STAT2361

ملاحظات المحاضرات

من إعداد موقع

BZU-HUB

Chapter 9

ملاحظة: المحاضرات من شرح الدكتور محمد مضية

Chapter (9): Hypothesis testing

- Sample
- probability

Hypothesis → (Claim / Statment) about unknown parameter

- True
- False

*** 2 Types of Hypothesis**

1. The null hypothesis H_0: Claim about a parameter (μ)
   → suppose to be true ما لم يثبت العكس

2. The alternative hypothesis H_a: Researcher Hypothesis

مثال ←

H_0: المتهم بريء حتى يثبت العكس

H_a: المتهم غير بريء

Conclusion

- Reject H_0
- Do not Reject H_0 (Accept)

* Hypothesis testing for μ : σ - known case (z)

* Hypothesis testing for μ : σ - unknown case (t)

*** Steps :-**

1. State the null and the alternative hypothesis (H_0 , H_a)

2. Select a level of significance
   - Type I error: Reject a true null hypothesis
   - Type II error: Do not Reject a false hypothesis

*The probability of type I error is called a level of significance

$P(\text{type I error}) = \alpha$

- α is given (select)
- Common values of α: 10%, 5%, 1%

Researcher

	Do not Reject H_0	Reject H_0
H_0 is True	correct Conclusion	Type I error
H_0 is False	Type II error	correct Conclusion.

P(Type II error) = β (calculated)

The power of the test = $1-\beta$

$\boxed{3}$ Identify a test statistic

- Hypothesis test for μ σ– Known case ⟹ $z = \dfrac{\bar{x}-\mu^{\circ}}{\sigma/\sqrt{n}} = \dfrac{\bar{x}-\mu^{\circ}}{\sigma}\sqrt{n}$
- Hypothesis test for μ σ– unknown case ⟹ $t = \dfrac{\bar{x}-\mu^{\circ}}{s/\sqrt{n}}$

The value of μ in H_0

$\boxed{4}$ Formulate a rejection Rule

Critical values approch

P-value approch

$\boxed{5}$ Conclusion

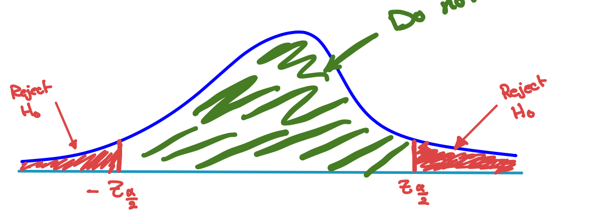
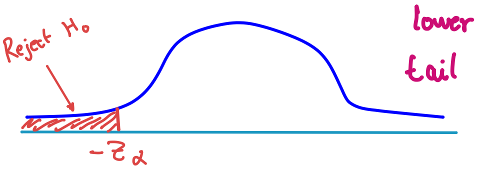
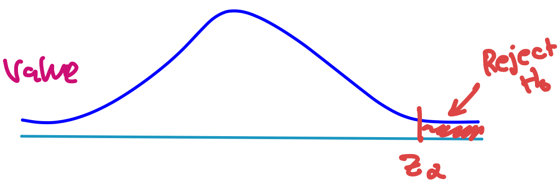
→ Reject No → the test is significant

→ Do not Reject

* Hypothesis testing for μ : σ– Known case

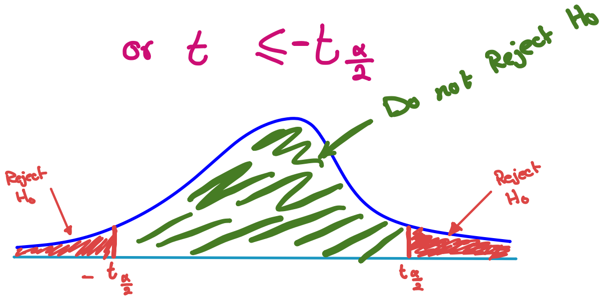
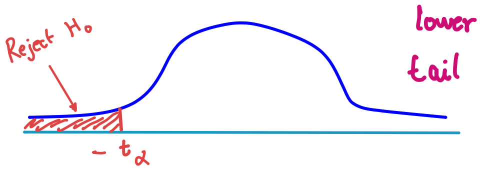
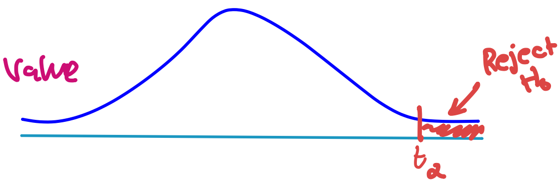
Given μ°, α

