad y_{\text{com}} = \frac{1}{M} \int y\, dm, \quad z_{\text{com}} = \frac{1}{M} \int z\, dm, \qquad \textbf{Equation (9-9)}$$

- Here M is the mass of the object

9-1 Center of Mass (9 of 12)

- We limit ourselves to objects of uniform density, ρ, for the sake of simplicity

$$\rho = \frac{dm}{dV} = \frac{M}{V},$$

Equation (9-10)

- Substituting, we find the center of mass simplifies:

$$x_{\text{com}} = \frac{1}{V}\int x\,dV, \quad y_{\text{com}} = \frac{1}{V}\int y\,dV, \quad z_{\text{com}} = \frac{1}{V}\int z\,dV.$$

Equation (9-11)

- You can bypass one or more of these integrals if the object has symmetry

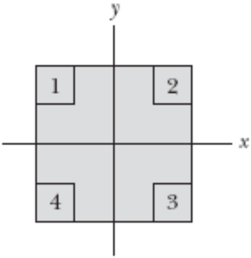
9-1 Center of Mass (10 of 12)

- The center of mass lies at a point of symmetry (if there is one)
- It lies on the line or plane of symmetry (if there is one)
- It needs not be on the object (consider a doughnut)

9-1 Center of Mass (11 of 12)

Checkpoint 1

The figure shows a uniform square plate from which four identical squares at the corners will be removed. (a) Where is the center of mass of the plate originally? Where is it after the removal of (b) square 1; (c) squares 1 and 2; (d) squares 1 and 3; (e) squares 1, 2, and 3; (f) all four squares? Answer in terms of quadrants, axes, or points (without calculation, of course).



Answer:

(a) at the origin

(b) in Q_4, along $y = -x$

(c) along the $-y$ axis

(d) at the origin

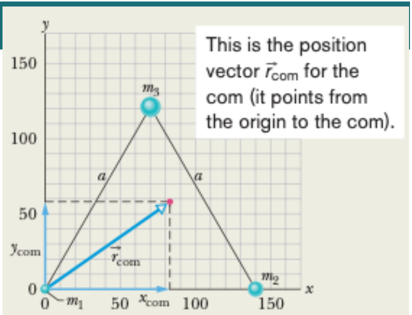
(e) in Q_3, along $y = x$

(f) at the origin

Sample Problem 9.01

$m_1 = 1.2$ kg, $m_2 = 2.5$ kg, and $m_3 = 3.4$ kg form an equilateral triangle of edge length $a = 140$ cm.

Where is the center of mass of this system?



$$x_{com} = \frac{1}{M}\sum_{i=1}^{3} m_i x_i = \frac{m_1x_1 + m_2x_2 + m_3x_3}{M}$$
$$= \frac{(1.2\text{ kg})(0) + (2.5\text{ kg})(140\text{ cm}) + (3.4\text{ kg})(70\text{ cm})}{7.1\text{ kg}}$$
$$= 83\text{ cm} \qquad \text{(Answer)}$$

$$y_{com} = \frac{1}{M}\sum_{i=1}^{3} m_i y_i = \frac{m_1y_1 + m_2y_2 + m_3y_3}{M}$$
$$= \frac{(1.2\text{ kg})(0) + (2.5\text{ kg})(0) + (3.4\text{ kg})(120\text{ cm})}{7.1\text{ kg}}$$
$$= 58\text{ cm}. \qquad \text{(Answer)}$$

The center of mass is located by the position vector :$\vec{r}_{com}$

Sample Problem 9.02

Example Subtracting

- Task: find com of a disk with another disk taken out of it:
- Find the com of each individual disk (start from the bottom and work up)
- Find the com of the two individual coms (one for each disk), treating the cutout as having negative mass
- On the diagram, com_C is the center of mass for Plate *P* and Disk *S* combined
- com_P is the center of mass for the composite plate with Disk *S* removed

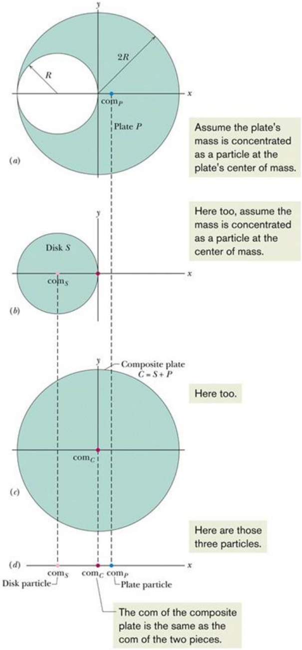


Figure 9-4

Sample Problem 9.02

use Eq. 9-2: $x_{S+P} = \dfrac{m_S x_S + m_P x_P}{m_S + m_P}$ (9-12)

and $x_{S+P} = x_C = 0$

We get $x_P = -x_S \dfrac{m_S}{m_P}$. (9-13)

mass = density x volume
= density x thickness x area.

then $\dfrac{m_S}{m_P} = \dfrac{\text{density}_S}{\text{density}_P} \times \dfrac{\text{thickness}_S}{\text{thickness}_P} \times \dfrac{\text{area}_S}{\text{area}_P}$

$$\frac{m_S}{m_P} = \frac{\text{area}_S}{\text{area}_P} = \frac{\text{area}_S}{\text{area}_C - \text{area}_S}$$

$$= \frac{\pi R^2}{\pi(2R)^2 - \pi R^2} = \frac{1}{3}$$

Substituting this and $x_S = -R$ in (9-13): $x_P = (1/3)\,R$.

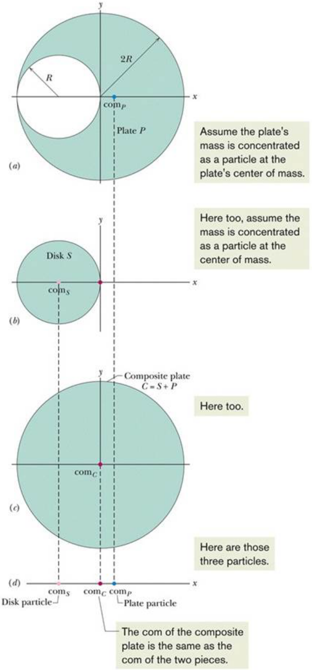


Figure 9-4

9-2 Newton's Second Law for a System of Particles (4 of 6)

- Center of mass motion continues unaffected by forces internal to a system (collisions between billiard balls)
- Motion of a system's center of mass:

$$\vec{F}_{\text{net}} = M\vec{a}_{\text{com}} \quad (\text{system of particles}). \qquad \textbf{Equation (9-14)}$$

$$F_{\text{net},x} = Ma_{\text{com},x} \quad F_{\text{net},y} = Ma_{\text{com},y} \quad F_{\text{net},z} = Ma_{\text{com},z}. \qquad \textbf{Equation (9-15)}$$

- Reminders:
  1. F_{net} is the sum of all external forces
  2. M is the total, constant, mass of the **closed** system
  3. a_{com} is the center of mass acceleration

9-2 Newton's Second Law for a System of Particles (5 of 6)

Examples Using the center of mass motion equation:

- Billiard collision: forces are only internal, $F = 0$ so $a = 0$
- Baseball bat: $a = g$, so com follows gravitational trajectory
- Exploding rocket: explosion forces are internal, so only the gravitational force acts on the system, and the com follows a gravitational trajectory as long as air resistance can be ignored for the fragments.

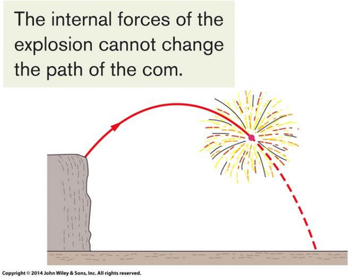


Figure 9-5

9-2 Newton's Second Law for a System of Particles (6 of 6)

Checkpoint 2

Two skaters on frictionless ice **hold opposite ends of a pole** of negligible mass. An axis runs along it, with the origin at the center of mass of the two-skater system. One skater, **Fred, weighs twice** as much as the other skater, Ethel. **Where do the skaters meet if** (a) Fred pulls hand over hand along the pole so as to draw himself to Ethel, (b) Ethel pulls hand over hand to draw herself to Fred, and (c) both skaters pull hand over hand?

Answer: The **system** consists of Fred, Ethel and the pole. All **forces are internal**. Therefore the com will remain in the **same place**. Since the origin is the com, they will meet at the origin in all three cases! (not that the **origin** where the com is located is **closer to Fred** than to Ethel.)