H_a → تحد

#	Hypothesis	Test	Rejection Rule (Conclusion)
1	$H_0: \mu = \mu^{\circ}$ $H_a: \mu \neq \mu^{\circ}$	$z = \dfrac{\bar{x}-\mu^{\circ}}{\sigma/\sqrt{n}}$	Reject H_0 if $z \geq z_{\frac{\alpha}{2}}$ or $z \leq -z_{\frac{\alpha}{2}}$ 
2	$H_0: \mu \geq \mu^{\circ}$ $H_a: \mu < \mu^{\circ}$	$z = \dfrac{\bar{x}-\mu^{\circ}}{\sigma/\sqrt{n}}$	Reject H_0 if $z < -z_{\alpha}$ lower (left) tail test Critical value ↳ $-z_{\alpha}$ 
3	$H_0: \mu \leq \mu^{\circ}$ $H_a: \mu > \mu^{\circ}$	$z = \dfrac{\bar{x}-\mu^{\circ}}{\sigma/\sqrt{n}}$	Reject H_0 if $z > z_{\alpha}$ upper (Right) tail test Critical value z_{α} 

* Hypothesis testing for μ : σ – unknown case $H_a \rightarrow$ تحدد

Given μ°, α

#	Hypothesis	Test	Rejection Rule (Conclusion)
1	$H_0: \mu = \mu^{\circ}$ $H_a: \mu \neq \mu^{\circ}$	$t = \dfrac{\bar{x} - \mu^{\circ}}{S / \sqrt{n}}$	Reject H_0 if $t \geq t_{\frac{\alpha}{2}}$ or $t \leq -t_{\frac{\alpha}{2}}$ 
2	$H_0: \mu \geq \mu^{\circ}$ $H_a: \mu < \mu^{\circ}$	$t = \dfrac{\bar{x} - \mu^{\circ}}{S / \sqrt{n}}$	Reject H_0 if $t < -t_{\alpha}$ lower (left) tail test Critical value $\hookrightarrow -t_{\alpha}$ 
3	$H_0: \mu \leq \mu^{\circ}$ $H_a: \mu > \mu^{\circ}$	$t = \dfrac{\bar{x} - \mu^{\circ}}{S / \sqrt{n}}$	Reject H_0 if $t > t_{\alpha}$ upper (Right) tail test Critical value t_{α} 

* ال H_a هي الي بتحدد إذا upper أو lower

*Ex:

H_0 : $\mu = 10$
H_a : $\mu > 10$ $\longrightarrow$ upper tail

***Example:** Consider the following hypothesis test

$H_0: \mu \le 20 \rightarrow \mu_0$ $\Rightarrow$ upper tail test

$H_a: \mu > 20$

A sample of size 50 provided a sample mean of 20.6
Assume that the population S.D is 2, use $\alpha = 5\%$. $(\sigma = 2)$

$\boxed{1}$ Compute the test Statistic

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{20.6 - 20}{2} \cdot \sqrt{50} = \boxed{2.12}$$

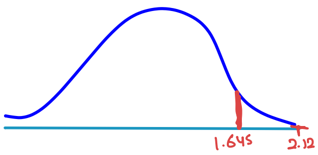
$\boxed{2}$ what is the rejection Rule

Reject Ho if $z > z_\alpha$

$\boxed{3}$ Critical Value(s)

$z_\alpha = z_{0.05} = 1.645$

$\boxed{4}$ Conclusion



$z = 2.12 > z_\alpha = 1.645$

$\Rightarrow$ Reject H_0

* نفس المثال بس إذا كانت σ- unknown case بنستخدم t

* Hypothesis test:-

1. H_0 , H_a
2. select α → 1%, 5%, 10%
3. test → z, t
4. Rejection Rule → Critical value(s), P- values
5. Conclusion → Reject, Do not Reject