9-3 Linear Momentum (3 of 7)

- The **linear momentum** is defined as:

$$\vec{p} = m\vec{v}$$ **Equation (9-22)**

- The momentum:
  - Points in the same direction as the velocity
  - Can only be changed by a net external force

The **time rate of change of the momentum** of a particle is **equal to the net force acting on the particle** and is **in the direction of that force.**

9-3 Linear Momentum (4 of 7)

- We can write Newton's second law thus:

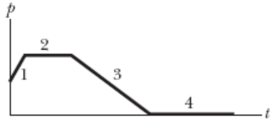
$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}. \qquad \textbf{Equation (9-23)}$$

9-3 Linear Momentum (5 of 7)

Checkpoint 3

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}.$$

The figure gives the magnitude p of the linear momentum versus time t for a particle moving along an axis. A force directed along the axis acts on the particle. (a) Rank the four regions indicated according to the **magnitude** of the force, greatest first. (b) In which region is the particle slowing?



Answer:

(a) 1, 3, 2 & 4

(b) region 3

9-3 Linear Momentum (6 of 7)

- We can sum momenta for a system of particles to find:

$$\vec{P} = M\vec{v}_{\text{com}} \quad \text{(linear momentum, system of particles)},$$

Equation (9-25)

The **linear momentum of a system** of particles is equal to the **product of** the total mass ***M*** **of the system and the velocity of the center of mass.**

9-3 Linear Momentum (7 of 7)

- Taking the time derivative we can write Newton's second law for a system of particles as:

$$\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt} \quad (\text{system of particles}), \qquad \textbf{Equation (9-27)}$$

- The net external force on a system changes linear momentum
- Without a net external force, the total linear momentum of a system of particles cannot change

9-4 Collision and Impulse (3 of 9)

- In a collision, momentum of a particle can change
- We define the **impulse *J*** acting during a collision:

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\, dt$$

Equation (9-30)

- This means that the applied impulse is equal to the change in momentum of the object during the collision

$$\Delta\vec{p} = \vec{J} \quad \left(\text{linear momentum-impulse theorem}\right).$$

Equation (9-31)

- This equation can be rewritten component-by-component, like other vector equations

9-4 Collision and Impulse (4 of 9)

- Given F_{avg} and duration:

$$J = F_{avg}\Delta t. \quad \textbf{Equation (9-35)}$$

- We are integrating: we only need to know the area under the force curve

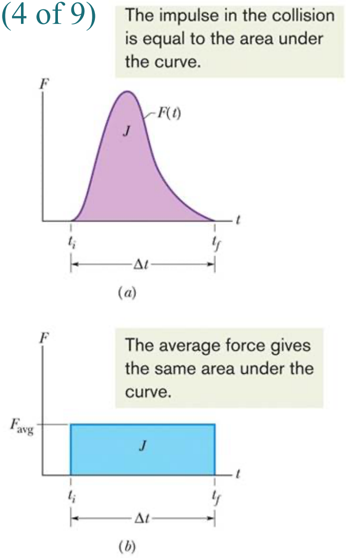


Figure 9-9

9-4 Collision and Impulse (5 of 9)

Checkpoint 4

A paratrooper whose chute fails to open lands in snow; he is hurt slightly. Had he landed on bare ground, the stopping time would have been 10 times shorter and the collision lethal. Does the presence of the snow increase, decrease, or leave unchanged the values of (a) the paratrooper's change in momentum, (b) the impulse stopping the paratrooper, and (c) the force stopping the paratrooper?

Answer:

(a) Unchanged (initial momentum is the same as final momentum)

(b) Unchanged (impulse is equal to change in momentum)

(c) Decreased (Δt increased 10 times)

$$J = F_{avg}\Delta t.$$

9-4 Collision and Impulse (6 of 9)

- For a steady stream of n projectiles, each undergoes a momentum change Δp

$$J = -n\,\Delta p,$$

Equation (9-36)

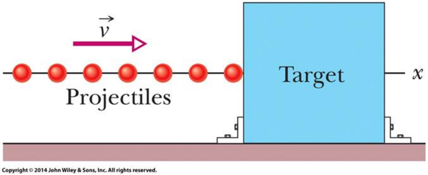


Figure 9-10

9-4 Collision and Impulse (7 of 9)

- The average force is:

$$F_{avg} = \frac{J}{\Delta t} = -\frac{n}{\Delta t}\Delta p = -\frac{n}{\Delta t} m\,\Delta v.$$

Equation (9-37)

- If the particles stop:

$$\Delta v = v_f - v_i = 0 - v = -v,$$

Equation (9-38)

- If the particles bounce back with equal speed:

$$\Delta v = v_f - v_i = -v - v = -2v.$$

Equation (9-39)

9-4 Collision and Impulse (8 of 9)

- The product nm is the total mass for n collisions so we can write:

$$F_{\text{avg}} = -\frac{\Delta m}{\Delta t}\Delta v.$$

Equation (9-40)