13. Consider the following hypothesis test:

$$H_0: \mu \le 50$$
$$H_a: \mu > 50$$

upper $z_c < z_\alpha$

A sample of 60 (n) is used and the population (σ- known) standard deviation is 8 (σ). Use the critical value approach to state your conclusion for each of the following sample results. Use $\alpha = .05$ (.01).

a. $\bar{x} = 52.5$
b. $\bar{x} = 51$
c. $\bar{x} = 51.8$

$z_c = \dfrac{52.5 - 50}{8} \sqrt{60}$: $z_c > z_\alpha$

① $\bar{x} = 52.5$

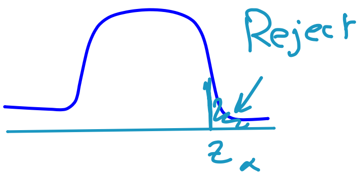
• test : $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{52.5 - 50}{8/\sqrt{60}} = 2.42$

• Rejection Rule : Reject if $z > z_\alpha$

$\alpha = 0.01 \Rightarrow z_{0.01} = 2.326$

• Conclusion

Reject



* P_Value

$Z_c \Rightarrow$ Z - calculated

$$Z_c = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

1. Upper tail : P_value $= P(Z > Z_c)$
2. Lower tail : P_Value $= P(Z < Z_c)$
3. two - tail : P_value $= 2P(Z > |Z_c|)$

* Reject H_0 if:-
  - P_value $< \alpha$

* Do not Reject :-
  - P_value $\geq \alpha$

* السؤال الي فوق

$Z = 2.42$ upper tail

$$\text{P-value} = P(Z > 2.42) = 1 - 0.9922 = 0.0078$$

$\alpha = 1\% \Rightarrow$ P_value $< \alpha \rightarrow$ reject

* You reject H_0 at $\alpha = 1\%$, what is your conclusion $\alpha = 5\%$, $\alpha = 10\%$

p_value $< 1\% < 5\% < 10\%$

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* Reject H_0 at $\alpha = 5\%$

10% $\Rightarrow$ Reject H_0

1% $\Rightarrow$?? لازم نجرب ونشوف

حسب

*Q(21) . page . 357 :-

$H_0 : \mu \leq 12$
$H_a : \mu > 12$ upper tail , $\mu_0 = 12$

a Sample of 25 provided a sample mean of 14 and S.D of 4.32 , $\alpha = 0.05$ (S)

→ σ - unknown Case ⇒ t

* By Critical Value(s)

- test Static $= t_c = \dfrac{\bar{x} - \mu_0}{S/\sqrt{n}} = \dfrac{14-12}{4.32}(5) = \boxed{2.31}$

- Rejection Rule:

Reject H_0 if $t_c > t_\alpha$

$t_{0.05} = 1.711$ $df = 25 - 1 = 24$

$t = 2.31 > t_\alpha = 1.711$
Reject H_0

* By p-Values: σ - unknown Case

Ex: Consider the following test

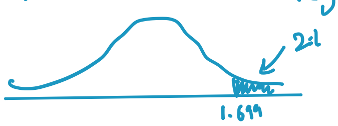
$H_0 : \mu \leq 100$ $\mu_0 = 100$
$H_a : \mu > 100$ upper tail

σ-unknown case ⇒ t-test

Sampl : $n = 65$, $\bar{x} = 103$, $S = 11.5$, $\alpha = 5\%$

(1) Test : $t = \dfrac{\bar{x} - \mu_0}{S}\sqrt{n} = \dfrac{103-100}{11.5}\sqrt{65} = 2.1$

(2) Critical Value : Reject H_0 if $t > t_\alpha$ ⇒ $t_{0.05} = 1.699$ $df = 64$

(3) 

$t = 1.699 < t = 2.1$ ⇒ Reject

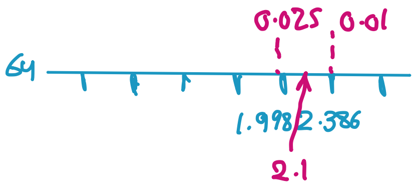
The test is significant

(4) P- Value??

→ Range of Values

$t_c = 2.1$ $df = 64$ from table

1.998 – 2.386 : بين قيمتين

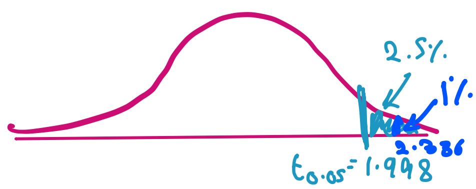


$1\% < p\text{-value} < 2.5\% < 5\%$ t_α

- Rejection Rule : Reject H_0

Because $t < t_\alpha$

if $\alpha = 1\%$ ⇒ Do not reject
$\alpha = 10\%$ ⇒ Reject



Question (1):

the population from which this sample was taken is significantly greater than 40.

36	50	36	55	55	45	38	42	53	65

a. Find the point estimate for the population mean.
b. Find the point estimate for the population standard deviation.
c. State the null and the alternative hypotheses.
d. Determine the test statistic.
e. Find the critical value(s).
f. Find the p-value.
g. Use 5% significance level to determine whether or not the mean of the population is significantly greater than 40.

- Values: 36, 50, 36, 55, 55, 45, 38, 42, 53, 65

- Significantly $\Rightarrow$ $H_a: \mu > 40$ σ– unknown case

(a) $\bar{x} = 47.5$ point estimate for μ

(b) $s = 9.69 = 9.7$ point estimator for σ

(c) $H_0: \mu \leq 40$

$H_a: \mu > 40 \rightarrow \mu_0$

(d) t-test

$$t_c = \frac{\bar{x} - \mu_0}{s}\sqrt{n} = \frac{47.5 - 40}{9.7}\sqrt{10} = 2.45$$

(e) Critical Value(s)

upper tail $\Rightarrow$ $t_{0.05} = 1.833$ $df = 9$

$t_c = 2.45 > t_\alpha = 1.833 \Rightarrow$ Reject H_0

(f) P-Value

$t_c = 2.45 \longrightarrow$ بين ال 2.262 و 2.821

$1\% < \text{p-value} < 2.5\% < 5\%$ 0.01   0.025 لـ مساحة

Reject H_0

(g) We accept Ha

The population mean is significantly greater than 40

* Question (2):

2. Your statistics instructor claimed that the average of STAT 2361 was at least 75. To test the instructor's claim, you as a student select sample of 30 students. The average grade in the sample was 73 with a standard deviation of 8.
   a. State the null and alternative hypotheses.
   b. Using the critical value approach, test the hypotheses at the 1% level of significance.
   c. Using the p-value approach, test the hypotheses at the 5% level of significance.

average $= 75$, $n = 30$, $\bar{x} = 73$, $\frac{s}{\sigma} = 8$

(a) $H_o: \mu \geq 75$ "Claim" σ-unknown case
$H_a: \mu < 75$

(b) Critical Value(s)

t-test $\Rightarrow t_c = \frac{\bar{x} - \mu_o}{s} \sqrt{n} = \frac{73 - 75}{8} \sqrt{30} = -1.37$

df = 29 $t_{0.01} = 2.462$

- Reject H_o if $t_c < -t_\alpha$

but $t_c > -t_\alpha$ so Do not Reject

(c) P-Value $\alpha = 5\%$

df = 29 $t_{0.05} = 1.699$ $t_c = |-1.37| \rightarrow$ from table

between 1.311 and 1.699

$t_\alpha = 5\% < \text{p-value} < 10\%$

Do not Reject

$\alpha = 10\% \Rightarrow$ Reject

*Question (4):

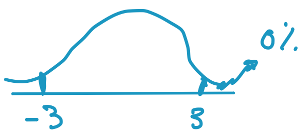
4. At a local university, a sample of 49 students was selected in order to determine whether the average age of the students is significantly different from 21. The average age of the students in the sample was 23. The population standard deviation of the ages is 3.5
   a. Formulate the hypotheses for this problem.
   b. Compute the test statistic.
   c. Determine the *p*-value and test these hypotheses. Let $\alpha = .05$.

$n = 49$, $H_a: \mu \neq 21$, $\bar{x}_{age} = 23$, $\underbrace{\sigma}_{\text{known case}} = 3.5$

(a) $H_0: \mu = 21$
$H_a: \mu \neq 21$

(b) Z-test:
$$Z_c = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{23 - 21}{3.5} \sqrt{49} = 4$$

(c) $P\text{-value} = P(Z > 4)^2$
$= 0$



$P\text{-value} < \alpha$
Reject H_0

unknown

9.5: Hypothesis testing about the population proportion p

1. H_0, H_a
2. Select α (level of significance)
3. test Statistic : $z = \dfrac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \dfrac{\bar{p} - p_0}{\sqrt{p_0(1-p_0)}} \cdot \sqrt{n}$
4. Rejection Rule
   - critical values
   - p-value
5. Conclusion
   - Reject H_0
   - Do not Reject H_0

9.5 Hypothesis testing about the population proportion p

unknown

Let p_0 be a given value for p in H_0. Let α be a given level of significant.

Critical Value(s) Approach

Hypothesis (1)	Statistic Test (3)	Rejection Rule (4), (5)	Critical value(s) $\alpha = 0.05$
1. $H_0: p = p_0$ $H_a: p \neq p_0$ (2 tailed)	$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$	Reject H_0 if $z > z_{\frac{\alpha}{2}}$ or $z < -z_{\frac{\alpha}{2}}$ ***two tailed test***	$z_{\frac{\alpha}{2}} = \pm 1.96$
2. $H_0: p \geq p_0$ $H_a: p < p_0$ (lower)	$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$	Reject H_0 if $z < -z_{\alpha}$ ***left tailed test*** (one tailed test)	$-z_{\alpha}$ =-1.645
3. $H_0: p \leq p_0$ $H_a: p > p_0$ (upper)	$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$	Reject H_0 if $z > z_{\alpha}$ ***right tailed test*** (one tailed)	z_{α} =1.645

p – value approach: Reject H_0 if p – value < α

EXAMPLE

35. Consider the following hypothesis test:

$$H_0: p = .20 \quad (p_0)$$
$$H_a: p \neq .20$$

2-tailed test

A sample of 400 (n) provided a sample proportion $\bar{p} = .175$.

a. Compute the value of the test statistic.
b. What is the p-value?
c. At $\alpha = .05$, what is your conclusion?
d. What is the rejection rule using the critical value? What is your conclusion?

a) $z = \dfrac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \dfrac{0.175 - 0.2}{\sqrt{(0.2)(0.8)}}\sqrt{400}$
$= -1.25$

b) $p\text{-value} = 2P(Z > |-1.25|)$
$= 2[1 - 0.8944) = (2)(0.1056)$
$= 0.21$

c) $p\text{-value} = 0.21 > \alpha = 0.05 \Rightarrow$ Do Not Rej.

(d) Critical values: Reject H_0 if $z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$

$\pm z_{\frac{\alpha}{2}} = \pm 1.96$

$-z_{\alpha/2} < z = -1.25 < z_{\alpha/2} \Rightarrow$ Do not Reject H_0

A Birzeit University professor claims that ~~75% or~~ more of students attend his on-line lectures. A student council is suspicious of the claim and thinks that the proportion is lower than 75%. A random sample of 120 students show that only 85 students have ever done so. Is there enough evidence to show that the true proportion is lower than 75%? Conduct the test at 1% significance level.

$H_0: p \geq 0.75$

$H_a: p < 0.75$

$n = 120$, $x = 85$

$\bar{p} = \frac{x}{n} = \frac{85}{120} = 0.71$

point estimate for p $\downarrow$ interval

$$\text{test} = \frac{0.71 - 0.75}{\sqrt{(0.75)(0.25)}} \cdot \sqrt{120} = -1.01$$

p-value: $P(z < -1.01) = P(z > 1.01)$

$= 1 - 0.8438$

$= 0.1562$

p-value $> \alpha \Rightarrow$ Do Not Reject H_0

Instructor claims is true.

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Chapter 8 & 9 problems

1. A simple random sample of 144 items resulted in a sample mean of 1257.85 and a standard deviation of 480. Develop a 95% confidence interval for the population mean.

$n = 144$, $\bar{x} = 1257.85$, $s = 480$

$n = 144$, $\bar{x} = 1257.85$, $S = 480$

$\bar{x} \pm t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}} \rightarrow 1257.85 \pm (1.96) \frac{(480)}{12}$

1257.85 ± 74.8

2. In a random sample of 400 BZU students, 240 students indicated that they are from Ramallah. Develop a 95% confidence interval estimate for the proportion of all BZU students who are from Ramallah.

$n = 400$, $x = 240$, 95% proportion

$\bar{p}$ = sample proportion $= \frac{x}{n} = \frac{240}{400} = 0.6$ → is a point estimate for p (proportion of All)

interval: $\bar{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

$: 0.6 \pm 1.96 \sqrt{\frac{(0.6)(0.4)}{400}} = 0.6 \pm 0.05 \rightarrow 0.55$ to 0.65

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3. The manager of a grocery store wants to determine what proportion of people who enter his store are his regular customers. What size sample should he take so that at 97% confidence the margin of error will not be more than 0.1?

$n = ?$, Error $= 0.1$, 97%

$\bar{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \rightarrow E$

$n = \frac{z^2}{E^2} \cdot p^*(1-p^*)$

$p^* = \frac{1}{2}$

$\alpha = 3\%$

$z_{1.5}$

$= \frac{(2.17)^2}{(0.1)^2} \cdot 0.5 \cdot 0.5$

$= 117.72 \rightarrow 118$

for the next integer

4. The manager of a grocery store has taken a random sample of 100 customers. The average length of time it took the customers in the sample to check out was 3.1 minutes with a standard deviation of 0.5 minutes. We want to test to determine whether or not the mean waiting time of all customers is significantly more than 3 minutes.

   a. State the null and the alternative hypothesis

   b. Find the test statistic is

   c. Find the *p*-value

   d. At 5% significance level, what is your conclusion?

$n = 100$, $\bar{x} = 3.1$, $s = 0.5$, $H_a: > 3$

(a) $H_0: \mu \leq 3$

$H_a: \mu > 3$ → upper tail test

(b) $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{3.1 - 3}{0.5} \; 10 = 2$

$df = 99$

(c) p-value t جدول : range of values

2 is between

$1.984 < 2 < 2.364$

$0.01 < \text{p-value} < 0.025$

(d) at $\alpha = 5\%$, Reject H_0 → $\text{p-value} < \alpha$

$\alpha = 1\%$? Do not Reject → $\text{p-value} > \alpha$

(f) Critical value : Reject H_0 if $t > t_{\frac{\alpha}{2}}$ $\quad df = 99 \Rightarrow t_{0.025} = 1.984$

$t = 2$

Reject H_0

So, mean waiting time is more than 3 (H_a)

5. A random sample of 100 people was taken. Eighty-five of the people in the sample favored Candidate A. We are interested in determining whether or not the proportion of the population in favor of Candidate A is significantly more than 80%.

   a. **State the null and the alternative hypothesis**
   b. **Find the test statistic is**
   c. **Find the *p*-value**
   d. **At 1% significance level, what is your conclusion?**

$n=100$, 85% of people favored Candiate A, $H_a: \bar{P} > 80\%$

$\bar{p} = 85$

a) $H_0: P_0 \leq 80\%$

$H_a: P_0 > 80\%$

b) $$z = \frac{\bar{p} - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} = \frac{0.85 - 0.80}{\sqrt{(0.8)(0.2)}} \cdot 10 = \boxed{1.25}$$

calculated

c) p-value: $p(z > 1.25) \rightarrow$ From table 0.8944

$: 1 - 0.8944 = 0.1056$

d) Conclusion: p-value $= 0.1656 > \alpha = 0.01$

Do not reject H_0

*Critical values: Reject H_0 if $z > z_\alpha$

$\alpha = 0.01 \Rightarrow z_{0.01} = 2.326 \Rightarrow$ Do not Reject

$\Rightarrow 2.326 > 1.25$